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Experimental Separation of Geometric and Dynamical Phases Using Neutron Interferometry

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We present results of the first experiment clearly demarcating geometric and dynamical phases. These two phases arise from two distinct physical operations, a rotation and a linear translation, respectively, performed on two identical spin flippers in a neutron interferometer. A reversal of the current in one flipper results in a pure geometric phase shift of π radians. This observation constitutes the first direct verification of Pauli anticommutation, implemented in neutron interferometry. [S0031-9007(96)02278-8]

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In general, a quantal system evolving under a time-dependent Hamiltonian $H(t)$ acquires, apart from the dynamical phase $-\text{Re}\int\langle H(t)\rangle dt/\hbar$, a nonintegrable and Hamiltonian-independent phase component called geometric phase, which depends only on the geometry of the curve traced in the ray space. Pancharatnam was the first to explicitly recognize geometric phase during his studies [1] of interference between optical beams of distinct polarizations. However, geometric phase attracted little further attention until Berry provided a general quantal framework [2] for geometric phase in the context of adiabatic evolutions, triggering an intense activity in this field. Geometric phase, already included in the standard formulation of quantum mechanics, can arise in any general evolution, be it nonadiabatic [3], noncyclic [4], or even nonunitary [4]. A completely general ray-space expression [5,6], in terms of just the pure state density operator, has been provided for geometric phase. Geometric phase has since been observed in a broad spectrum [7–15] of physical phenomena.

In the first quantal prescription [2] of geometric phase an adiabatic evolution was considered. The early neutron experiments [10,11] therefore observed geometric phase for an eigenstate of a slowly rotating magnetic field. However, an adiabatic evolution generates a dynamical phase background which is much larger than the geometric phase signal. An ideal geometric phase experiment should therefore effect *not* an adiabatic evolution, but a parallel transportation [16], an intrinsically *nonadiabatic* evolution, that eliminates dynamical phase and yields a pure geometric phase. In an evolution which does not parallel transport the state, it is still possible to generate a pure geometric phase by arranging for a null dynamical phase [16–18] at the end of the evolution.

The wave function of a spin $\frac{1}{2}$ particle changes sign [19,20] when the spin precesses through 2π radians. This 4π spinor symmetry has been directly verified in neutron interference experiments [21–23]. The spinor phase also depends on the *orientation* [17] of the precession axis. While elucidating this dependence, Wagh and Rakhecha proposed the first experiment effecting a clear separation [18] of geometric and dynamical phases. Here a $|z\rangle$-polarized neutron beam incident on an interferometer (Fig. 1) permeated by a uniform guide field $B_0\hat{\mathbf{z}}$ splits into subbeams I and II. The subbeams pass through identical spin flippers F_1 and F_2 which take the neutron state to

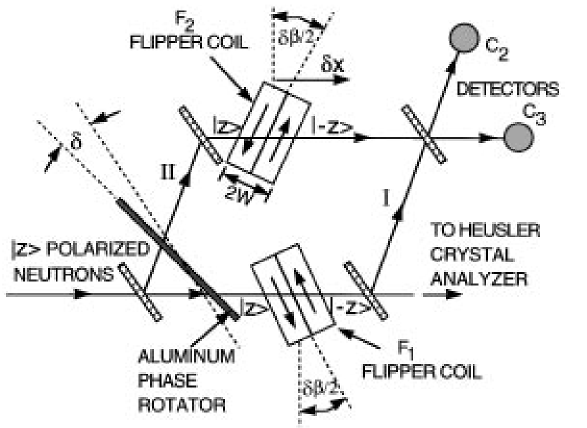

FIG. 1. Schematic diagram of the experiment demarcating geometric and dynamical phases. A uniform guide field $B_0\hat{\mathbf{z}}$, transverse to the plane of the diagram, is applied over the Si (220) skew symmetric LLL interferometer. A relative rotation $\delta\beta$ between the identical dual flippers F_1 and F_2 produces a pure geometric phase Φ_G, equal to $\delta\beta$, for the incident $|z\rangle$-polarized neutron beam; their relative translation δx results in a pure dynamical phase Φ_D, proportional to δx.

$|-z\rangle$. The relevant ray space for the spinor is the unit sphere of spin directions. For a relative rotation $\delta\beta$ between F_1 and F_2 about $\hat{\mathbf{z}}$, the closed spin trajectory traced during the evolution subtends a solid angle $\Omega = -2\delta\beta$ [cf. Fig. 2(b) in [18]] at the center of the spin sphere, yielding a pure geometric phase [18]

$$\Phi_G = -\Omega/2 = \delta\beta\,. \tag{1}$$

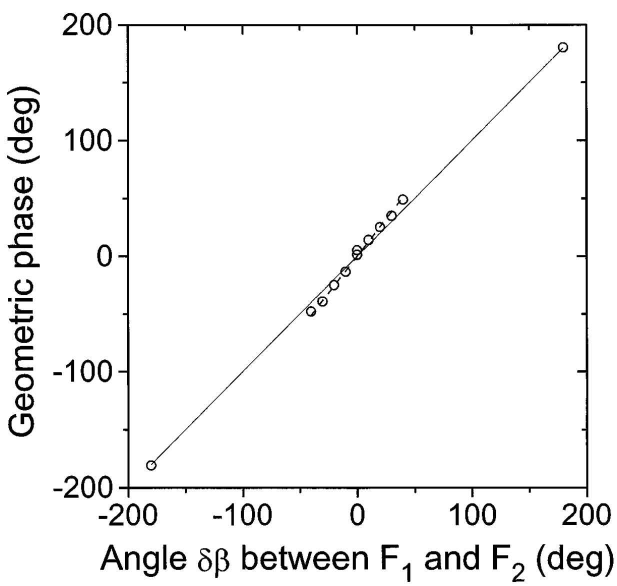

FIG. 2. Geometric phase Φ_G arising from the angle $\delta\beta$ between the flippers F_1 and F_2. Error bars are not shown since they are smaller than the size of points. The solid line is the theoretical prediction [Eq. (1)], and the dashed line is the fit to the data for angles $\delta\beta$ from $-40°$ to $+40°$ achieved by rotating the flippers mechanically. The second point at 0° is the phase measured with both flipper currents reversed.

This relation brings out the geometric nature of Φ_G, which is given by just the *angle* $\delta\beta$ [24] regardless of the Hamiltonian.

A linear translation δx of F_2, say, along the respective subbeam, on the other hand, changes the precession [18] about the guide field $B_0\hat{\mathbf{z}}$ by $\delta\phi$, say, in the $|z\rangle$ state and $-\delta\phi$ in the $|-z\rangle$ state. The translation thus leaves the spin trajectory and hence Φ_G unaltered, generating a pure dynamical phase shift [18,25]

$$\Phi_D = \delta\phi = 2|\mu|B_0\delta x/\hbar v\,. \tag{2}$$

Here μ and v denote the neutron's (negative) magnetic moment and speed, respectively. A translation of F_1 would produce an identical dynamical phase, but with the opposite sign. In contrast with the geometric phase [cf. (1)], the dynamical phase Φ_D depends on the field B_0 in the Hamiltonian.

In our experiment, each flipper was a dual flipper [17] consisting of two successive rectangular coils producing horizontal magnetic fields of magnitude B_0, in opposite directions. Along with the guide field $B_0\hat{\mathbf{z}}$, they produced net magnetic fields of magnitude $\sqrt{2}B_0$ along mutually orthogonal axes, $\hat{\mathbf{p}}$ and $\hat{\mathbf{q}}$, say, in a vertical plane, subtending angles $+\pi/4$ and $-\pi/4$, respectively, with $\hat{\mathbf{z}}$. The magnitude of these fields was set so that over the neutron path length through each coil, the spin precessed through an azimuthal angle π. The dual flipper thus effects two successive π precessions about axes $\hat{\mathbf{p}}$ and $\hat{\mathbf{q}}$. Its operation,

$$e^{-i\sigma_q\pi/2}e^{-i\sigma_p\pi/2} = (-i\sigma_q)(-i\sigma_p) = -\sigma_q\sigma_p\,, \tag{3}$$

brings the $|+z\rangle$ state to $|-z\rangle$. Here σ_p and σ_q represent the components of the Pauli spin operator $\vec{\sigma}$ along $\hat{\mathbf{p}}$ and $\hat{\mathbf{q}}$, respectively. On reversing the current in the two coils of a dual flipper, the neutron is subjected to a field along $\hat{\mathbf{q}}$ followed by a field along $\hat{\mathbf{p}}$, i.e., to the operation

$$e^{-i\sigma_p\pi/2}e^{-i\sigma_q\pi/2} = -\sigma_p\sigma_q = \sigma_q\sigma_p\,, \tag{4}$$

since σ_p and σ_q anticommute, being orthogonal components of the Pauli spin operator. Thus, the reversed flipper also takes $|+z\rangle$ to $|-z\rangle$, but with a change of sign as compared to the original operation (3). This *sign change manifests itself as a π phase shift* [6,17] and *can only be observed interferometrically*, as will be reported below; a polarimetric experiment is incapable [26] of detecting a current reversal in the flipper.

The current reversal in a dual flipper is equivalent to a π rotation [6,17] of the flipper about $\hat{\mathbf{z}}$. The spin trajectory for the reversed current, comprising two semicones of polar angles $\pi/4$ and $3\pi/4$ about $\hat{\mathbf{q}}$ and $\hat{\mathbf{p}}$, respectively, is thus obtainable from that for the forward current by a π rotation about $\hat{\mathbf{z}}$. These two trajectories enclose half the surface of the spin sphere, i.e., a solid angle of $\pm 2\pi$, resulting in a pure geometric [18] phase of $\mp\pi$, generated without physically rotating the flippers.

The experiment was carried out at the beam port C interferometry facility [27] at the 10 MW Research

Reactor of the University of Missouri (MURR) in a BARC-Vienna-MURR collaboration. A 2.35 Å neutron beam from a focusing pyrolitic graphite monochromator, was polarized vertically upwards, i.e., along $\hat{\mathbf{z}}$, by a reflection from a Fe-Si magnetic supermirror and passed through a 2 mm (wide) $\times$ 6 mm (high) slit to illuminate a skew symmetric (220) LLL silicon interferometer (Fig. 1). With a Heusler spin-state analyzing crystal downstream of the interferometer, the $|z\rangle$ fraction in this beam was measured to be 0.925, corresponding to a polarization $P = 0.85$. The interferometer was enclosed in an aluminum box (providing an isothermal enclosure) inside a heavy Benelex-70 box, which rests upon a 550 kg black granite slab, floated on four Firestone air cushions. Excluding the polarizing mirror, the entire setup is enclosed within a large Plexiglas box for general environmental isolation. With this isolation from the ambient mechanical vibrations and thermal variations, phase drifts typically less than 5° over a day were achieved.

A pair of water-cooled Helmholtz coils produced a fairly uniform guide field. The two rectangular coils of each dual flipper were connected in series and operated at 7 A dissipating about 2 W each. Each coil was made of a 25 mm wide and 90 μm thick anodized Al foil wound on an aluminum fork, which was firmly mounted onto a TeCu-145 heat sink block. A special low temperature Al-Cu brazing technique provided excellent electrical contacts between the coil ends and the Cu current leads. Two 1 mm thick copper sheets screwed on the front and back sides of the heat sink conducted the heat produced in the coils away from the interferometer. The flippers were suspended in the 40 mm spaces between the interferometer blades with a precision rotation/translation gadget specially constructed in the Missouri Physics Machine shop. Field mapping carried out with each flipper turned on and off, using a precision Hall probe within the interferometer, revealed that fields produced by the flippers in the relevant region exterior to them were well below 1 G. The temperature of the flipper heat sinks was maintained within ± 0.01 °C of the ambient air temperature with a controller operating through a closed-cycle water loop. Special precautions had to be taken to ensure that no vibrations were transmitted to the interferometer by the water flow.

The Heusler alloy analyzer crystal was used to ascertain a π spin flip in the dual flippers by adjusting the fields B_0 produced by the Helmholtz coils and the flipper F_1. The optimum B_0 was about 30 G, in agreement with the calculated field for 2.35 Å neutrons traversing a path length of about 7 mm through each coil (at normal incidence). Because of the space constraints within the interferometer, the maximum mechanical rotation of each flipper was limited to $\pm 22°$. Larger angles $\delta\beta$ were therefore achieved electrically. With the flippers normal to the respective subbeams, a reversal of current in F_1 (F_2) yields $\delta\beta = 180°$ ($-180°$). In each of the $\delta\beta$ settings, the two flippers were oriented symmetrically relative to the neutron subbeams, presenting nearly equal pathlengths and hence nuclear phases. Any residual difference between the nuclear phases acquired in traversing the two flipper materials was eliminated as described later (see also [26]).

The interferograms recorded in the He-3 detectors C_2 and C_3 were obtained by rotating a 1.05 mm thick Al phase flag in the interferometer (Fig. 1), which varies the nuclear (spin-independent) phase. The C_3 interference contrast of 64% for the empty interferometer reduced to about 32% on inserting the water-cooled dual flippers. This contrast dropped slightly to 28% when the currents in the flippers were switched on.

In each run, two interferograms, one with flippers on and the other with flippers off, were recorded simultaneously by periodically switching the flippers on and off after every 6×10^4 monitor counts, taking about 5 s, at each angular setting of the phase flag. Since this period is much shorter than the time constants for thermal variations and mechanical phase drifts, the phase shift *difference* between the on and off interferograms eliminates nuclear phase variations within the flippers and spurious phase drifts. To obtain the desired spin-dependent phase (1) or (2), corrections were applied to this phase difference for a small difference in the quantity $\int B_0\, dl$ over paths I and II, the incomplete polarization P of the incident beam and, when the flippers were not normal to the subbeams, for precessions $\pi/\cos(\delta\beta/2)$ about $\hat{\mathbf{p}}$ and $\hat{\mathbf{q}}$ instead of π. P corrections to the observed phase lay within $\pm 5°$. The correction for the excess spin flip ranged between 0° and 1°. The present analysis assumes identical operations in the two flippers and ignores the measured small difference ($\sim$1 G) between the guide fields at the two flippers.

Interferograms for geometric phase were recorded for 11 fixed angles $\delta\beta$ between F_1 and F_2 held at fixed positions. The resultant pure geometric phases Φ_G are shown in Fig. 2. The data points lie close to the

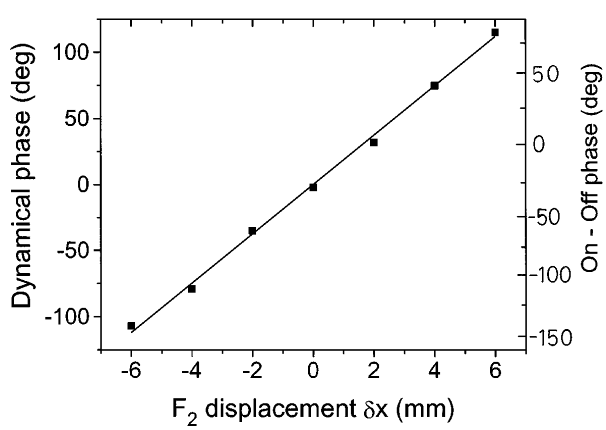

FIG. 3. Pure dynamical phase Φ_D as a function of the translation δx of the flipper F_2, obtained by correcting (see text) the observed phase shift between the scans recorded with the flippers on and off. Error bars are smaller than the size of points.

theoretical (solid) line [Eq. (1)]. The phases for $\delta\beta$ values between $-40°$ and $+40°$, obtained by rotating the flippers mechanically, fall on the dashed line which has a slope of 1.23 ± 0.03. The discrepancy of this slope with the theoretically expected slope of 1 may have arisen due to a possible dynamical phase contamination. With our flipper mounting gadget, it was not possible to position the rotation axis of each flipper accurately on the centerline of the respective subbeam. A rotation of such an off-center flipper would be accompanied by a displacement δx and a consequent dynamical phase [cf. (2)]. An offset of about 1 mm only of one flipper axis can generate a Φ_D contamination accounting for the observed deviation. The phases for $\delta\beta = -180$, 180, and 360 (shown as the second point at 0) degrees which are free from such a contamination, since they are measured by reversing the current first in one, then in the other, and then in both flippers, while both flippers remain normal to the subbeams, agree to within 2% with theory.

The flippers were then made normal to the respective subbeams ($\delta\beta = 0$) and interference patterns recorded as a function of the linear translation of first F_2 and then F_1. Figure 3 displays the consequent variation of the pure dynamical phase obtained by translating F_2. The slope 18.7 ± 0.2 deg/mm of the best fit corresponds to [cf. Eq. (2)] a guide field of 29.9 ± 0.3 G which agrees with its measured value of 29.9 ± 0.1 G in the vicinity of F_2. The translation of F_1 yielded a straight line for Φ_D with a negative slope of similar magnitude, as predicted.

Figure 4 depicts the interference patterns recorded with the current in the flipper F_1 switched between the forward (F) and reverse (R) directions. On reversing the current, the pattern just gets reflected about the line representing its average, as expected. The observed pure geometric phase shift (cf. Fig. 2) equals $180° \pm 3°$, confirming the anticommutivity [(3)–(4)] between σ_p and σ_q. If the current in F_2 is also reversed, the interferogram shifts further by 180° becoming identical to the initial interferogram. This π phase shift observed with a mere flick of a switch reversing the flipper current constitutes the first direct verification of Pauli anticommutation, accomplished here interferometrically with neutrons.

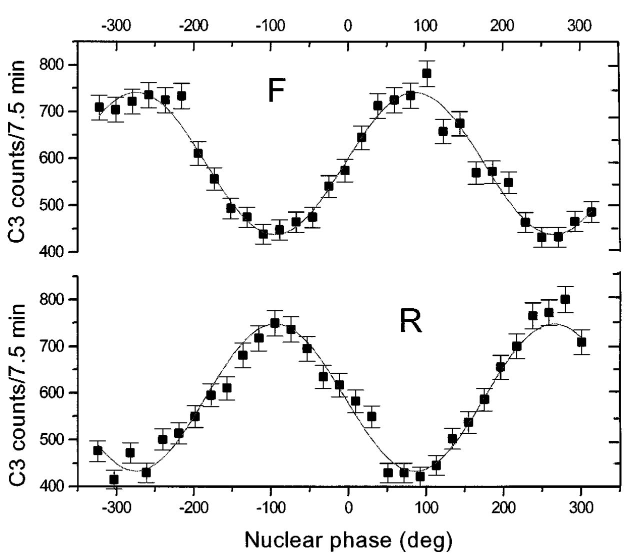

FIG. 4. The interferogram has shifted by $180.5° \pm 3.0°$ on switching the current in F_1 from the forward (F) to the reverse (R) sense. This result verifies Pauli anticommutation to within about 2%.

In conclusion, we have observed the spinor phase dependence on the orientation of the precession axis in a polarized neutron interferometric experiment. This is the first experiment effecting a clean separation of geometric and dynamical phases. Here, a relative rotation of two π flippers gives rise to a pure geometric phase; their relative translation produces a pure dynamical phase. A reversal of current in a flipper, equivalent to a π rotation of the flipper, generates a pure geometric phase of π, and its observation has confirmed the anticommutivity of orthogonal components of the Pauli spin operator.

The success of this work depended on the skilled workmanship of Clifford Holmes and his staff at the physics machine shop at the University of Missouri-Columbia. This research was carried out under the Austrian Fonds (FWF) Project No. P9266-PHY, the U.S. NSF-Physics Grant No. 9024608 and a UM Research Board Grant No. RB-94-003. One of us (A. G. W.) wishes to acknowledge the support for two months from the Austrian Fonds and for one month from the University of Missouri-Columbia during the experiment.

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