Physical Regimes and Dimensional Structure of Rotating Turbulence

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Numerical simulations of rotating turbulence have given rise to "unexpected results": An increasing Ω did not lead to the "expected" route to a 2D state. A recent model of turbulence leads to a new number $N = K(\nu\Omega)^{-1}$ (K and ν are turbulent kinetic energy and viscosity) so that DNS (direct numerical simulation) and LES (large eddy simulation) correspond to $N < 1$ and $N > 1$. In the first case, the energy cascade is suppressed, while in the second case there exists an inertial spectrum which is an equilibrium of quasi–2D-3D modes. With these ingredients, we reproduce DNS and LES data. [S0031-9007(96)02204-1]

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Recent numerical simulations data on rotating turbulence are difficult to interpret if one adopts the Taylor-Proudman theorem; to wit, as Ω increases, rotating turbulence should tend to a 2D state with L_v(vertical) $\gg L_h$(horizontal). DNS (direct numerical simulation) and LES (large eddy simulation) results do not confirm such expectations. *First*, early DNS [1–3] and experiments [4] confirmed the trend toward 2D but further DNS work [5] with larger Ω yielded the opposite result: L_v/L_h first grows with Ω but then decreases returning toward a 3D state. *Second*, using LES, the tendency toward 2D was seen, and it was thought that lateral ($L_{11,3}$) and longitudinal ($L_{33,3}$) vertical length scales would *both* be larger than L_h. It was, however, found [6,7] that $L_{11,3} \gg L_h$, but $L_{33,3} \sim L_h$. It was stated that [7] "the decoupling was unexpected especially considering the strong coupling between vertical and horizontal fluctuations," and that [6] "most striking is the large growth rate in $L_{11,3}$ which attains values between 5–10 larger than $L_{33,3}$."

We show that both DNS-LES results can be reproduced and understood on the basis of a new hierarchy of regimes which we construct using a recent model previously tested on a variety of other data [8,9]. For large Ω, we show that there exist two quite different regimes separated by the new number $N = K/\nu\Omega$ (K is the turbulent kinetic energy and ν is the viscosity). For $N < 1$, strong rotation suppresses the energy cascade altogether. No inertial regime, defined by the constancy of the energy flux, develops. In a freely decaying case, viscosity remains the only operating mechanism, and, in the absence of energy transfer, an initially isotropic 3D turbulence remains thus and never tends towards a 2D state. This explains the DNS data. For $N > 1$, the energy cascade is restored and the flow consists of mutually interacting 2D and 3D states. We further show that $L_{33,3}$ belongs to a 3D state (where all lengths are of the same order), while $L_{11,3}$ belongs to a 2D state and, thus, $L_{11,3} \gg L_{33,3} \sim L_h$. This explains the different behavior vs Ω found in LES [6,7]. We also compute the power law exponents for energy and length scales for freely decaying rotating turbulence and show that the results reproduce LES data [7,10]. The nonlinear interactions, though weakened, are the main cause of 2D state which is not of the Proudman-Taylor type since the latter requires negligible nonlinearities.

Stationary, homogeneous, rotating turbulence.—Before applying the model of [8,9] to rotating turbulence, it is necessary to give a brief sketch of its physical content. Generally speaking, the interaction of an eddy of wave number k is contributed by two processes: the interaction with all the smaller eddies with wave numbers larger than k (the ultraviolet part, UV) and with the larger eddies with wave numbers smaller than k (infrared part, IR). As Wyld [11] showed long ago, both parts can be described by an infinite set of Feynman diagrams. The UV part is divergent but renormalizable, and thus the total sum can be obtained, for example, using RNG (renormalization group) techniques. The result, expressed in terms of a dynamical (turbulent) viscosity, is given by Eq. (4). The IR presents a major problem since it has been known for many years [12] that it diverges and, contrary to the UV part, the divergence is not renormalizable. One approach is to truncate the infinite series and retain only the first diagram, the so-called one-loop approximation. Wyld [11] showed that this gives rise to the DIA (direct-interaction approximation) model [and by inference the EDQNM (eddy-damped quasinormal Markovian) model]. Thus, while DIA contains no adjustable parameters, it neglects a large set of diagrams. For the problems encountered in extending DIA to anisotropic, inhomogeneous flows, see [13].

The model presented in [8,9] employs the RNG technique to compute the UV part but departs substantially from the one-loop, DIA model. Since the series of IR diagrams cannot be summed, a physical model was suggested based on the assumption that the nonlinear transfer of energy is mostly a local process. This assumption alone (made first by Kolmogorov) allows one to derive a closed set of equations for $E(k)$ and the Reynolds stresses (the contribution of nonlocality, the so-called backscatter, was also computed and included in all calculations). Since the assumption of locality is an heuristic one, the reliability of the model results can only be assessed on the

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basis of its performance on a wide variety of flows. The model was thus tested against more than fifty turbulence statistics, including among others, shear, plane, axisymmetric, and high Rayleigh number convection [8,9]. The equations relevant to the present case are

$$\frac{\partial}{\partial t}E(k) = T(k) - 2\nu k^2 E(k) + A_s(k), \tag{1}$$

where the last term represents the work done by the stirring forces and $T(k)$ is the transfer,

$$T(k) = -\frac{\partial \Pi(k)}{\partial k}, \tag{2a}$$

$$\Pi(k) = E(k) r\big(k, E(k)\big). \tag{2b}$$

In analogy with $j = \rho v$, (2b) represents the energy flux $\Pi(k)$ (the analog of j) in terms of a rapidity r (the analog of v), and of the energy spectrum $E(k)$ (the analog of ρ). The rapidity $r\big(k, E(k)\big)$ is a highly nonlinear function of $E(k)$ since

$$r(k) \equiv 2\int_0^k p^2 \nu_t(p)\,dp, \tag{3}$$

$$\nu_t(k) = \nu_d(k) - \nu, \quad \nu_d(k) = \left(\nu^2 + \tfrac{2}{5}\int_k^\infty p^{-2}E(p)\,dp\right)^{1/2}. \tag{4}$$

Here, ν_t is the turbulent viscosity. How can one include rotation? There is ample evidence, DNS-LES simulations [1,6,7,10], experimental work [14,15], and closure models [16] that the energy flux $\Pi(k)$ is inhibited by rotation. Using the helical formalism of [17], we derive the following energy flux

$$\Pi_\Omega(k,t) = -\int \Sigma_\alpha C(\alpha_n|\mathbf{p}_n) \times \langle b^*_{\alpha_1}(\mathbf{p}_1) b^*_{\alpha_2}(\mathbf{p}_2) b_{\alpha_3}(\mathbf{p}_3)\rangle e^{-i\omega t}\theta(k - p_1) \times \Pi_{n=1}^{n=3} d\mathbf{p}_n + \text{c.c.}$$

Here, $\omega = \mathbf{\Omega}\cdot(\mathbf{p}_1/p_1 + \mathbf{p}_2/p_2 - \mathbf{p}_3/p_3)$, θ is the Heaviside function, the b's and the C's represent the velocity field and the structure functions in the helical representation [17], and $\alpha_n = \pm 1$. Instead of ensemble averaging in, we average over the lifetime of coherent triads, that is, $\Pi(k) \rightarrow \tau^{-1}\int_0^\tau dt\,\Pi(k,t)$. Because of the rapidly oscillating factor $\exp(-i\omega t)$, the main contribution is for $t \sim \Omega^{-1}$. The only physically acceptable candidate for τ is $(k^2\nu_t)^{-1}$ and, thus,

$$\Pi_\Omega(k) = \frac{k^2}{\Omega}\nu_t(k)\Pi(k). \tag{5}$$

An analogous expression holds for $r_\Omega(k)$. Next, we consider a steady state and assume that the stirring forcing is concentrated in the low k region. In the case of a high Reynolds number flow ($\nu \rightarrow 0$), there is an extended inertial region in which $\Pi(k) = \epsilon$. Using Eqs. (1), (2b), (4), and (5), we obtain

$$E(k) = \left(\frac{45}{8}\right)^{1/2}(\epsilon\Omega)^{1/2}k^{-2}, \tag{6a}$$

$$r_\Omega(k) = \left(\frac{8}{45}\right)^{1/2}\left(\frac{\epsilon}{\Omega}\right)^{1/2}k^2. \tag{6b}$$

The spectrum $E(k) \sim k^{-2}$ has been obtained phenomenologically in [18] and in [19] via the solution of the model [20]. In the presence of Ω, the turbulent viscosity $\nu_\Omega(k)$ is given by

$$\nu_\Omega(k) = \frac{1}{2k^2}\frac{\partial}{\partial k}r_\Omega(k) = \left(\frac{8}{45}\right)^{1/2}\left(\frac{\epsilon}{\Omega}\right)^{1/2}k^{-1} = \frac{K^2}{\epsilon}g, \quad g \equiv \frac{8}{45}\frac{\epsilon}{K\Omega}. \tag{7}$$

The last equality corresponds to $k = k_0 \sim L$. The condition that the energy flux is inhibited by rotation, $\Pi_\Omega < \Pi$, is equivalent, to using Eqs. (4)–(6),

$$k < k_\Omega, \quad k_\Omega \equiv 2\epsilon^{-1/2}\Omega^{3/2}. \tag{8}$$

Thus, we envisage a spectrum that in the interval $k_0 < k < k_\Omega$ is given by (6a), while for $k_\Omega < k < k_d$ is given by Kolmogorov. Here, k_d has the usual expression $k_d \equiv (\epsilon\nu^{-3})^{1/4}$. However, since k_Ω increases with Ω, it may become larger than k_d. In that case, Kolmogorov no longer attains, and one goes directly from (6a) into a dissipation region which begins at a wave number k_d^* defined by the condition $\nu = \nu_\Omega(k_d^*)$ which gives

$$k_d^* \equiv 2(\epsilon\Omega^{-1})^{1/2}\nu^{-1}. \tag{9}$$

The condition for a Kolmogorov spectrum to exist, $k_\Omega < k_d^*$, translates into

$$\frac{\Omega^2\nu}{\epsilon} < \frac{1}{4}, \quad \text{Ro}^\omega > 1, \tag{10}$$

where $\text{Ro}^\omega \equiv \omega/2\Omega$ is the Rossby micronumber, $\omega \equiv (\epsilon/\nu)^{1/2}$. The condition for the spectrum (6a) to exist is instead (L is the size of the system)

$$k_\Omega > k_0 \sim L^{-1}, \quad \text{Ro}^L < 1, \tag{11}$$

where $\text{Ro}^L \equiv K^{1/2}/\Omega L$ and from (6a), $K \sim (\epsilon\Omega)^{1/2}L$. In the case of strong rotation, the new dissipation wave number k_d^* can become even smaller than $k_0 \sim L^{-1}$. This will happen when

$$N \equiv \frac{K}{\nu\Omega} < 1. \tag{12}$$

When this occurs, there is no inertial region where $\Pi(k) = \epsilon$. The spectrum is no longer universal, and it depends on the specific type of external forcing. We suggest N as a new number to characterize rotating turbulence. We summarize the results as follows: If $\text{Ro}^L > 1$, weak rotation does not affect turbulence. If $\text{Ro}^L < 1$, rotation affects turbulence, and we must distinguish two regimes: $N > 1$, an inertial regime sets in. The specific form of $E(k)$ depends on whether condition (10) is satisfied or not. If it is, we have two inertial branches $E \sim k^{-2}$ in the $k_0 < k < k_\Omega$ interval and Kolmogorov in the $k_\Omega < k < k_d$ region. If not, we have only one inertial regime, the $E \sim k^{-2}$ spectrum from $k_0 < k < k_d^*$. In the latter, 2D and 3D modes are in equilibrium (see below). $N < 1$, no inertial regime sets in since strong rotation inhibits the transfer of energy. An initially isotropic flow remains thus and only viscous and external forces operate. This resolves the paradox in [5] (the conjecture of a 2D state as Ω increases), since as Ω increases, N decreases and the 3D → 2D gets suppressed.

$N > 1$: quasi-2D, 3D equilibrium.—The Coriolis force, $2\mathbf{\Omega_k} \times \mathbf{u}(\mathbf{k})$, $\mathbf{\Omega_k} = k^{-2}(\mathbf{\Omega} \cdot \mathbf{k})\mathbf{k}$, affecting an eddy $\mathbf{u}(\mathbf{k})$, vanishes if $\mathbf{k} \perp \mathbf{\Omega}$. This implies that eddies with $k_h \gg k_z$ form a quasi-2 mode which is weakly affected by rotation. Since in 3D the energy cascade is mostly forward, while it is mostly backward in 2D, the energy flux from 3D→2D occurs mostly at large k's, whereas the 2D→3D transition occurs mostly at low k's where the inhibition of energy transfer is the largest. Thus, if the initial energy densities $e(\text{2D}) = e(\text{3D})$, flux(3D → 2D) > flux(2D → 3D), which leads to a flow of energy from 3D to 2D until equilibrium is reached in which $e(\text{2D}) \gg e(\text{3D})$. As Ω increases, so does e(2D), while there is a corresponding decrease of the volume of the mode whose boundary is defined by the condition $\Omega_\mathbf{k} \sim k^2 \nu_\Omega(k)$. The use of $\Omega_\mathbf{k}$ and the second equality (7) yields the boundary value k_z^* that separates 2D and 3D modes,

$$k_z^* \sim \Omega^{-1}(\epsilon\Omega^{-1})^{1/2}k^2. \tag{13}$$

As a result of the two opposing tendencies, the total 2D energy can either increase, decrease with Ω, or tend to a fixed asymptotic value. LES data [7,10] exhibit a symmetry of the Reynolds stress tensor for quite a long decay time during which the Rossby number decreases quite significantly . We can thus assume that the ratio of the energies in the 2D-3D modes tends to a finite value as $\Omega \to \infty$. In addition, local interactions among eddies of either mode limit the value of the ratio $E(\text{2D})/E(\text{3D})$ at each k so as to prevent an infinite energy flux from one mode to the other. This, in turn, can be viewed as an indication that the two energy spectra must have similar shapes.

Decaying turbulence.—The LES results [6,7,10,21] correspond to the strong rotation limit, $N > 1$. We consider an initial small k behavior of $E(k)$ of the form $E(k, t = 0) = B_{s\to 2}k^s$, where $s = 2, 4$ [22]. Carrying out a similarity analysis similar to the one in [22], but with Eq. (6a), we obtain

$$K(t) = B_0^{2/5}\Omega^{3/5}t^{-3/5}, \quad K(t) = B_2^{2/7}\Omega^{5/7}t^{-5/7}, \tag{14}$$

for $s = 2, 4$ respectively. The results agree with LES data [7,10]. Furthermore, k_0, the value at which the initial spectrum and (6a) coincide, is

$$k_0(t) = (B_0 t\Omega^{-1})^{-1/5}, \quad k_0(t) = (B_2 t\Omega^{-1})^{-1/7}, \tag{15}$$

for $s = 2, 4$; these functions are used in evaluating the length scales

$$L_{\alpha\alpha,\beta}(t) = \langle u_\alpha^2\rangle^{-1}\int \langle u_\alpha(\mathbf{r})u_\alpha(\mathbf{r} + s\mathbf{e}_\beta)\rangle ds \tag{16}$$

(no summation over α); $\mathbf{e}_\beta$ is the unit vector along the X_β axis and $\mathbf{r}$ is the radius vector. Since turbulence is axially symmetric (with respect to the x_3 axis, $\mathbf{\Omega} = 0, 0, \Omega$), there are three horizontal $L_{\alpha\alpha,1} (\alpha = 1, 2, 3)$ and two vertical independent scales, $L_{11,3}$ and $L_{33,3}$. In a 3D mode, all five length scales are of the same order of magnitude k_0^{-1}

$$L_{\alpha\alpha,1} \sim L_{33,3} \sim k_0^{-1} \sim t^{1/5}, t^{1/7} \tag{17}$$

depending on $s = 2, 4$. The results are in agreement with LES data [7]. In a 2D mode, there is an horizontal $L_{11,1} \sim k_0^{-1}$ and a vertical length scale, $L_{11,3}$, defined by the 2D/3D boundary value (13) of k_z^* at $k = k_0$

$$L_{11,3} \sim (k_{0z}^*)^{-1} = k_0^{-2}(\Omega^3\epsilon^{-1})^{1/2}. \tag{18}$$

Using Eqs. (15), we obtain (for $s = 2, 4$)

$$L_{11,3} \sim (B_0\Omega^4 t^6)^{1/5}, \quad L_{11,3} \sim (B_2\Omega^6 t^8)^{1/7}, \tag{19}$$

which grow much faster than $L_{33,3}$. The results agree with LES data [7] and explain the decoupling of the vertical length scales $L_{33,3}$ and $L_{11,3}$ that surprised the authors of [6,7].

In conclusion, the main feature of rotating turbulence is the inhibition of the energy transfer [1]. The authors of [7,10] suggested that this causes the dominant time scale of triple correlations τ to be $\sim\Omega^{-1}$. Substitution in the relation $\epsilon \sim \tau[k^2E(k)]^2$ suggested in [23] yields $E(k) \sim \epsilon^{1/2}\Omega^{1/2}k^{-2}$ [18]. The turbulence model [8,9], which was previously tested on several turbulent statistics, shows that such a spectrum exists only in the $N > 1$ case. The number N differentiates two very different regimes: a total suppression of the energy transfer (with a viscosity dominated decay) and a weakened energy cascade. The first regime has $N < 1$; the second is $N > 1$. *DNS data correspond to $N < 1$, while LES data correspond to $N > 1$.* This allows us to explain the seemingly paradoxical DNS results [5], whereby as Ω increases, turbulence exhibits 3D, rather than 2D, features. Since DNS corresponds to $N < 1$, the energy transfer is strongly suppressed as is the 3D→2D transition. With $E(k)$ and similarity analysis, we show that the time dependence of both kinetic energy and length scales reproduce the power law exponents obtained by LES.

A major new feature is that in the $N > 1$ case, there exists an equilibrium between 3D and quasi-2D states separated by k_z^*, Eq. (13). The existence of the latter, though arrived at heuristically, explains a further "unexpected" LES result, the very different time behavior of the two vertical length scales: one belongs to 2D, the other to 3D.

When this work was completed, we received a manuscript [24] in which a similar (but not identical), hierarchy of rapidly rotating turbulent states was considered for the freely decaying case. The work is based on the numerical solution of the EDQNM-2 model [25]. In [24], the condition for a 2D state, $\mathrm{Ro}^\omega > 1$ was proposed which differs from our condition $N > 1$. In fact, for the spectrum (6a), $N \sim (\mathrm{Ro}^\omega/\mathrm{Ro}^L)^2$ so that $N > 1$ implies $\mathrm{Ro}^\omega > \mathrm{Ro}^L$, which is much weaker than the $\mathrm{Ro}^\omega > 1$ condition of [24].

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