Theory of Incoherent Self-Focusing in Biased Photorefractive Media

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(Received 30 September 1996)

A theory describing propagation and self-focusing of partially spatially incoherent light beams in nonlinear media is developed. It is shown that this process is effectively governed by an infinite set of coupled nonlinear Schrödinger-like equations, provided they are initially appropriately weighted with respect to the incoherent angular power spectrum of the source. The particular case of spatially incoherent beam propagation in biased photorefractive media is considered in detail. Numerical simulations indicate that spatial compression as well as self-trapped states are possible under appropriate conditions. Our results are in good agreement with recent experimental observations. [S0031-9007(96)02222-3]

PACS numbers: 42.65.Jx, 42.65.Hw

Optical self-focusing has been a subject of considerable interest in the last three decades or so [1–10]. Over the years this process has been systematically investigated in all states of matter with the aid of laser sources [3]. Thus far several physical systems have been identified that can lead to optical self-trapping. These include, for example, $\chi^{(3)}$ or Kerr-like media [4,5], the $\chi^{(2)}$ family of materials [6,7], as well as the class of photorefractives [8–10]. Highly relevant to this topic is of course the very existence of optical spatial solitons [11]. These latter entities can occur provided that diffraction effects are exactly balanced by optical self-trapping. At this point it may be fair to say that both coherent self-focusing as well as the coherent excitation of solitons are by now in principle well understood. In a very recent study, however, a successful observation of incoherent self-trapping has been reported for the first time [12]. In this experiment, partially spatially incoherent light was found to self-trap in a biased strontium barium niobate (SBN) photorefractive crystal. Apart from its possible scientific and technological implications, this observation [12] in turn poses a new fundamental challenge. More specifically, a theory of incoherent self-focusing and possibly of incoherent solitons now needs to be developed. Unlike their coherent beam counterparts, for which the phase at all points varies in unison with time, the phases at different points across an incoherent beam vary in an uncorrelated manner [13,14]. This introduces an important new element in the nonlinear theory of self-focusing. Even though the properties of speckle-inhomogeneous fields have been considered in the past in connection with optical phase conjugation [15], the propagation behavior of partially incoherent (multimode) beams in nonlinear media has not yet been explored.

In this Letter, a theory of incoherent self-focusing in noninstantaneous nonlinear media, and in particular in biased photorefractive crystals, is presented. We consider nonlinear media with response times much longer than the characteristic phase fluctuation time across the optical beam, which in turn will experience only the time-averaged intensity. In the case of stationary spatial source fluctuations, we show that this process can be effectively described by means of an infinite set of coupled nonlinear Schrödinger-like equations provided that they are appropriately weighted with respect to the source incoherent angular power spectrum. Our simulations demonstrate that spatial compression as well as self-trapped states are possible under appropriate conditions. In these cases, self-trapping can be intuitively understood as fusion of multiparticles. Pertinent examples are provided to further elucidate this behavior. A possible soliton solution to this system is also presented.

To start, let us consider, for example, a SBN crystal with its optical c axis oriented in the x direction. Let us also assume that the optical beam propagates along the z axis and it is allowed to diffract only along the x direction. For simplicity, we limit our analysis to one transverse dimension (x) and we assume uniformity in y. Furthermore, an external bias electric field is applied along x (i.e., the c axis), in which case the perturbed extraordinary refractive index is given by [16,17], $(n_e')^2 = n_e^2 - n_e^4 r_{33} E_{\rm sc}$, where n_e is the unperturbed index of refraction, r_{33} is the electrooptic coefficient involved, and $E_{\rm sc}$ is the static space charge field in this photorefractive crystal. Under strong bias conditions (0.2–4 kV/cm) and for relatively broad brightlike beam configurations ($\gtrsim 25\lambda_0/n_e$), the steady-state space charge field is approximately given by [16,17] $E_{\rm sc} = E_0 I_d [I_d + I(x,z)]^{-1}$ where $I = I(x,z)$ is the power density of the optical beam, I_d is the so-called dark irradiance of the crystal, and E_0 is the value of the space charge field at $x \to \pm\infty$. If the spatial extent of the

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optical wave involved is much less than the x width W of the crystal, then, for a constant bias voltage V, E_0 is approximately given by $\pm V/W$ [17].

Next we consider the diffraction behavior of this incoherent beam. Let us assume, as in the experiment reported [12], that the incoherent wave front results from a quasithermal quasimonochromatic source such as, for example, laser light after passing through a rotating diffuser [12,14,18]. Let the $\hat{x}$ polarized optical wave be expressed in terms of a slowly varying electric field envelope $\phi(x,z)$, i.e., $\vec{E}(x,z) = \hat{x}\phi(x,z)\exp(ikz)$ where $k = (2\pi/\lambda_0)n_e$. In the paraxial limit ($k_x/k \ll 1$) and in the linear regime, the envelope ϕ evolves according to

$$\phi(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk_x\, \hat{\Phi}(k_x)\exp\{i[k_x x - (k_x^2/2k)z]\}, \quad (1)$$

where $\hat{\Phi}(k_x)$ is the Fourier transform of the field right at the input, i.e., at $z = 0$. Let the optical field at the origin also be written as $\phi(x, z = 0) = f(x)\phi_0(x)$, where $f(x)$ is a spatial modulation function and $\phi_0(x)$ is the field before modulation which implicitly contains all the spatial statistical properties of the source. If the source fluctuations obey a stationary random process [19], then the spatial statistical autocorrelation function of $\phi_0(x)$ is given by $\langle\phi_0(x)\phi_0^*(x')\rangle = R(x - x')$. In turn, the autocorrelation function of the source spectrum can be obtained, i.e., $\langle\hat{\Phi}_0(k_x)\hat{\Phi}_0^*(k_x')\rangle = 2\pi\delta(k_x - k_x')G(k_x')$ where $\hat{\Phi}_0(k_x)$ and $G(k_x)$ are the Fourier transforms of $\phi_0(x)$ and $R(x)$, respectively, and $\delta(x)$ is a delta function. Physically, the real function [19] $G(k_x)$ represents the incoherent angular power spectrum of the source. Using the frequency convolution theorem and the fact that $\hat{\Phi}(k_x)$ is the Fourier spectrum of the product $f(x)\phi_0(x)$, one can readily obtain

$$\langle\hat{\Phi}(k_x)\hat{\Phi}^*(k_x')\rangle = (1/2\pi) \times \int_{-\infty}^{\infty} d\eta\, G(\eta)F(k_x - \eta)F^*(k_x' - \eta), \quad (2)$$

where $F(k_x)$ is the Fourier transform of the spatial modulation function $f(x)$. Keeping in mind that the intensity of this wave is given by $I(x,z) = \langle|\phi|^2\rangle$ and by employing Eqs. (1) and (2) we finally obtain

$$I(x,z) = \int_{-\infty}^{\infty} d\theta\, G_N(\theta) \times \left|\frac{1}{2\pi}\int_{-\infty}^{\infty} dk_x\, F(k_x)\exp[ik_x(x - \theta z)] \times \exp[-ik_x^2(z/2k)]\right|^2, \quad (3)$$

where $G_N(\theta)$ in Eq. (3) has been normalized for convenience and $\theta = k_x/k$ represents an angle (in radians) with respect to the z axis. At this point it may be useful to make a few remarks. First, the quantity in the brackets of Eq. (3) represents in fact the intensity profile resulting from an otherwise coherent beam when its initial field profile is $f(x)$. Furthermore, this "coherent component" propagates at an angle θ with respect to the z axis by obeying the paraxial equation of diffraction: $i(U_z + \theta U_x) + (1/2k)U_{xx} = 0$ where $U_z = \partial U/\partial z$, etc. In essence, Eq. (3) leads to the following important conclusion: The diffraction behavior of an incoherent beam can be effectively described by the sum of the intensity contributions from all its coherent components provided that their field profiles at the origin have been appropriately scaled with respect to the incoherent angular power spectrum $G_N(\theta)$, i.e., $U(x, z = 0) = G^{1/2}(\theta)f(x)$. As one may anticipate, in the limit $G_N(\theta) \to \delta(\theta)$, the result of Eq. (3) reduces to that of the coherent case. Note that similar arguments have been previously employed in connection with incoherent imaging [20], theory of speckle-inhomogeneous fields [15], and incoherent wave propagation in dispersive media [21].

Up to this point our treatment is quite general. On the other hand, when an incoherent optical beam propagates in a slowly responding nonlinear medium, one should also expect that each of these coherent components or quasiparticles will be influenced by the nonlinearity involved. In turn, an intensity-dependent nonlinearity will follow the incoherent (intensity) superposition of all these components. In the particular case of a biased photorefractive crystal, the nonlinear index change $\Delta n = (n_e^3/2)r_{33}E_{sc}$ can then be readily incorporated into the underlying equations of motion by following the procedure of [16,17]. By doing so and by discretizing the diffraction integral of Eq. (3) (i.e., $\theta \to j\Delta\theta$), then under steady-state conditions we are finally led to the following infinite set of coupled nonlinear Schrödinger-like equations:

$$i\left\{\frac{\partial U_j}{\partial z} + (j\Delta\theta)\frac{\partial U_j}{\partial x}\right\} + \frac{1}{2k}\frac{\partial^2 U_j}{\partial x^2} - \frac{k_0}{2}n_e^3 r_{33}E_0\frac{U_j}{1 + I(x,z)} = 0, \quad (4)$$

where $I(x,z)$ is the intensity profile of the incoherent beam which is given by

$$I(x,z) = \sum_{j=-\infty}^{\infty}|U_j(x,z)|^2 \quad (5)$$

and the discrete index $j = 0, \pm1, \pm2, \ldots$. Effectively, Eqs. (4) and (5) describe the process of incoherent self-focusing in biased photorefractives in the limit $\Delta\theta \to 0$. Moreover, in these equations each coherent fragment has been scaled with respect to I_d, and at the origin we have assumed that $U_j(x,0) \propto G_N^{1/2}(j\Delta\theta)f(x)$.

As an example let us consider a biased SBN:75 crystal. Here the parameters used will be very similar to those reported in [12]. In particular, let $r_{33} = 1022$ pm/V, $\lambda_0 = 488$ nm, $n_e = 2.3$, $W = 6$ mm, and let the length of propagation be 6 mm. Let us also assume that the input intensity profile as well as the incoherent angular power spectrum are Gaussian, i.e., $f(x) = \exp(-x^2/2x_0^2)$

and $G_N(\theta) = (\pi^{1/2}\theta_0)^{-1}\exp(-\theta^2/\theta_0^2)$. In our numerical computations we use 201 components ($-100 \le j \le 100$) equidistantly spanning the range $\pm 2.5\theta_0$. From our previous discussion, the field of each coherent component at the origin is set to be $U_j(x,0) = \hat{r}^{1/2} f(x)\exp[-(j\Delta\theta)^2/2\theta_0^2]$ where $\hat{r}$ is an appropriate constant which is related to the maximum intensity ratio r_T (with respect to I_d) of the input incoherent beam. More specifically, $r_T = \hat{r}\sum_j \times \exp[-(j\Delta\theta/\theta_0)^2]$. Equations (4) and (5) are solved by means of standard split-step Fourier methods. The accuracy of our results was then checked against the conservation laws of Eqs. (4) and (5) and by increasing the number of coherent components. As a first example let us consider an incoherent Gaussian beam with $x_0 = 18\ \mu\text{m}$, in which case its input intensity FWHM is 30 μm. Moreover, let the width of the source angular power spectrum be $\theta_0 = 9.56$ mrad or 0.548°. In this case the beam linearly diffracts to a FWHM of 102 μm after 6 mm of propagation. Note that if this beam were spatially coherent, it would have diffracted to 35.4 μm after 6 mm, as has been observed in [12]. Once the crystal is appropriately biased, self-trapping effects start to emerge. Figure 1(a) depicts the intensity evolution of this beam ($x_0 = 18\ \mu\text{m}$, $\theta_0 = 0.548°$) when the applied voltage is 400 V and $r_T = 3$. After 6 mm, the beam has developed a rectangularlike profile with a FWHM of $\sim$34.7 μm. Evidently self-focusing played an important role in this example even though it was not enough to balance diffraction effects. Figure 1(b), on the other hand, shows what would happen if the bias is increased to 550 V. In this case the incoherent beam propagates almost undistorted and it behaves like a quasisoliton. In other words, all the coherent components or quasiparticles have appropriately fused together thus producing a stationary beam. At an even higher voltage $V = 1000$ V, the beam starts to exhibit considerable spatial compression. Figure 1(c) shows that cycles of compression and expansion are now possible during propagation. At the output, i.e., $z = 6$ mm, its intensity FWHM is $\sim$18 μm. Note that behavior of this sort [including the rectangularlike beam features of Figs. 1(a) and 1(c)] is consistent with previous experimental observations [22]. Next, let us consider what will happen at higher intensity ratios r_T. Figure 2(a) shows the propagation of an incoherent Gaussian beam when $V = 550$ V, $\theta_0 = 0.548°$, and $r_T = 40$. At $z = 6$ mm, the beam now expands from 30 μm FWHM to 87 μm. As in the coherent limit [16,17], this expansion is attributed to the saturation of the photorefractive nonlinearity when $r_T \gg 1$. In our simulations the angular width θ_0 was also found to play an important role. Figure 2(b) depicts the propagation of such an incoherent Gaussian beam (30 μm FWHM at the input) when $V = 550$ V, $r_T = 3$, and $\theta_0 = 0.8°$. As the figure shows, the beam expands in a rectangularlike fashion to a FWHM of 51.3 μm after 6 mm of propagation. Clearly, for $\theta_0 = 0.8°$ a higher bias voltage is required to overcome diffraction effects and indeed at $\sim$950 V self-trapping is reestablished.

We also point out that the infinite system of coupled nonlinear partial differential equations of Eq. (4) does in fact admit solitary wave solutions. To obtain such a solution we write $U_j(x,z) = u_j(x,z)\exp\{i[(j\Delta\theta)^2(kz/2) - j\Delta\theta kx]\}$ and $u_j = r_j^{1/2} Q(x)\exp(i\mu z)$, where $Q(x)$ is a normalized function, i.e., $0 \le Q(x) \le 1$. Direct substitution of these latter forms in Eq. (4) leads to the following

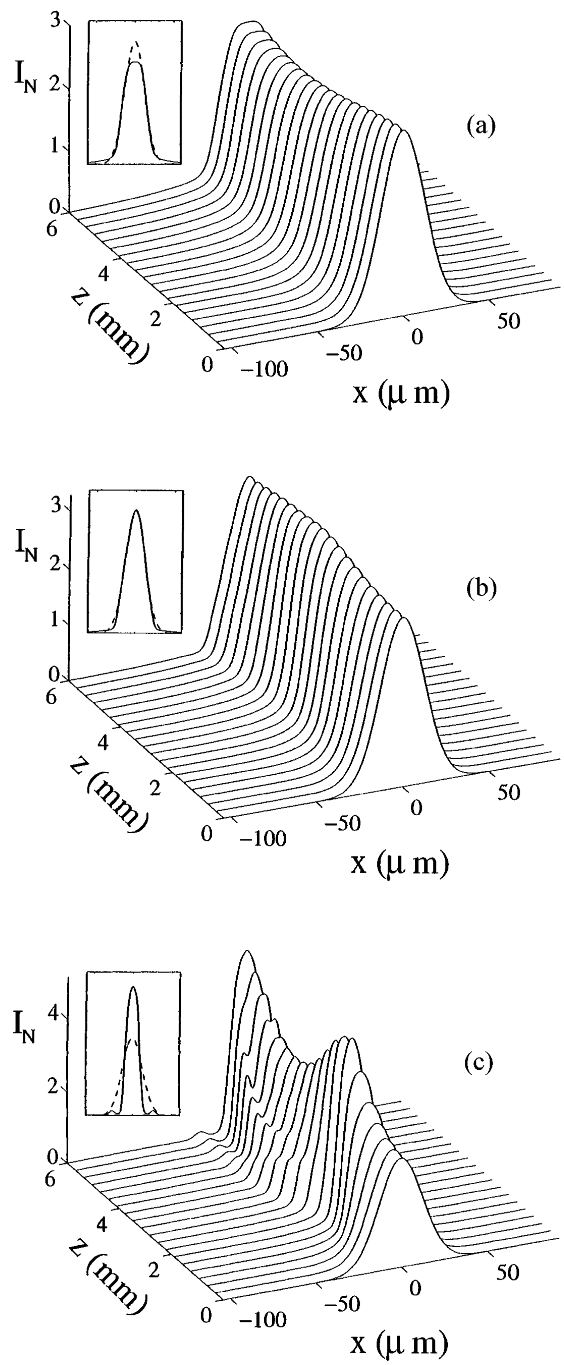
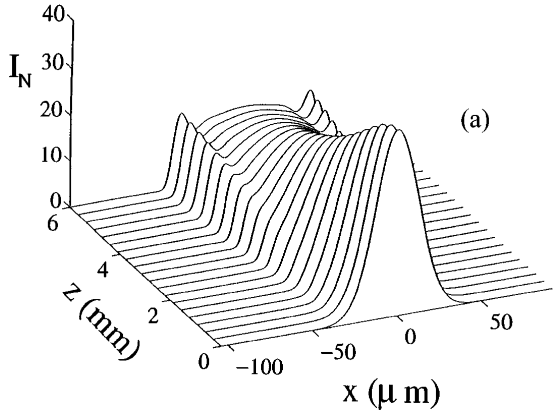


FIG. 1. Evolution of the normalized intensity profile resulting from an incoherent Gaussian beam when its initial FWHM is 30 μm, $\theta_0 = 0.548°$, $r_T = 3$, and when the applied voltage is (a) 400 V, (b) 550 V, and (c) 1000 V. The insets show the input (dashed curve) and output at $z = 6$ mm (solid curve) intensities.



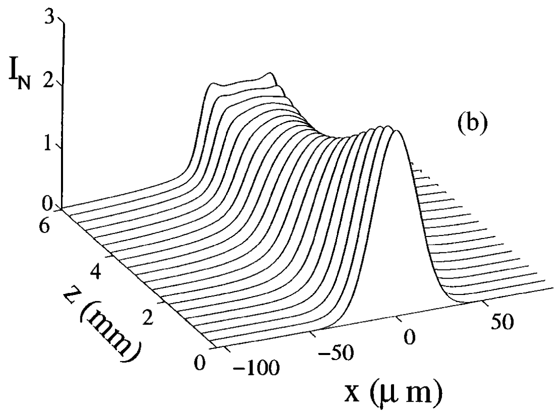


FIG. 2. Propagation of an incoherent Gaussian beam in a biased SBN:75 crystal when the applied voltage is 550 V, its initial FWHM is 30 μm, and (a) $\theta_0 = 0.548°$, $r_T = 40$, and (b) $\theta_0 = 0.8°$, $r_T = 3$.

ordinary differential equation

$$\frac{d^2Q}{dx^2} - 2\mu k Q - (k^2 n_e^2 r_{33} E_0) \frac{Q}{1 + r_T Q^2} = 0\,, \quad (6)$$

which is known to allow bright solitons [16,17] when $E_0 > 0$ and $\mu = -(k n_e^2 r_{33} E_0 / 2 r_T) \ln(1 + r_T)$. In Eq. (6), r_T is the total intensity ratio, i.e., $r_T = \sum_j r_j$. We emphasize, however, that physically this solitary wave solution has a limited range of applicability. More specifically the employed $U_j - u_j$ transformation implies in reality that all the coherent components propagate in parallel along z. Thus a solitary solution of this sort may be applicable only when θ_0 is quite small in which case all quasiparticles may tend to propagate almost in parallel after fusion. In fact, the above mentioned solution represents a generalization of the so-called incoherent coupled photorefractive soliton pairs previously discussed in the literature [23].

In conclusion, a theory of incoherent self-trapping in biased photorefractives has been developed. It has been shown that this process can be effectively described by an infinite set of coupled nonlinear Schrödinger-like equations provided they have been initially weighted with respect to incoherent angular power spectrum of the source. Relevant examples have been provided. Our numerical computations were in close agreement with recent experimental data. We wish to emphasize that our theoretical approach applies not only to photorefractive media but also to any other nonlinear material whose temporal response time is much longer than the phase fluctuation time of an optical beam.

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