

Gauge-Invariant Decomposition of Nucleon Spin

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I introduce a gauge-invariant decomposition of the nucleon spin into quark helicity, quark orbital, and gluon contributions. The total quark (and hence the quark orbital) contribution is shown to be measurable through virtual Compton scattering in a special kinematic region where single quark scattering dominates. This deeply virtual Compton scattering has much potential to unravel the quark and gluon structure of the nucleon. [S0031-9007(96)02221-1]

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The spin structure of the nucleon reflects interesting nonperturbative physics in quantum chromodynamics (QCD). From the recent data on polarized deep-inelastic scattering [1], one finds that about $(20 \pm 15)\%$ of the nucleon spin is carried by quark spin or helicity [2]. Natural questions are then where is the remainder of the nucleon spin? How can it be measured or calculated? This Letter attempts to provide answers to them.

Intuitively, the candidates for the “missing” spin are the quark and gluon orbital angular momenta and gluon helicity. In QCD, they can be identified with matrix elements of certain quark-gluon operators in the nucleon state [3]. The problem, however, is that these operators take free-field expressions and are not gauge invariant in an interacting gauge theory. Hence it is doubtful that their matrix elements have any experimental significance, although they can be calculated in theory, for instance, on a lattice, with a fixed gauge.

In this Letter I show that there exists a gauge-invariant decomposition of the QCD angular momentum operator into quark and gluon contributions. The quark part can be separated further into the usual quark helicity plus the *gauge-invariant orbital* contribution. There exists, however, no gauge-invariant separation of the gluon part into helicity and orbital contributions, although high-energy scattering favors such a separation in the lightlike gauge and infinite momentum frame. The gauge-invariant quark and gluon contributions to the nucleon spin are shown to asymptotically approach ratio $16:3n_f$, where n_f is the number of active fermion flavors. This result is incidentally the same as what Hoodbhoy, Tang, and I have derived previously in a gauge noninvariant formulation [4]. The gauge-invariant expression for the angular momentum operator allows one to calculate meaningfully fractions of the nucleon spin carried by quarks and gluons. Furthermore, it allows them to be measured in deeply virtual Compton scattering (DVCS) in which the virtual photon momentum approaches the Bjorken limit. DVCS gives an access to a new class of nucleon observables—the off-forward parton distributions—which are a generalization of ordinary parton distributions and elastic form factors.

The angular momentum operator in QCD is defined according to the generators of Lorentz transformation,

$$J^i = \frac{1}{2} \epsilon^{ijk} \int d^3x M^{0jk}, \quad (1)$$

where M^{0ij} is the angular momentum density, expressible in terms of the energy-momentum tensor $T^{\mu\nu}$ through

$$M^{\alpha\mu\nu} = T^{\alpha\nu} x^\mu - T^{\alpha\mu} x^\nu. \quad (2)$$

$T^{\mu\nu}$ has the Belinfante-improved form and is symmetric, gauge invariant, and conserved [5]. It can be separated into gauge-invariant quark and gluon contributions,

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}, \quad (3)$$

where the quark part is

$$T_q^{\mu\nu} = \frac{1}{2} [\bar{\psi} \gamma^{(\mu} i \overleftrightarrow{D}^{\nu)} \psi + \bar{\psi} \gamma^{(\mu} i \overleftarrow{D}^{\nu)} \psi], \quad (4)$$

and the gluon part is

$$T_g^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^\nu{}_\alpha, \quad (5)$$

where $(\mu\nu)$ denotes symmetrization with respect to μ, ν indices. I will ignore the issues of regularization, gauge fixing, and trace anomaly [6], as they do not affect the following discussion.

For the above equations, one sees that \vec{J} can be written as a gauge-invariant sum, $\vec{J}_{\text{QCD}} = \vec{J}_q + \vec{J}_g$, where

$$J_{q,g}^i = \frac{1}{2} \epsilon^{ijk} \int d^3x (T_{q,g}^{0k} x^j - T_{q,g}^{0j} x^k). \quad (6)$$

In pure gauge theory, \vec{J}_g by itself is a conserved angular momentum charge, generating spin quantum numbers for glueballs. It is clear that \vec{J}_q and \vec{J}_g are interaction dependent and thus differ from the corresponding expressions in free-field theory.

To understand the physical content of the gauge invariant \vec{J}_q and \vec{J}_g , one can reexpress them using QCD equations of motion and superpotentials [3,5]. After some algebra, one finds

$$\begin{aligned} \vec{J}_q &= \int d^3x \psi^\dagger [\vec{\gamma} \gamma_5 + \vec{x} \times (-i\vec{D})] \psi, \\ \vec{J}_g &= \int d^3x [\vec{x} \times (\vec{E} \times \vec{B})] \end{aligned} \quad (7)$$

where color indices are implicit.

It is instructive to compare the above gauge-invariant form of the angular momentum operator with the free-field expression [3]. Because the interaction between quarks and gluons contains no derivative, one can write $\vec{J}_{\text{QCD}} = \vec{J}'_q + \vec{J}'_g$, where $\vec{J}'_{q,g}$ are interaction independent (except in the definition of \vec{E}),

$$\begin{aligned}\vec{J}'_q &= \int d^3x \psi^\dagger [\vec{\gamma} \gamma_5 + \vec{x} \times (-i\vec{\partial})] \psi, \\ \vec{J}'_g &= \int d^3x [(\vec{E} \times \vec{A}) - E_i(\vec{x} \times \vec{\partial}) A_i].\end{aligned}\quad (8)$$

Here each term has a straightforward interpretation: Both the quark and gluon angular momenta are sums of spin and orbital contributions. In the presence of gauge interactions, \vec{J}'_q and \vec{J}'_g , as well as the individual terms in them except the quark helicity, are gauge independent.

The gluon angular momentum \vec{J}'_g does not admit further gauge-invariant decomposition as spin and orbital contributions, contrary to the quark case. If one disregards the issue of gauge invariance, \vec{J}'_g can be viewed as a sum of three terms: two terms in J'_g in Eq. (8) and an interaction-dependent term $-g \int d^3x \psi^\dagger \vec{x} \times \vec{A} \psi$. The term $\vec{S}_g = \int d^3x \vec{E} \times \vec{A}$ has the simple interpretation as the spin of gluons in the $A^0 = 0$ gauge [3,7,8]. In the infinite momentum frame (IMF), the gauge condition becomes $A^+ = 0$ and the rotational generators are constructed from the density M^{+ij} . The nucleon matrix element of \vec{S}_g in the lightlike gauge and IMF is measurable in high-energy scattering. In fact, the first moment of the polarized gluon distribution $\Delta g(x)$ gives [7,8]

$$\int_0^1 \Delta g(x) dx 2\vec{S} = \langle PS | \hat{O} | PS \rangle, \quad (9)$$

where \hat{O} is a gauge-invariant operator which reduces to S_g in the $A^+ = 0$ gauge and IMF. To maximally utilize this piece of experimental information, one can define the gluon orbital angular momentum \vec{L}_g as the difference $\vec{J}'_g - \vec{S}_g$. The nucleon matrix element of \vec{L}_g in the lightlike gauge and IMF can be deduced from the matrix elements of \vec{S}_g and \vec{J}'_g and might offer some insights on the spin structure of the nucleon.

According to Eq. (6), the Q^2 evolution of the quark and gluon contributions to the nucleon spin is the same as that of matrix elements of $T_{q,g}^{\mu\nu}$. The reason is quite simple: forming spatial moments of $T_q^{\mu\nu}$ and $T_g^{\mu\nu}$ does not change the short-distance singularity of the operators. On the other hand, in Ref. [4], it was shown that the matrix elements of \vec{J}'_q and \vec{J}'_g have the same leading-log evolution as the quark and gluon contributions to the nucleon momentum. If both results above are consistent, the interaction-dependent term, $-g \int d^3x \psi^\dagger \vec{x} \times \vec{A} \psi$, shall not affect the leading-log evolution in the lightlike gauge. Indeed, an explicit calculation confirms this. If one defines

$$J_{q,g}(Q^2) 2\vec{S} = \langle PS | \vec{J}'_{q,g}(Q^2) | PS \rangle, \quad (10)$$

the leading-log Q^2 dependence is simply,

$$\begin{aligned}J_q(Q^2) &= \frac{1}{2} \frac{3n_f}{16 + 3n_f} + \left(\frac{\ln Q_0^2/\Lambda^2}{\ln Q^2/\Lambda^2} \right)^{2(16+3n_f)/(33-2n_f)} \\ &\times \left[J_q(Q_0^2) - \frac{1}{2} \frac{3n_f}{16 + 3n_f} \right], \\ J_g(Q^2) &= \frac{1}{2} \frac{16}{16 + 3n_f} + \left(\frac{\ln Q_0^2/\Lambda^2}{\ln Q^2/\Lambda^2} \right)^{2(16+3n_f)/(33-2n_f)} \\ &\times \left[J_g(Q_0^2) - \frac{1}{2} \frac{16}{16 + 3n_f} \right],\end{aligned}\quad (11)$$

where n_f is the number of quark flavors ($J_q + J_g = 1/2$). As $Q^2 \rightarrow \infty$, the partition of the nucleon spin between quarks and gluons approaches the ratio $16:3n_f$, the same as the asymptotic partition of the nucleon momentum derived by Gross and Wilczek [9].

The gauge-invariant form of the QCD angular momentum operator allows one to meaningfully calculate and measure the fraction of the nucleon spin carried by quarks and gluons. Recently, Balitsky and I have estimated $J_{q,g}$ at the scale of 1 GeV using the QCD sum rule method [10]. To see how they can be measured in an experiment, I define the form factors of the quark and gluon energy-momentum tensors

$$\begin{aligned}\langle P' | T_{q,g}^{\mu\nu} | P \rangle &= \bar{U}(P') [A_{q,g}(\Delta^2) \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g}(\Delta^2) \bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha / 2M + C_{q,g}(\Delta^2) (\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2) / M \\ &+ \bar{C}_{q,g}(\Delta^2) g^{\mu\nu} M] U(P),\end{aligned}\quad (12)$$

where $\bar{P}^\mu = (P^\mu + P^{\mu'})/2$, $\Delta^\mu = P^{\mu'} - P^\mu$, and $U(P)$ is the nucleon spinor. (An analogous equation for the full tensor was considered by Jaffe and Manohar [3].) Taking the forward limit for $\mu = 0$ and integrating over 3-space, one finds that $A_{q,g}(0)$ give the momentum fractions of the nucleon carried by quarks and gluons [$A_q(0) + A_g(0) = 1$]. On the other hand, substituting the above into the nucleon matrix element of Eq. (6), one finds

$$J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)]. \quad (13)$$

Thus, to find the quark and gluon contributions to the nucleon spin, one has to measure the B -form factor, which is analogous to the Pauli form factor for the vector current.

Since there is no fundamental probe that couples separately to the quark and gluon energy-momentum tensors, it appears hopeless to measure the form factors. On the other hand, $T_{q,g}^{\mu\nu}$ does appear in the operator product expansion (OPE) for the product of vector currents $TJ_\alpha(\xi)J_\beta(0)$, and thus $B_{q,g}(0)$ is accessible through deep-inelastic sum rules, like what $A_{q,g}(0)$ is measured. The complication,

however, is that the $B_{q,g}(0)$ term does not contribute to the forward matrix element and so the usual inclusive deep-inelastic process is useless in this regard. A way to get around this is to measure the off-forward matrix element of $TJ_\alpha(\xi)J_\beta(0)$ and extrapolating the form factors to the forward limit. The natural process to do this is the Compton scattering [11]. To ensure there is an OPE, one has to have one of the photons far off-shell and single-quark scattering dominating the process. To emphasize this special kinematic region, I call the process DVCS.

Consider virtual Compton scattering with a virtual photon of momentum q^μ absorbed by a nucleon of momentum P^μ , and an outgoing real photon of momentum $q'^\mu = q^\mu - \Delta^\mu$, and a recoil nucleon of momentum $P'^\mu = P^\mu + \Delta^\mu$. The deeply virtual kinematics refer to q^μ in the Bjorken limit, namely, $Q^2 = -q^2 \rightarrow \infty$, $P \cdot q \rightarrow \infty$, and $Q^2/P \cdot q$ finite. Given the kinematics above, it is easy to show that the only dominant scattering subprocess, modulo radiative corrections, involves a single quark absorbing the deeply virtual photon and subsequently radiating a real photon and falling back to the nucleon. I henceforth concentrate on this dominant subprocess shown in Fig. 1.

To calculate the scattering amplitude, it is convenient to define a special system of coordinates. I choose q^μ and $\bar{P}^\mu = (P + P')^\mu/2$ to be collinear and in the z direction.

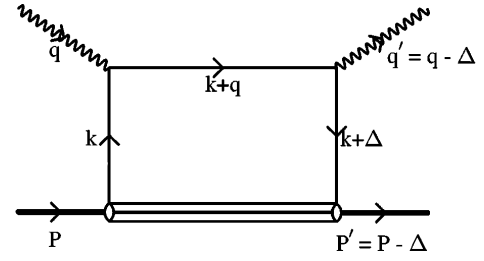


FIG. 1. Dominant scattering process in deeply virtual Compton scattering.

Introduce two lightlike vectors, $p^\mu = \Lambda(1,0,0,1)$ and $n^\mu = (1,0,0,-1)/(2\Lambda)$, with $p^2 = n^2 = 0$, $p \cdot n = 1$, and Λ arbitrary. I expand other vectors according to p^μ , n^μ , and transverse vectors,

$$\begin{aligned} \bar{P}^\mu &= p^\mu + (\bar{M}^2/2)n^\mu, \\ q^\mu &= -\xi p^\mu + (Q^2/2\xi)n^\mu, \end{aligned} \quad (14)$$

$$\Delta^\mu = -\xi[p^\mu - (\bar{M}^2/2)n^\mu] + \Delta_\perp^\mu,$$

where $\bar{M}^2 = M^2 - \Delta^2/4$ and $\xi = Q^2/(2\bar{P} \cdot q)$. ξ is bounded by 0 and $\sqrt{-\Delta^2}/\sqrt{M^2 - \Delta^2/4}$. The Compton amplitude $T^{\mu\nu} = i \int d^4y e^{-iq \cdot y} \langle P' | T J^\nu(y) J^\mu(0) | P \rangle$, where ν and μ are the polarization indices of the initial and final photons, is

$$\begin{aligned} T^{\mu\nu}(P, q, \Delta) &= -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu}) \int_{-1}^1 dx \left(\frac{1}{x - \xi/2 + i\epsilon} + \frac{1}{x + \xi/2 - i\epsilon} \right) \\ &\times \left[H(x, \Delta^2, \xi) \bar{U}(P') \not{n} U(P) + E(x, \Delta^2, \xi) \bar{U}(P') \frac{i\sigma^{\alpha\beta} n_\alpha \Delta_\beta}{2M} U(P) \right] \\ &- \frac{i}{2} \epsilon^{\mu\nu\alpha\beta} p_\alpha n_\beta \int_{-1}^1 dx \left(\frac{1}{x - \xi/2 + i\epsilon} - \frac{1}{x + \xi/2 - i\epsilon} \right) \\ &\times \left[\tilde{H}(x, \Delta^2, \xi) \bar{U}(P') \not{n} \gamma_5 U(P) + \tilde{E}(x, \Delta^2, \xi) \frac{\Delta \cdot n}{2M} \bar{U}(P') \gamma_5 U(P) \right]. \end{aligned} \quad (15)$$

Thus only 4 of the 12 helicity amplitudes survive the Bjorken limit [12]. All photon helicity flipping and longitudinal photon amplitudes vanish. H , \tilde{H} , E , and \tilde{E} are new, off-forward, twist-two parton distributions defined through the following light-cone correlation functions,

$$\begin{aligned} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}(-\lambda n/2) \gamma^\mu \psi(\lambda n/2) | P \rangle &= H(x, \Delta^2, \xi) \bar{U}(P') \gamma^\mu U(P) + E(x, \Delta^2, \xi) \bar{U}(P') \frac{i\sigma^{\mu\nu} \Delta_\nu}{2M} U(P) + \dots, \\ \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}(-\lambda n/2) \gamma^\mu \gamma_5 \psi(\lambda n/2) | P \rangle &= \tilde{H}(x, \Delta^2, \xi) \bar{U}(P') \gamma^\mu \gamma_5 U(P) + \tilde{E}(x, \Delta^2, \xi) \bar{U}(P') \frac{\gamma_5 \Delta^\mu}{2M} U(P) + \dots, \end{aligned} \quad (16)$$

where I have neglected the gauge link and the dots denote higher-twist distributions. From the definition, H and \tilde{H} are nucleon helicity-conserving amplitudes and E and \tilde{E} are helicity flipping. When QCD radiative corrections are taken into account, Eq. (15) will be modified by analogous gluon distributions.

Extracting the parton distributions from an experimental cross section is more intricate than in the usual case of deep-inelastic scattering, as it is bilinear in the Compton amplitude which involves an integration of Feynman

x. In practice, one might have to parametrize the parton distributions and fit them to the cross section.

The off-forward parton distributions have the characters of both ordinary parton distributions and nucleon form factors. In fact, in the limit of $\Delta^\mu \rightarrow 0$, we have

$$H(x, 0, 0) = q(x), \quad \tilde{H}(x, 0, 0) = \Delta q(x), \quad (17)$$

where $q(x)$ and $\Delta q(x)$ are quark and quark helicity distributions. On the other hand, forming the first moment of

the new distributions, one gets the following sum rules

$$\begin{aligned} \int_{-1}^1 dx H(x, \Delta^2, \xi) &= F_1(\Delta^2), \\ \int_{-1}^1 dx E(x, \Delta^2, \xi) &= F_2(\Delta^2), \\ \int_{-1}^1 dx \tilde{H}(x, \Delta^2, \xi) &= G_A(\Delta^2), \\ \int_{-1}^1 dx \tilde{E}(x, \Delta^2, \xi) &= G_P(\Delta^2), \end{aligned} \quad (18)$$

where F_1 and F_2 are the Dirac and Pauli form factors and G_A and G_P are the axial-vector and pseudoscalar form factors. The most interesting sum rule relevant to the nucleon spin is

$$\begin{aligned} \int_{-1}^1 dx x [H_q(x, \Delta^2, \xi) + E_q(x, \Delta^2, \xi)] &= A_q(\Delta^2) \\ &+ B_q(\Delta^2), \end{aligned} \quad (19)$$

where luckily the ξ dependence, or $C_q(\Delta^2)$ contamination, drops out. Extrapolating the sum rule to $\Delta^2 = 0$, the total quark (and hence quark orbital) contribution to the nucleon spin is obtained. By forming still higher moments, one gets form factors of various high-spin operators.

It is important to comment on practical aspects of the experiment. First of all, from the cross section, one finds that E and H can be measured either in unpolarized scattering, or in electron single-spin asymmetry through interference with the Bethe-Heitler amplitude [12], or in polarized electron scattering on a transversely polarized target. A detailed examination of various possibilities, together with some numerical estimates will be published elsewhere [13]. Second, the DVCS cross section is down by an order of α_{em} compared with the deep-inelastic cross section, but has the same scaling behavior. So the cross section is appreciable, but statistics would be a challenging requirement. The ideal accelerator for the experiment is ELFE [14]. Finally, the extrapolation of Δ^2 from order M^2 to 0 requires dispersive study of the form factors of the tensor currents.

The off-forward parton distributions can be defined for quark helicity-flip (chiral-odd) correlations, for higher twists, and for gluons. There are other processes one can consider to measure them. For instance, the diffractive ρ or J/ψ production studied recently by Brodsky *et al.* [15] can be used to measure the off-forward gluon distributions. Thus there is now a new territory to explore the quark and gluon structure of the nucleon besides the traditional inclusive (parton distributions) and exclusive (form factors) processes.

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