Which Chiral Symmetry is Restored in High Temperature Quantum Chromodynamics?

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Sigma models for the high temperature phase transition in quantum chromodynamics suggest that at high temperature the $\mathrm{SU}(N_f) \times \mathrm{SU}(N_f)$ chiral symmetry becomes exact, but the anomalous axial U(1) symmetry need not be restored. In numerical lattice simulations, traditional methods for detecting symmetry restoration have sought multiplets in the screening mass spectrum. However, these methods were imprecise and the results, so far, incomplete. With improved statistics and methodology, we are now able to offer evidence for a restoration of the SU(2) × SU(2) chiral symmetry just above the crossover, but not of the axial U(1) chiral symmetry. [S0031-9007(96)02251-X]

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A high temperature phase transition from a deconfined quark plasma to a confined phase is thought to have occurred as the early Universe cooled. This phenomenon is under investigation in high energy heavy-ion collisions. Through numerical simulations of quantum chromodynamics (QCD) we hope to gain an understanding of the qualitative and quantitative characteristics of this phase transition. The phase transition (perhaps only a crossover at physical quark masses) is associated with the spontaneous breaking of the chiral symmetry and formation of chiral condensates. Sigma models suggest that in the limit of zero up and down quark masses, the SU(2) × SU(2) chiral symmetry is exact in the high temperature phase [1], and a phase transition separates it from a cold phase in which this symmetry is spontaneously broken. The gauge anomaly, present at low temperature, may persist at high temperature, however, breaking the U(1) axial symmetry at all temperatures.

Early efforts to detect symmetry restoration looked for chiral multiplets in the screening mass spectrum [2]. For example, the following channels are related according to the indicated symmetries:

$$\begin{array}{ccc} \leftarrow & \mathrm{SU}(2) \times \mathrm{SU}(2) & \rightarrow \\ & \uparrow & \\ \mathrm{U}(1)_A & f_0 & \pi \\ \downarrow & \eta & a_0 \,. \end{array}$$

The screening mass spectrum is found from the spacelike hadron propagators. The restoration of the SU(2) × SU(2) symmetry requires a degeneracy between the lowest pion screening mass and that of its chiral partner, the $J^P = 0^+, I = 0$ f_0 meson (also known as the σ). The determination of the f_0 screening mass through numerical simulation is complicated by the presence of quark-line disconnected graphs. Computing them requires an expensive determination of the quark propagator from multiple origins. In early simulations, therefore, it was common to keep only connected graphs. This practice, applied to the f_0, results instead in a determination of the screening mass for the $J^P = 0^+, I = 1$ a_0 meson (also known as the δ) [3]. This meson is the axial U(1) chiral partner of the pion. Thus a degeneracy in the π and a_0 screening masses would imply a suppression of the gauge anomaly and a partial restoration of the axial U(1) symmetry, but does not test restoration of the SU(2) × SU(2) symmetry.

New simulations with large data samples make it possible to revisit the question of which symmetry is restored [4,5]. Further statistical improvement can be obtained by studying the susceptibilities related to the propagators, rather than just the screening masses: for example, from the pion susceptibility

$$\chi_\pi = \int d^4 r \, \langle \pi(0) \pi(r) \rangle \tag{1}$$

and the related susceptibilities, χ_{f_0} and χ_{a_0}, we can define two order parameters

$$\chi_{\mathrm{SU(2)\times SU(2)}} = \chi_\pi - \chi_{f_0} \text{ and } \chi_{\mathrm{U(1)}} = \chi_\pi - \chi_{a_0} \,. \tag{2}$$

Restoration of either symmetry requires that the corresponding order parameter vanish.

We use the staggered fermion scheme. This scheme breaks all but one generator of chiral SU(4) × SU(4).

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The full symmetry is expected to be recovered in the continuum limit. The one surviving generator, however, can be used to explore symmetry restoration at the phase transition at nonzero lattice spacing. The staggered fermion treatment of the axial U(1) symmetry is less satisfactory. That symmetry, formulated in the conventional manner, is broken explicitly on the lattice. It, too, is expected to be recovered in the continuum limit. Since our analysis treats only one lattice spacing, namely $a \approx 1/6T_c$, further study will be required to distinguish between effects of the lattice approximation and continuum effects of the gauge anomalies.

A preliminary report of our results was presented in Ref. [5]. A number of other groups have also taken up this question and have also reported preliminary results [6–8].

We simulate the N_f-flavor staggered fermion action with the standard partition function at temperature T on a hypercubic Euclidean lattice with spacing a, quark matrix $M(U, m_q)$, quark mass m_q, and gauge link matrices U [9]:

$$Z = e^{-VF(T,am_q)/T} = \int [dU] \times \exp[-S_g(U)][\det M(U, m_q)]^{N_f/4}. \quad (3)$$

As is well known, the fermion determinant can be expressed as $\det M(U, m_q) = \det[D^2 + (2am_q)^2]$, where the latter determinant is taken on the even lattice sites only and D^2 is the square of the fermion hopping matrix. Thus the free energy is manifestly even in the quark mass.

We will be concerned with a variety of susceptibilities related to the singlet chiral order parameter,

$$\langle f_0 \rangle \equiv \langle \overline{\psi}\psi \rangle = \partial F(T, m_q)/\partial m_q = TN_f a/2V\langle \mathrm{Tr} M^{-1} \rangle, \quad (4)$$

where the expectation values are defined on the ensemble (3). The associated susceptibility is

$$\chi_{f_0} = \partial\langle f_0 \rangle/\partial m_q = \int d^4x [\langle f_0(0) f_0(x) \rangle - \langle f_0(0) \rangle^2] = \chi_{\mathrm{conn}} + \chi_{\mathrm{disc}}. \quad (5)$$

The quark-line connected and disconnected contributions are

$$\chi_{\mathrm{conn}} = TN_f a^2/V\langle \mathrm{Tr} M^{-2} \rangle \text{ and } \chi_{\mathrm{disc}} = (T/V)[\langle (aN_f/2\mathrm{Tr} M^{-1})^2 \rangle - \langle aN_f/2\mathrm{Tr} M^{-1} \rangle^2]. \quad (6)$$

It can be seen from this result that the disconnected contribution to the susceptibility is just proportional to the "configuration variance" of $\langle f_0 \rangle$, that is, $\chi_{\mathrm{disc}} = (V/T)[\langle f_0^2 \rangle - \langle f_0 \rangle^2]$.

All of our simulations are carried out with two dynamical (sea) quark flavors. However, in measuring susceptibilities, we can adjust the valence flavor number to suit the observable. If we stick with only the four flavors forced upon us by fermion doubling in the staggered fermion scheme, all isospin components of the a_0 meson are generated by a nonlocal fermion bilinear [10]. However, at the expense of increasing the flavor degeneracy to eight, we can create an a_0 analog from a diagonal fermion bilinear operator. In any case all such a_0 components are expected to be degenerate in the continuum limit and any of them can be used to test symmetry restoration. The susceptibility of the diagonal a_0 operator is exactly the connected part of the f_0 susceptibility:

$$\chi_{a_0} = \chi_{\mathrm{conn}}. \quad (7)$$

We measure this susceptibility directly from the connected part of the f_0 correlator: $\chi_{\mathrm{conn}} = \int d^4x \langle f_0(0) f_0(r) \rangle|_{\mathrm{conn}}$, while Chandrasekharan and Christ measure it by taking the derivative of $\langle f_0 \rangle$ with respect to the valence quark mass [6]. Finally, a well-known Ward identity relates the pion susceptibility to the chiral order parameter [11]:

$$\chi_\pi = N_f T a^2/V\langle \mathrm{Tr}(M^\dagger M)^{-1} \rangle = \langle f_0 \rangle/2m_q. \quad (8)$$

In practice we measure the order parameters (2) through

$$\chi_{\mathrm{SU(2)}\times SU(2)} = \langle f_0 \rangle/2m_q - \chi_{\mathrm{conn}} - \chi_{\mathrm{disc}} \text{ and } \chi_{\mathrm{U(1)}} = \langle f_0 \rangle/2m_q - \chi_{\mathrm{conn}}. \quad (9)$$

The simulation consisted of a subset of configurations generated in an extensive study of the equation of state for $N_t = 6$ and $N_f = 2$ at $6/g^2 = 5.45$ and quark masses $am_q = 0.0075$, 0.01, 0.0125, 0.015, 0.02, and 0.025 [4,5]. This parameter range lies in the high temperature phase slightly above the phase transition, as illustrated in Fig. 1, and was selected to permit an extrapolation of the measured quantities to zero quark mass in the high temperature phase. The simulation sample at each mass covered a molecular dynamics time span of at least

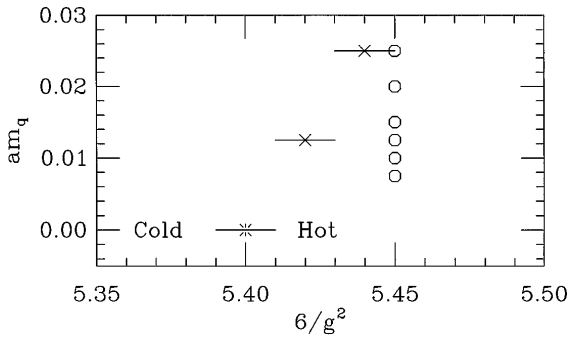

FIG. 1. Phase diagram for the standard SU(3) Wilson gauge plus two-flavor staggered fermion action showing the approximate $N_t = 6$ crossover location (crosses and burst) as a function of gauge coupling $6/g^2$ and quark mass am_q. Data sample points are indicated by octagons.

2000 time units with the first 400 omitted. Measurements were taken at intervals of at most 50 time units. The chiral order parameter $\langle f_0 \rangle \equiv \langle \overline{\psi}\psi \rangle$ was measured using the random source method [12] with 33 random sources. These measurements, with care taken to avoid biases inherent in the noisy source technique, in turn, provided an estimate of $\chi_{\rm disc}$ through the configuration variance.

Results are shown in Fig. 2 and Table I. We have indicated a linear extrapolation in $(am_q)^2$. Because they are closer to the crossover (Fig. 1), where curvature may be expected, we chose to exclude the two highest mass points from the fit. The zero mass intercepts are

$$\chi_{\rm SU(2)\times SU(2)} = -0.04(31) \text{ and } \chi_{\rm U(1)} = 0.75(22)\,, \tag{10}$$

with $\chi^2/df = 2.6/2$ and $2.5/2$, respectively. Fits to all points gave $\chi_{\rm SU(2)\times SU(2)} = 0.33(20)$ with $\chi^2/df = 5.6/4$ and $\chi_{\rm U(1)} = 0.81(11)$ with $2.7/4$.

It is surprising that a fit of the same points to an expression *linear* in am_q gives a result consistent with a zero intercept for *both* order parameters: $\chi_{\rm SU(2)\times SU(2)} = -0.40(56)$ with $\chi^2/df = 2.4/2$ and $\chi_{\rm U(1)} = 0.15(38)$ with $\chi^2/df = 1.8/2$. So which fit is correct? As we have emphasized, the free energy is rigorously even in the quark mass. In consequence the order parameters are also even. Thus if the free energy is analytic at zero quark mass, a quadratic fit is required. Now some gauge field configurations give rise to fermion zero modes or near-zero modes. In a two-flavor simulation, those modes contribute terms in $[(am_q)^2]^{N_f/4} = |am_q|$ to the free energy—terms are linear but nonanalytic. Such behavior, if not suppressed by a vanishing probability for encountering zero modes, would imply a phase transition or infrared singularity at zero quark mass. However, measurements of screening masses for $T > T_c$ give no indication of infrared singularities for small am_q. A phase transition at zero quark mass for $T > T_c$ is likewise unexpected in sigma models.

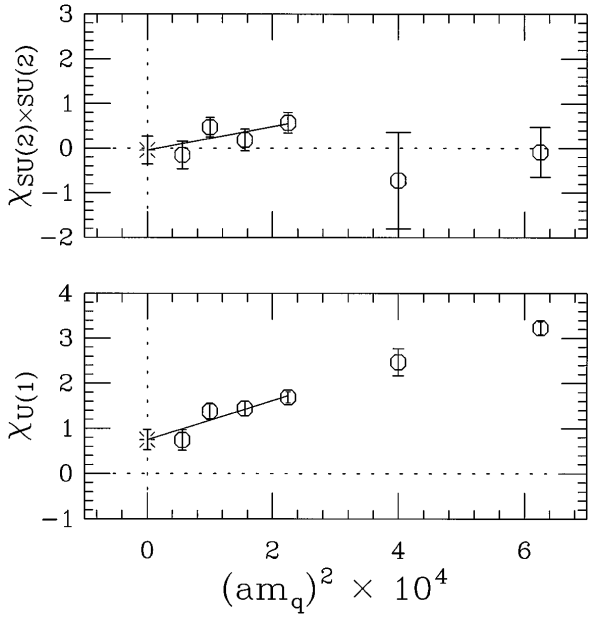

FIG. 2. Chiral order parameters extrapolated in quark mass squared.

TABLE I. Susceptibilities and order parameters in lattice units.

am_q	$\langle \overline{\psi}\psi \rangle$	$\chi_{\rm conn}$	$\chi_{\rm U(1)}$	$\chi_{\rm disc}$	$\chi_{\rm SU(2)\times SU(2)}$
0.0075	0.0446(12)	5.21(17)	0.74(23)	0.89(21)	−0.15(31)
0.01	0.0599(16)	4.61(9)	1.38(18)	0.91(12)	0.47(22)
0.0125	0.0724(16)	4.35(9)	1.44(16)	1.25(18)	0.19(22)
0.015	0.0885(15)	4.21(7)	1.69(12)	1.12(20)	0.57(23)
0.02	0.121(5)	3.59(14)	2.5(3)	3.1(1.0)	−0.7(1.1)
0.025	0.157(3)	3.04(8)	3.23(14)	3.3(5)	−0.1(6)

In conclusion, our results are consistent with the sigma model scenario: a restoration of SU(2) × SU(2) but not of $U(1)_A$ (approximately 3σ). Whether the apparent breaking of the axial U(1) symmetry is a lattice artifact or a consequence of the anomaly remains to be established by future measurements at smaller lattice spacing and with improved actions.

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