Kinetic Model of Particle Resuspension By Drag Force

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A kinetic model of particle reentrainment by a turbulent drag force is presented. The model uses the potential well approach and a definition of a tangential pull-off force necessary to separate a particle from a surface. A formula for the resuspension rate from surfaces where there is a spread of adhesive forces due to surface roughness is obtained. The rate turns out to be significantly larger than that resulting from the action of a turbulent lift force. [S0031-9007(96)02180-1]

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The removal of small particles from surfaces is important in many engineering problems. It arises, for example, in semiconductor manufactoring operations, where the particles affect the performance of integrated circuits. Other practical areas of importance are clean room technologies, indoor air contamination, particle behavior in respiratory tracts, etc.

When set on a surface, small particles are held by very strong surface forces which are a combination of physical attractions, chemical bonds, and mechanical stresses. This combination is usually referred to as the adhesion force. Considering the magnitude of the adhesion force, Bowling [1] noted that emphasis should be placed on prevention of particle deposition on surfaces rather than on subsequent removal. However, total prevention of deposition cannot be achieved in practice. This makes it necessary to consider particle removal by various means, including fluid flow over the particle laden surface [2] by a high velocity air jet [3] or by applying high-frequency sonic waves to the medium in which the surface is submerged [4].

The fact that resuspension occurs in spite of the very strong surface forces, has led Reeks, Reed, and Hall [5] to develop an essentially new approach to the analysis of the problem. Using the concept of a fluctuating lift force, they analyze the problem by means of an energy balance instead of a force balance. Their model for resuspension is based on the assumption that, when a particle is exposed to turbulent flow, there is turbulent energy transferred to the particle. A particle can be resuspended from a substrate when it accumulates enough energy to allow detachment from the adhesive potential well. Based on a dynamic analysis, Reeks, Reed, and Hall [5] predicted the reentrainment rate of particles as a function of time for a log-normally distributed adhesion force. An empirical kinetic model of particle reentrainment was presented by Wen and Kasper [6]. In that paper, coefficients of the model were evaluated by comparison of the reentrainment rate with experimental data collected from high-purity gas systems.

The present work deals with a kinetic model of particle reentrainment by a turbulent fluid drag force. It employs the potential well model developed in [5] and introduces a definition of a tangential load necessary to separate the particle from a surface. The determination of the tangential pull-off force uses a nonlinear model of particle streamwise oscillations on the surface restrained by a linear spring (Nayfeh and Mook [7], Ziskind [8]), shown in Fig. 1.

It turns out that a condition of a force balance between the nonlinear restoring force and the drag force is equivalent to the condition of a moment balance (Wang [9], Ziskind, Fichman, and Gutfinger [10]), where the drag-force moment equals the adhesion-force moment. It is shown that the reentrainment rate caused by the drag force is considerably larger than that of the lift force determined in [5]. This is consistent with the fact that in turbulent flow the mean and fluctuating drag forces are significantly larger than the mean and fluctuating lift forces.

Consider a flow over a particle on a smooth surface. The direction of the drag force coincides with that of the mean flow and is parallel to the surface (x direction). The surface reaction to a hydrodynamic moment may be expressed in terms of a moment of the adhesion force F_a, considered as acting at the center of the contact circle, the radius of which at equilibrium is r_{ae} [9]. Equilibrium means that the adhesive attraction is balanced by the elastic tension. Note that this approach overestimates the magnitude of the moment necessary to rotate the particle by a factor $\frac{3}{2}$ since in the adhesion models the contact radius decreases when the applied force increases [10]. For the Johnson, Kendall, and Roberts (hereafter referred to as JKR) adhesion model [11], the moment for a smooth surface can be presented in the form

$$M_a = F_a \times r_{ae} = \frac{3}{2}\pi\Delta\gamma R \times 1.26(3\pi\Delta\gamma R^2/k)^{1/3}, \tag{1}$$

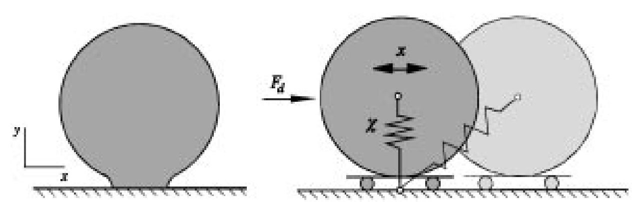

FIG. 1. A nonlinear model for particle oscillations.

where $\Delta\gamma$ is the surface energy, k is the elastic constant, and R is the particle radius.

The hydrodynamic moment of the drag force F_d may be written in the form [10]

$$\begin{aligned} M_d &= 1.399 F_d R\,, \\ F_d &= 6\pi\mu\dot{\gamma}R^2 f\,, \quad f = 1.7\,, \end{aligned} \tag{2}$$

where μ is the dynamic viscosity, and $\dot{\gamma}$ is the shear rate. The condition for detachment in terms of fluid, particle, and surface properties becomes

$$\frac{M_a}{M_d} = 0.3\,\frac{\Delta\gamma^{4/3}}{R^{4/3}k^{1/3}\mu\dot{\gamma}} < 1\,. \tag{3}$$

Let us show that the ratio of the moments in (3) has the same dependence on the relevant parameters as that of the tangential pull-off force and the drag force, where the pull-off force is defined by a nonlinear model [8] of particle streamwise vibrations on the surface, shown in Fig. 1. For this purpose, recall that Reeks, Reed, and Hall [5] consider linear oscillations only, where the stiffness of the spring, found from the JKR adhesion model, is represented by its equilibrium value as

$$\chi_e = \tfrac{9}{10}\,(6\pi\Delta\gamma)^{1/3}k^{2/3}R^{2/3}. \tag{4}$$

The elastic constant is given by

$$k = \frac{4}{3}\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)^{-1},$$

where ν_i, E_i $(i = 1,2)$ are Poisson's ratio and Young's modulus of a particle and a substrate, respectively. We know that the spring is actually nonlinear, and its stiffness changes from the maximum value at equilibrium to zero at the separation point. However, since we are going to compare our results with those of Reeks, Reed, and Hall [5], we use their approach. Following [5], we introduce the maximum vertical displacement, corresponding to separation of the particle from the surface, at point B, based on the constant stiffness,

$$y_B = \frac{F_a}{\chi_e} = 1.96\,\frac{\Delta\gamma^{2/3}R^{1/3}}{k^{2/3}}\,. \tag{5}$$

The result of Eq. (5) has the same form as the exact value of the displacement, found in [8]; however, the coefficient there is 3.3.

Let the particle perform streamwise oscillations according to the model presented in Fig. 1. We assume that the length of the spring is equal to the particle radius. The particle kinetic and potential energies are given, respectively, by

$$T = \frac{m\dot{x}^2}{2}, \quad U = \frac{\chi_e}{2}\left(\sqrt{x^2+R^2} - R\right)^2, \tag{6}$$

where m is the mass of the particle. Using the Lagrange equation of particle motion and the condition of small oscillations, $x \ll R$, which in this case is satisfied up to the separation point, one obtains the equation of particle motion in the form,

$$m\ddot{x} + \frac{\chi_e}{2R^2}\,x^3 = F_d\,. \tag{7}$$

Note that fluid damping which opposes particle motion and mechanical damping caused by the propagation of elastic waves in the solid substrate are negligible at low frequencies. The first inertial term in the equation is also negligible at low frequencies. Hence, the response is determined by the balance of the driving force with the restoring force, i.e.,

$$\frac{\chi_e}{2R^2}\,x^3 = F_d\,, \tag{8}$$

We assume that the maximum spring extension during oscillations should not exceed y_B. Hence, the maximum displacement in the x direction is determined by

$$\sqrt{x_B^2 + R^2} = y_B\,, \tag{9}$$

leading to

$$x_B \approx \sqrt{2Ry_B} = 1.98\,\frac{\Delta\gamma^{1/3}R^{2/3}}{k^{1/3}}\,. \tag{10}$$

For the special case $x = x_B$, the driving force F_d in Eq. (8) equals the tangential pull-off force $F_{a\tau}$, leading to

$$F_{a\tau} = \frac{\chi_e}{2R^2}\,x_B^3 = 9.3\,\frac{\Delta\gamma^{4/3}R^{2/3}}{k^{1/3}}\,. \tag{11}$$

This force is much smaller than the adhesion force F_a.

The ratio of this tangential pull-off force and the drag force, given by Eq. (2), may be represented in the form,

$$\frac{F_{a\tau}}{F_d} = 0.3\,\frac{\Delta\gamma^{4/3}}{R^{4/3}k^{1/3}\mu\dot{\gamma}}\,. \tag{12}$$

As seen, the condition for detachment $F_{a\tau}/F_d < 1$, obtained from the force balance in the nonlinear model, coincides with the condition of moment balance (3). This is a sufficient argument for the application of the instantaneous force balance (8) to evaluate the resuspension rate constant, which is defined as the probability per unit time of particle release from a surface, using the kinetic model outlined in [5]. In the case of a harmonic potential, Reeks, Reed, and Hall [5] obtained the equation for the resuspension rate constant p which, for a low-frequency response, reflects a force balance between the harmonic restoring force and the lift force. Note that by the condition for detachment, Eq. (12), particles leave the surface more easily, e.g., at lower flow velocities, than by the condition used in [5]. However, a straightforward substitution of the force ratio of Eq. (12) into the equation for p (see Eq. (38) in [5]), which corresponds to a harmonic potential, has no sufficient foundation for a nonharmonic potential describing nonlinear oscillations. For a general conservative potential well the equation suggested in [5] is

$$p = f_0 \exp\left(-\frac{Q}{2\langle PE\rangle}\right), \tag{13}$$

where f_0 is a typical frequency of vibration, Q the height of the potential well, and $\langle PE\rangle$ the average potential energy of a particle in the well. By definition, the potential energy in terms of the displacement is

$$U = \frac{\chi_e}{2R^2}\int_0^x x^3 dx = \frac{\chi_e}{2R^2}\frac{x^4}{4}. \tag{14}$$

It may be expressed in terms of the force F_d using the force-displacement relation, Eq. (8). Similarly, the height of the potential well, Q, in Eq. (13), may be found as a function of $F_{a\tau}$ carrying out the integration in Eq. (14) from 0 to x_B together with Eq. (11). Hence, Eq. (13) may now be rewritten as

$$p = f_0 \exp\left(-\frac{F_{a\tau}^{4/3}}{F_d^{4/3}}\right). \tag{15}$$

The turbulent drag force in (15) is determined from the mean value of the fluctuating component of the shear rate [2]

$$\langle\dot{\gamma}\rangle = 0.3\,\frac{u_\tau^2}{\nu}, \tag{16}$$

where u_τ is the friction velocity, and ν the kinematic viscosity.

We have limited the maximum value of p to the bursting frequency in a turbulent boundary layer [5], i.e., we have assumed that weakly bound particles have to encounter a burst before they resuspend. Hence, we use the same typical frequency as in [5]:

$$f_0 = \frac{u_\tau^2}{300\nu}. \tag{17}$$

Most surfaces involved in resuspension are rough. A small surface asperity may play the role of a particle, and the particle itself the role of a flat surface. The force of adhesion, F_a, and the tangential pull-off force, $F_{a\tau}$, come out to be smaller than those for a smooth surface, since they should be calculated using the asperity radius, r_a, rather than the particle radius, R. The surface topography can be characterized by a distribution in r_a. Following [5], we define

$$r_a' = \frac{r_a}{R} \tag{18}$$

and let $\varphi(r_a')$ be the probability density for the occurrence of r_a'.

The fraction of particles, $f_R(t)$, remaining on the surface at time t, is given by

$$f_R(t) = \int_0^\infty \exp[-p(r_a')t]\varphi(r_a')dr_a'. \tag{19}$$

Using this expression for $f_R(t)$, we investigate the behavior of the remaining fraction for a log-normal distribution of normalized adhesive radii, r_a'. Thus, $\varphi(r_a')$ is of the form given in [5]

$$\varphi(r_a') = \frac{1}{(2\pi)^{1/2}}\frac{1}{r_a'}\frac{1}{\ln\sigma_a'}\exp\left(-\frac{[\ln r_a'/\ln\bar{r}_a']^2}{2(\ln\sigma_a')^2}\right). \tag{20}$$

Here, $\bar{r}_a'$ is the geometric mean of r_a' and a measure of the reduction in adhesion due to surface roughness; σ_a' is a measure of the spread in adhesive forces. The typical values $\bar{r}_a' = 0.1$ and $\sigma_a' = 4$ are used in the calculations below. For illustration we consider the resuspension under the same conditions as in [5], namely, spherical glass particles on a stainless steel substrate exposed to a fully developed turbulent air flow in a channel. The relevant fluid and material properties are

$$\begin{gathered}\rho_p = 2470\ \mathrm{kg\,m^{-3}}, \quad \rho_f = 1.18\ \mathrm{kg\,m^{-3}}, \quad \rho_2 = 7.8\times10^3\ \mathrm{kg\,m^{-3}},\\ \nu = 1.54\times10^{-5}\ \mathrm{m^2\,s^{-1}}, \quad \Delta\gamma = 0.15\ \mathrm{J\,m^{-2}}, \quad E_1 = 8.01\times10^{10}\ \mathrm{Pa},\\ E_2 = 2.15\times10^{11}\ \mathrm{Pa}, \quad \nu_1 = 0.27, \quad \nu_2 = 0.28, \quad R = 20\ \mu\mathrm{m},\end{gathered} \tag{21}$$

where ρ_p, ρ_f, ρ_2 are particle, gas, and substrate densities, respectively.

Figure 2 shows the variation of f_R after a 1 second exposure as a function of the friction velocity. The transition of f_R from unit to effectively zero with increasing flow is not extremely sharp because of the influence of surface roughness, characterized by a reduction and spread in the force of adhesion. A comparison with [5] (see curve A in their Fig. 5) clearly demonstrates that the rate of resuspension resulting from the action of the drag force is much higher than the rate predicted there. For example, at a friction velocity $u_\tau = 2.19$ m/s the remaining fraction equals 0.08 in our calculations compared to 0.5 in the calculations of Reeks, Reed, and Hall [5]. Hence, the drag force is a more effective agent for the transfer of turbulent energy from the flow to a particle on a surface. This energy

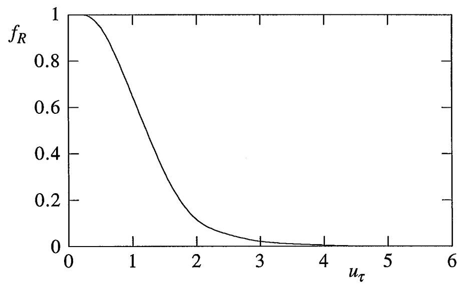

FIG. 2. The remaining fraction after a 1 second exposure as a function of the friction velocity.

transfer takes place at frequencies which are significantly lower than the resonant frequency of vibration, introduced in [5].

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