Dissipative Dynamics of Collisionless Nonlinear Alfvén Wave Trains

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The nonlinear dynamics of collisionless Alfvèn trains, including resonant particle effects, is studied using the kinetic nonlinear Schrödinger (KNLS) equation model. Numerical solutions of the KNLS reveal the dynamics of Alfvén waves to be sensitive to the sense of polarization as well as the angle of propagation with respect to the ambient magnetic field. The combined effects of both wave nonlinearity and Landau damping result in the evolutionary formation of *stationary S*- and arc-polarized directional and rotational discontinuities. These wave forms are frequently observed in the interplanetary plasma. [S0031-9007(97)03463-7]

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Numerous satellite observations of magnetic activity in the solar wind have exhibited the nonlinear nature of MHD waves [1,2]. Recent observations indicate the existence of directional (DD) and rotational (RD) discontinuities, i.e., regions of rapid phase jumps where the amplitude also varies [1,3], which are thought to be a result of the nonlinear development and evolution of MHD waves. Several types of RD/DD's which might be distinguished by their phase portraits have been observed. There are (i) discontinuities of the *"S-type,"* at which the magnetic field vector rotates first through an angle less than or approaching 90° in one direction, followed by rotation in the opposite direction through an angle larger than 180° (typically, $180° < \Delta\phi \leq 270°$) [3,4], and (ii) *arc-polarized* discontinuities, where the magnetic field vector rotates along an arc through an angle less than 180° [1,5]. At DD's, the fast phase jump is accompanied by moderate amplitude modulation ($\delta B \sim B$). At RD's, the amplitude modulation is small or negligible ($\Delta B \ll B$).

The envelope dynamics of nonlinear Alfvén waves at small β are thought to be governed by the derivative nonlinear Schrödinger (DNLS) equation, which describes parametric coupling with acoustic modes [6]. The theory of nondissipative Alfvén waves governed by the conservative DNLS equation predicts nonlinear wave steepening and formation of wave forms with steep fronts. Thus, spiky, many-soliton structures are emitted from the steep edge. It was shown that the nonlinear wave relaxes to a shock train and constant-B RD's where the field rotates through exactly 180° [6,7], when the linear damping due to finite plasma conductivity is taken into account. In spite of this, the DNLS theory was unable to explain the existence and dynamics of both (i) the S-polarized DD's and RD's and (ii) arc-polarized RD's, with rotation of less than 180°. It is believed (and confirmed by recent particle code simulations [8–10]) that the dynamics of Alfvén waves in the $\beta \sim 1$, isothermal solar wind plasma are intrinsically *dissipative*, on account of Landau damping of ion-acoustic oscillations [16,17]. We should comment here that particle code simulations (e.g., Refs. [8–10]) model "microscopic" behavior of plasma particle motions, thus may be referred to as "numerical experiments." A numerical solution of a "macroscopic" evolution equation is a complementary way which allows one to get a theoretical insight into the underlying physics and theoretically explain the observed (experimental) data.

The dynamics of magnetic fields in the solar wind has been extensively investigated using different analytical approaches. Various (e.g., beat, modulational, decay) instabilities of Alfvén waves were shown [11] to be sensitive to the wave polarization and the value of plasma β. The turbulence-based linear model which describes the radial evolution of magnetic fluctuations in the solar wind was successfully developed [12], taking into account the effects of advection and solar wind expansion, along with mixing effects. The simple nonlinear noisy-KNLS (kinetic nonlinear Schrödinger) model of turbulence was proposed and investigated in [13]. Arc-polarized waves, which had been first discussed in [14], were explained [15] via coupling of obliquely propagating circular Alfvén waves and a driven fast/slow wave. Some damping is necessary in this case in order to (consistently) select the arc-type solutions.

In this Letter, we use a recently developed [17] analytical model of the kinetic DNLS (KNLS) equation to investigate the influence of Landau damping on the (strongly) nonlinear dynamics of Alfvén waves. The main claim of this Letter is that *all* the discontinuous wave structures discussed above are *distinct solutions* of the same *simple* analytical model for different initial conditions, e.g., initial wave polarization and wave propagation angle. One should note that the term which describes the resonant particle effect is usually integral (nonlocal) in nature, on account of the finite ion transit time through the envelope modulation of an Alfvén train. Thus, the envelope evolution equation obtained is a nonlocal, integro-differential equation which is not amenable to *analytical* solution. We should comment here on a case which extends the traditional paradigm of shock wave forms. There are two known types of shocks, namely (i) *collisional* (hydrodynamic) *shocks*, in which nonlinear steepening is limited by *collisional* (viscous) dissipation, which sinks energy from small scales and (ii) *collisionless shocks* (common in astrophysical plasma), in which nonlinear steepening is limited by *dispersion*, resulting in the formation of soliton-type structures with energy content in high-k harmonics. We add a new class of shock, namely (iii) *dissipative structures* (which also can be referred to as *collisionless dissipative shocks*), for which nonlinear steepening is limited by *collisionless* (scale independent, i.e., acting on all scales) damping. They emerge only from quasiparallel, (nearly) linearly polarized waves. Of course, to obtain familiar shock-like wave forms, the cyclotron damping at large k must be incorporated, as usual, for collisionless shocks.

The KNLS equation may be written [16,17] as

$$\frac{\partial b}{\partial t} + \frac{v_A}{2}\frac{\partial}{\partial z}(\{M_1[|b|^2 - \langle |b|^2\rangle] + M_2\hat{\mathcal{H}}[|b|^2 - \langle |b|^2\rangle]\}b) + i\frac{v_A^2}{2\Omega_i}\frac{\partial^2 b}{\partial z^2} = 0, \tag{1}$$

where $b = (b_x + ib_y)/B_0$ is the wave magnetic field, v_A and Ω_i are the Alfvén speed and proton ion-cyclotron frequency, $\langle\cdots\rangle$ means average over space and fast (Alfvénic) time. Here the constants M_1 and M_2 depend on β only (see Ref. [17] for details) and $\hat{\mathcal{H}}$ is the integral Hilbert operator $\hat{\mathcal{H}}[f](z) = \pi^{-1}\int_{-\infty}^{\infty}(z' - z)^{-1}f(z')\,dz'$. This equation was solved for periodic boundary conditions using a predictor-corrector scheme and a fast Fourier transform; 1024 harmonics and spatial points were taken. The dimensionless spatial coordinate and time were introduced, respectively, as $\zeta = z/\bar{z}$ and $\tau = t_e/\bar{t}$, where $\bar{z} = 50c/\omega_p$ and $\bar{t} = 200/\Omega_i$. For $\beta = 0$, kinetics do not impact the wave dynamics [17], so Eq. (1) reduces to the familiar DNLS equation ($M_1 = 0.5, M_2 = 0$). The DNLS is integrable and has an exact (soliton) solution. The test run has shown an excellent agreement with the analytical solution during the time of computation (up to $\tau = 40$, i.e., 8000 cyclotron periods).

High-amplitude magnetic perturbations in plasmas typically evolve from small-amplitude (linear) ones. Thus the most general approach is to examine the nonlinear evolution of finite-amplitude periodic waves of different polarizations. The initial wave profiles are given by two initially excited Fourier harmonics, b_k. For linear polarizations, we pick b_k's equal $b_{-1} = b_1 = 1$, all others are zeroes, for circular polarizations we pick $b_{-2} = b_{-1} = 1$, for elliptical polarizations, we pick $b_{-1} = 1.1, b_1 = 0.9$. Thus, the waves are left-hand polarized. Results of a reference run for $\beta = 0$, with amplitude modulated linear polarization (the circular polarization and oblique propagation cases look similar) are shown in Fig. 1(a). The wave exhibits the nonlinear steepening

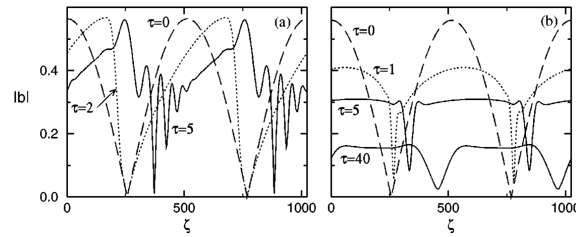

FIG. 1. Wave profile evolution of a quasiparallel, linearly polarized, sinusoidal wave initial condition for $\beta = 0$ (a) and $\beta = 1$ (b).

phase of a front at early times ($\tau \sim 2$). Dispersion further limits steepening and produces (at times $\tau \sim 5$) small-scale, oscillatory, circularly polarized wave structures (even for initial linear polarization). Significant high harmonic energy content is generated. At later times, $\tau \sim 40$ (not shown), nonlinear processes result in a wave magnetic field which is completely irregular, indicating strong, large-amplitude Alfvénic turbulence.

For $\beta \neq 0$, we first compare the wave form profiles and harmonic energy spectra obtained from the KNLS with the previous case. From now on, $\beta = 1$ and $T_e = T_i$ ($M_1 = 0.75, M_2 = -0.83$) unless stated otherwise. Figure 1(b) depicts the time evolution of a parallel propagating wave with the same initial conditions. In contrast to the $\beta = 0$ case, localized *quasistationary* structures are seen to form very rapidly; the formation time is $\tau_f \sim 2$. The harmonic spectrum of the dissipative structures (also referred to as S-type DD's, see below) is *narrow*, indicating that energy accumulates in low-k harmonics, i.e., at large scales. It worthwhile to emphasize the quasistationary character of such wave forms. They preserve their shape for thousands cyclotron periods and thus may be indentified as "structures". Meanwhile, the wave energy decays strongly, as seen from Fig. 2. Thus, the dissipative structures emerge via the competition of *nonlinear steepening* of the wave and scale invariant *collisionless damping*. The fact that energy dissipates *in* the dissipative structures (and not somewhere inbetween) is readily seen from the simple fact that $\hat{\mathcal{H}}[\text{const}] \equiv 0$.

Figure 3 is a snapshot of the dissipative structures at $\tau = 15$ which emerged from a quasiparallel, linearly

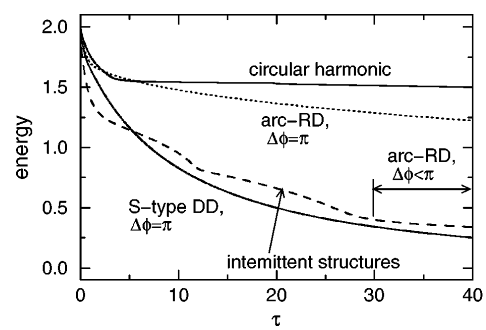

FIG. 2. Wave energy evolution for different initial conditions.

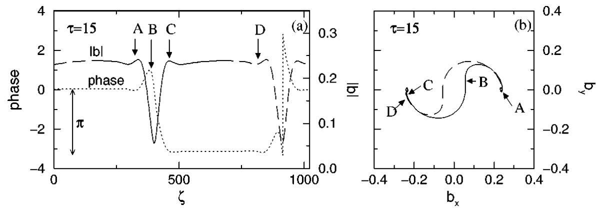

FIG. 3. *S*-polarized DD (quasiparallel case): (a) amplitude and phase profiles; (b) hodograph.

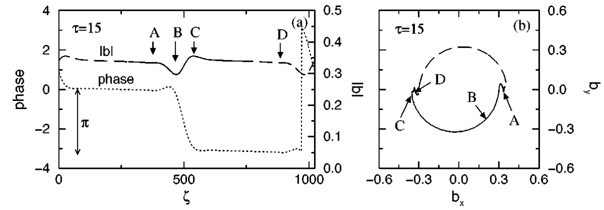

FIG. 4. Same as Fig. 3 for the $\Delta\phi = \pi$ RD (quasiparallel case).

polatized wave. It is seen that regions of significant field variations are accompanied by fast phase rotation. However, in the regions of negligibly varying $|b|$, linear polarization is preserved. The dissipative structures exhibit an easily distinguishable "*S*-shaped" phase portrait, namely that *at* the discontinuity (solid path *A*-*B*-*C*), the magnetic field vector completes a rotation through π radians. During the subsequent quescent region (path *C*-*D*), the magnetic field vector resides at the "tip" of the left arm, indicating pure linear polarization. At the next discontinuity, the vector returns to the initial position, similarly completing a π radian rotation as shown by the dashed path. Thus, the KNLS dissipative structures have the requisite properties of localized, *RD's/DD's*, as recently observed in the solar wind [3,4]. Note, however, these KNLS RD's are associated with the regions of *varying* $|b|$, unlike the conventional definition that $|b| =$ const across the RD. There is no sharp difference between an RD and a (weak) DD. One may be transformed into another by changing the initial wave polarization and propagation angle. Hence, we use both terms for the *S*-type KNLS discontinuity. We should also note the remarkable similarity of hodographs obtained by solution of the KNLS equation and from full numerical plasma simulations [8]. Such KNLS discontinuities occur commonly and are not restricted to β's close to unity. These structures are quite evident in a wide interval of β, of approximately 0.5–0.6 to 1.4–1.6. The dissipative structures still form at smaller M_2; however, the formation time increases when M_2 decreases.

In contrast to the case of linear polarization, circularly polarized, quasiparallel waves evolve in a few τ to a single (almost purely) circularly polarized harmonic at the lowest k and do not form discontinuities. Energy decay (Fig. 2) is negligible in the stationary state.

Figure 4 depicts a snapshot of the quasiparallel, initially elliptically polarized wave, an intermediate case between purely circular and linear polarizations. Sudden phase jumps (by π radians), which are localized at regions of varying wave amplitude (typical of linear polarizations), are easily seen. However, these discontinuities are not accompanied by wave amplitude discontinuities. Thus, they are the $\Delta\phi = \pi$ *RD's*. Note, these discontinuities [which are the semicircles in Fig. 4(b)] are separated by extended regions of linear polarization. Energy dissipation (Fig. 2) is weak, in comparison to the case of linear polarization.

Obliquely propagating waves are still described by the KNLS equation. However, a new wave field which (formally) contains a perpendicular projection component of the ambient magnetic field should be introduced. Assuming the ambient field lies in the *x*-*z* plane, we write the new field as $b = (b_x + B_0 \sin\Theta + ib_y)/B_0$. The nonlinear evolution of the linearly and highly elliptically polarized waves is strongly sensitive to the angle between the polarization plane and the plane defined by the ambient magnetic field vector and the direction of wave propagation. This angle is set by initial conditions. When this angle is small, the oscillating wave magnetic field has a longitudinal component along the ambient field. Thus, we refer such waves to as *longitudinal*. In the opposite case, the wave magnetic field oscillates (nearly) perpendicularly to the ambient field. Thus, such waves are called *transverse*. Note that this classification scheme fails for circularly polarized waves, since a polarization plane cannot be defined in this case.

Figure 5 shows a typical (quasi-) stationary, *arc-polarized* discontinuity which evolved from an obliquely propagating ($\Theta \sim 40°$), amplitude modulated, circularly polarized wave at $\tau = 40$. The discontinuity is associated with minor (almost negligible) amplitude modulation. The magnetic field vector makes a fast clockwise rotation through less than π radians (solid path *A*-*B*-*C*). The ends *A* and *C* are connected by a sector of circularly polarized wave packet (slow counterclockwise rotation in the phase diagram, along the dashed, perfect arc *C*-*D*-*A*). Circular polarization is also indicated by the smoothly decreasing phase outside the discontinuity [Fig. 5(a)]. Since $|b|^2 \simeq$ const across the discontinuity (as well as for a pure circularly polarized harmonic), it is nearly decoupled from dissipation. Note the remarkable similarity of this solution of the KNLS equation to the structures detected in the solar wind and observed in computer simulations [5,10].

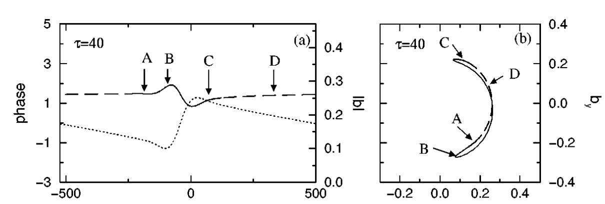

FIG. 5. Same as Fig. 3 for the $\Delta\phi < \pi$ RD (oblique case).

The wave evolution of the linearly polarized, obliquely propagating, transverse and longitudinal waves differs drastically. The transverse waves evolve very quickly (in a few τ) to form a perfect arc-polarized RD. Energy dissipation is negligibly small in this process. The longitudinal waves instead form two S-type DD's propagating with different group velocities. Thus, they can merge and annihilate each other almost completely, yielding small-amplitude, residual magnetic perturbations.

The sharp contrast between these three quasistationary solutions is a direct consequence of the unique harmonic scaling of collisionless (Landau) dissipation in the KNLS equation. It is crucial to understand that *collisionless damping enters at all* k, in contrast to hydrodynamic systems where diffusion (viscosity) yields dissipation only at large k (i.e., small scales, or steep gradient regions). However, higher-k harmonics are strongly damped, which is typical of a phase-mixing process (i.e., smaller scales mix faster). For quasiparallel propagating waves, Landau damping enters symmetrically for $+k$ and $-k$ spectrum components. It does not change the symmetry of a spectrum, so that the initial helicity (set by initial spectrum symmetry) is preserved. Since linear polarizations have spectra symmetric upon $k \longrightarrow -k$, they couple more strongly to dissipation than circular polarizations do. Thus, S-polarized discontinuities, which consist of predominantly two low-k harmonics of nearly equal amplitude, emerge. For the circularly polarized wave, the (initial) spectrum is highly asymmetric. Thus, such a wave evolves to a single harmonic final state, which is, itself, a stationary (and exact) solution of the KNLS equation (i.e., it experiences no steepening and minimal damping). No discontinuities emerge in this case. For the oblique and quasiperpendicular cases, there is asymmetry between b_x and b_y components induced by the ambient magnetic field. This allows the formation of a wave packet with nearly constant $|b|^2$ (i.e., decoupled from dissipation). Such wave packets are the arc-polarized RD's with $\Delta\phi < \pi$. We should emphasize the fact that since there is no characteristic dissipation scale in the system, the ultimate scale of the dissipative structures is set by *dispersion*, alone (*a là* collisionless solitons and shock waves). Accordingly, one can suggest that (given initial equal populations of isotropically distributed circular and linear polarizations) quasiparallel magnetic field fluctuations will consist of predominantly circularly polarized waves and lower amplitude S-polarized KNLS DD's, while oblique perturbations are predominantly arc-polarized discontinuities, separated by pieces of oppositely circularly polarized waves.

To conclude, the influence of the effect of Landau damping was investigated in this Letter. (A more complete study will be published elsewhere [18].) A tractable analytic evolution equation, the KNLS equation, was numerically solved to study nonlinear dynamics of finite-amplitude coherent Alfvénic trains in a $\beta \sim 1$, isothermal plasma, natural to the solar wind. Current studies shows that *all* the discontinuous wave structures observed in the solar wind are *distinct solutions* of the same *simple* analytical KNLS model for different initial conditions, e.g., initial wave polarization and wave propagation angle with no *a priori* assumptions or special initial conditions used.

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