Parity Nonconservation in Relativistic Hydrogenic Ions

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Possibilities of precision weak-interaction measurements in relativistic hydrogenic ions are considered. It is shown that using high-energy ion storage rings (RHIC, SPS, and LHC), and utilizing relativistic Doppler tuning and laser cooling, it is possible to achieve sensitivity necessary for testing physics beyond the standard model. [S0031-9007(97)03437-6]

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Hydrogenic systems are attractive objects for precision measurements of parity nonconservation (PNC) since their simple atomic structure allows for accurate theoretical calculations necessary for relating the observed effects to the standard model. Various possibilities for PNC experiments with hydrogenic ions have been discussed in the literature. Most of the early work dealt with the PNC contribution to the $2S \rightarrow 1S + 1\gamma$ spontaneous decay [1,2] (an experimental upper limit on PNC mixing in hydrogenlike argon was obtained in [3]), while PNC in transitions between metastable states [4,5] was discussed more recently. Unfortunately, none of the possibilities discussed in these papers seem to offer realistic ways to observe PNC effects, much less to achieve the high precision desirable today.

Here, we show that high precision PNC experiments with ions become possible, in view of recent developments in relativistic ion colliders, high-brightness ion sources, laser cooling methods of ions in storage rings, etc. In particular, PNC experiments in relativistic hydrogenic ions involving laser-induced $1S + 1\gamma \rightarrow 2S$ transitions can be carried out using the heavy ion accelerators RHIC, SPS, and LHC. Relativistic Doppler tuning is used to tune laser light (photon energy $\sim$ a few eV) in resonance with the ion transition ($\sim$keV in the ion rest frame) also allowing laser cooling, and polarization of ions by optical pumping. It is estimated that the PNC effect can be measured with statistical uncertainty $\sim 10^{-3}$ in about a week of running time. At this level, these measurements provide a quantitative test of the standard model sensitive to its various possible extensions [6].

We consider the conceptually simplest variant of a PNC experiment in a hydrogenic system: circular dichroism on the $1S \rightarrow 2S$ transition in the absence of external electric and magnetic fields. Dichroism arises due to interference between the $M1$ and the PNC-induced $E1$ amplitudes of the transition. This approach allows one to estimate the required ion beam and laser parameters, and the achievable statistical sensitivity. In practice, it will be necessary to use one of the schemes involving ion polarization and external electric and magnetic fields [2]. These schemes do not change statistical sensitivity in an idealized experiment. In practice, however, they often allow one to reduce the influence of technical noise. They also allow for reversals providing efficient control of systematic effects. A detailed discussion of possible interference schemes will be given elsewhere.

Because of the PNC interaction, the $2S$ state acquires an admixture of the $2P_{1/2}$ state. The magnitude of the PNC admixture is probed by tuning the laser in resonance with the highly forbidden $1S \rightarrow 2S$ $M1$ transition and observing circular dichroism, i.e., the difference in transition rates for right- and left-circularly polarized light. In order to consider PNC experiments with hydrogenic ions, it is convenient to trace Z dependences of various atomic parameters. For convenience of reference, approximate formulas for these parameters including Z dependences are collected in Table I. A more detailed discussion can be found, e.g., in [1,2,7].

Consider an ion with relativistic factor $\gamma = 1/\sqrt{1-\beta^2} \gg 1$ colliding head-on with a photon of frequency $\omega_{\rm lab}$ in the laboratory frame. In the ion rest frame, the frequency is given by $\omega_{\text{ion-frame}} = \gamma(1+\beta)\omega_{\rm lab} \approx 2\gamma\omega_{\rm lab}$. In order to tune to the $1S \rightarrow 2S$ resonance for a hydrogenic ion, it is necessary to satisfy the condition $\Delta E_{2S\text{-}2P} \approx Z^2 \times 10.2\ \text{eV} = 2\gamma\hbar\omega_{\rm lab}$. The relevant parameters of the storage rings RHIC [8], SPS [9], and LHC [9] are collected in Table II. With visible and near-UV lasers, it is possible to access $1S \rightarrow 2S$ transitions for ions with Z up to ≈ 11 (Na) at RHIC. For LHC, the corresponding Z range is up to $Z \approx 48$ (Cd). In the following, we use RHIC parameters for numerical estimates.

Consider circularly polarized laser light tuned in resonance with $1S \rightarrow 2S$ transition of an ion. The width of this transition is dominated by the Doppler width (Γ_D) due to the ion energy spread,

$$\Gamma_D = (\omega\Delta\beta)_{\text{ion-frame}} \approx \omega_{\text{ion-frame}}\frac{\Delta\gamma}{\gamma}. \qquad (1)$$

The fraction of ions excited from the ground state to $2S$ is given by

$$\chi_{M1} = (M1\,\tilde{B}\tau)^2\frac{1}{\Gamma_D\tau}. \qquad (2)$$

Here $\tilde{B} = \tilde{E}$ is the laser field, τ is the ion-laser interaction time. Here and below, we write all quantities in the ion

TABLE I. Z dependence of atomic characteristics for hydrogenic ions. In the given expressions, α is the fine structure constant, $\hbar = c = 1$, m_e is the electron mass, G_F is the Fermi constant, θ_w is the Weinberg angle, and A is the ion mass number.

Parameter	Symbol	Approximate Expression
Transition energy	$\Delta E_{n-n'}$	$\frac{1}{2}(\frac{1}{n^2} - \frac{1}{n'^2})\alpha^2 m_e Z^2$
Lamb shift	$\Delta E_{2S\text{-}2P}$	$\frac{1}{6\pi}\alpha^5 m_e Z^4 F(Z)$ [a]
Weak interaction Hamiltonian	H_w	$i\sqrt{\frac{3}{2}}\,\frac{G_F m_e^3 \alpha^4}{64\pi}\{(1 - 4\sin^2\theta_w) - \frac{(A-Z)}{Z}\}Z^5$
Electric dipole amplitude ($2S \rightarrow 2P_{1/2}$)	$E1_{2S\rightarrow 2P}$	$\sqrt{\frac{3}{\alpha}}\, m_e^{-1} Z^{-1}$
Electric dipole amplitude ($1S \rightarrow 2P_{1/2}$)	$E1$	$\frac{2^7}{3^5}\sqrt{\frac{2}{3\alpha}}\, m_e^{-1} Z^{-1}$
Forbidden magn. dipole ampl. ($1S \rightarrow 2S$)	$M1$	$\frac{2^{5/2}\alpha^{5/2}}{3^4}\, m_e^{-1} Z^2$
Radiative width	Γ_{2P}	$(\frac{2}{3})^8 \alpha^5 m_e Z^4$

[a]The function $F(Z)$ is tabulated in [1]. Some representative values are $F(1) = 7.7$; $F(5) = 4.8$, $F(10) = 3.8$; $F(40) = 1.5$.

bunch center of mass frame, unless specified otherwise. In this expression, the first term gives the transition probability for a group of ions in resonance with the laser field; the width of this group in the energy distribution is given by $1/\tau$. The second term gives the fraction ($\ll 1$) of ions in the resonant group compared to the entire energy distribution. In the ongoing, we also assume that the laser spectral width is no greater than $1/\tau$.

From the point of view of sensitivity to PNC, it is desirable to have as many $1S \rightarrow 2S$ events as possible; thus, one desires to use as high as possible laser power. However, there is an upper limit on laser power. In addition to the $1S \rightarrow 2S$ transition, there is off-resonant excitation of the $1S \rightarrow 2P_{1/2}$ transition, which leads to optical pumping into the $1S$ Zeeman component decoupled from the laser light (for simplicity, we consider an ion with zero nuclear spin). The optical pumping saturation parameter (for $\Gamma_D \ll \Delta E_{2S\text{-}2P}$) is given by

$$\chi_{E1} = \frac{(E1\,\tilde{E})^2}{4(\Delta E_{2S\text{-}2P})^2}\,\Gamma_{2P}\tau\,. \tag{3}$$

In order to avoid significant bleaching due to optical pumping, it is necessary to have $\chi_{E1} \lesssim 1$. Later we will see that one actually has to operate at a somewhat lower saturation parameter in order to avoid ion loss due to photoionization. From (2) and (3) and expressions from Table I, we have

$$\chi_{M1} = \frac{3^8 \alpha^9 F^2(Z) Z^8}{2^{15}\pi^2(\Delta\gamma/\gamma)}\,\chi_{E1}\,. \tag{4}$$

The total number of ions excited into the $2S$ state by light of a given circular polarization during the total time of experiment T (assuming the polarization is in each state half of the time) is $N_\pm \approx \chi_{M1}\dot{N}_{\mathrm{ions}}T/2$. Here $\dot{N}_{\mathrm{ions}}$ is the average number of ions entering the interaction region per unit time. The PNC effect is given by the asymmetry,

$$P = \frac{N_+ - N_-}{N_+ + N_-} = \frac{2H_w}{\Delta E_{2S\text{-}2P}}\,\frac{E1}{M1} \tag{5}$$

(e.g., for $Z = 10$, $P \approx 4 \times 10^{-6}$). This corresponds to a statistical uncertainty,

$$\delta H_w = \frac{1}{4}\sqrt{\frac{\Gamma_D \Gamma_{2P}}{\dot{N}_{\mathrm{ions}} T \chi_{E1}}}\,. \tag{6}$$

This shows that for an optimally designed PNC experiment, the statistical sensitivity is completely determined

TABLE II. Parameters of relativistic ion storage rings.

Parameter	RHIC	SPS	LHC
$\gamma_{\max}$ for protons [a]	250	450	7000
Number of ions/ring [b]	$\sim 5 \times 10^{11}$	$\sim 2 \times 10^{11}$	$\sim 5 \times 10^{10}$
Number of bunches/ring	57	128	500–800
R.m.s. bunch length	84 cm	13 cm	7.5 cm
Circumference	3.8 km	6.9 km	26.7 km
Energy spread without laser cooling	2×10^{-4}	4.5×10^{-4}	2×10^{-4}
Normalized emittance (N.E.)	$\approx 4\ \pi\ \mu$m rad	$\approx 4\ \pi\ \mu$m rad	$\approx 4\ \pi\ \mu$m rad
Dipole field	3.5 T	1.5 T	8.4 T
Vacuum, cold	$<10^{-11}$ Torr (H_2, He)	$\cdots$	$<10^{-11}$ Torr (H_2, He)

[a]For hydrogenic ions, $\gamma_{\max}^{\mathrm{ions}} = \gamma_{\max}^{p}(Z - 1)/A$.
[b]Estimated from proton and heavy ion data.

by the total number of available ions and by the transition widths. Combining Eqs. (6) and (1) and the expressions in Table I, one finds that in order to obtain a certain sensitivity to $\sin^2\theta_w$, it is necessary to have exposure $\dot{N}T$ which is conveniently represented in units of particle ampere year,

$$\text{Exposure[part A yr]} \geq \frac{\Delta\gamma}{\gamma}\,\frac{0.1}{Z^4(\delta\sin^2\theta_w)^2\chi_{E1}}\,. \quad (7)$$

In order to proceed with the numerical estimate of the necessary exposure, we now briefly consider processes specific to nonbare ions leading to reduction of ion lifetime in a storage ring, and discuss the effect of these processes on the proposed PNC experiments.

Residual gas ionization.—For relativistic ions, the stripping cross section in a collision with a residual gas atom is given by [10]

$$\sigma = 4\pi\alpha^2 a_B^2\frac{Z_a(Z_a+1)}{Z^2}\,. \quad (8)$$

Here a_B is the Bohr radius, Z_a and Z are atomic numbers of the residual gas atom, and the ion, respectively. Estimates based on Eq. (8) show that with residual gas pressure as given in Table II, ion lifetime limited by this process is $\approx\frac{1}{2}$ h for $Z = 10$.

Laser-induced photoionization.—Ions excited to the virtual $2P_{1/2}$ state can absorb another photon from the laser beam and photoionize. The corresponding photoionization cross sections are given in [11]. For the photon energy corresponding to the $1S \to 2S$ resonance, we have

$$\sigma_{2P}^{\text{ioniz}} \approx 1.4\times10^{-2}\frac{a_B^2}{Z^2}\,. \quad (9)$$

Using this cross section, Eq. (3), and the formulas in Table I, one estimates the photoionization probability in one laser-pulse interaction,

$$W \approx 1.2\times10^3\frac{a_B}{z_R}\,\frac{\gamma F^2(Z)}{Z^2}\,\chi_{E1}^2\,. \quad (10)$$

Here we assume that the laser and the ion beam profiles are matched, and the Rayleigh range for the optical beam is z_R. [Since the ion beam emittance (see Table II) is less than that of a laser beam: $\varepsilon_{\text{tr}} = (\text{N.E.})/\gamma < \lambda_{\text{laser}}/4\pi$, the relevant Gaussian beam parameters are determined by the optical beam.] Using the RHIC parameters, and choosing $z_R = 1$ m, $Z = 10$, and requiring the lifetime due to photoionization to be 1 h, one finds the necessary value of the saturation parameter: $\chi_{E1} \approx 6\times10^{-2}$. Note that photoionization losses due to population of the $2S$ state rapidly scale with Z ($\propto Z^{10}$ relative to photoionization from $2P$), and may become important for $Z \approx 40$. It may also become important in a Stark-PNC interference scheme if a sufficiently strong electric field is applied increasing the $1S \to 2S$ transition rate.

Let us now return to the estimate of the necessary exposure in a PNC experiment [Eq. (7)]. As an example, for $\delta\sin^2\theta_w = 10^{-3}$, using Ne ions ($Z = 10$) in RHIC, and substituting $\chi_{E1} \approx 6\times10^{-2}$, one obtains the necessary running time $\sim$1 week. In this estimate we assumed $\Delta\gamma/\gamma = 10^{-6}$, which is possible to achieve using laser cooling [12] as we discuss below.

Let us now estimate the laser power required to maintain a given value of χ_{E1} [see Eq. (3)]. We first assume that the laser operates in pulsed mode with repetition rate matched to the rate of arrival of ion bunches into the interaction region. The beam cross section at the waste is $2\pi\sigma^2 = \lambda_{\text{laser}}z_R/2$. Using the formulas in Table I, one obtains an estimate for the required number of photons in a laser pulse,

$$N_{\text{phot}} = \frac{\tilde{E}^2 2\pi\sigma^2 2\tau}{8\pi\Delta E_{1S\text{-}2P}} \approx 10^2F^2(Z)\gamma^2\frac{z_R}{\lambda_{\text{laser}}}\,\chi_{E1}\,. \quad (11)$$

For $\gamma = 100$, $Z = 10$, $z_R = 1$ m, and $\chi_{E1} = 6\times10^{-2}$, we have $N_{\text{phot}} \approx 1.2\times10^{12}$. The number of scattered laser photons is equal to the number of ions in a bunch $\times\chi_{E1}$ ($\approx10^{10}\times\chi_{E1}$ for RHIC). This corresponds to light pulse energy $\approx$1.2 μJ, absorbed light energy $\approx$0.6 nJ, and the average absorbed light power $\approx$3 mW (57 bunches $\times$ 76 kHz). Laser parameters necessary for the proposed experiments are achievable with present technology.

In hydrogen experiments, atoms in the $2S$ state are conveniently detected by applying electric field which mixes $2S$ and $2P_{1/2}$, and observing the resulting fluorescence. An alternative to the dc electric field is laser excitation to one of the higher-lying P states. Unfortunately, it becomes increasingly difficult to use these techniques for the proposed experiment when the ion beam is ultrarelativistic. Indeed, essentially all fluorescence photons are emitted in the forward direction in a narrow angle $\theta\sim1/\gamma$. It is difficult to discriminate these photons from the (orders of magnitude more numerous) laser photons backscattered due to the $1S \to 2P$ resonance, since all photons arrive at a detector within a narrow time interval. It is also impossible to turn ions with a bending magnet by an angle $\sim1/\gamma$, because ions in the $2S$ state quench by the electric field arising in the ion's frame before the rotation is accomplished.

A detection scheme involving $2S$ quenching can be realized for relatively low γ. For example, in the case of RHIC where $\gamma\approx100$ and the length of a straight section is 18 m, the experimental arrangement could be the following. The ion-laser interaction region $\sim$1 m long is arranged in the beginning of the straight section. About 10 m downstream from the interaction region, the ion beam passes through a collimator $\sim$1 cm in diameter. This collimator serves to absorb the backscattered laser photons traveling along the beam axis in an angle greater than $\sim10^{-3}\ll1/\gamma$. The $2S$-quenching region is located after the collimator; a position-sensitive x-ray detector located several meters from the quenching region detects photons resulting from quenching with nearly 100% efficiency (the remaining backscattered photons in the narrow angle $\sim 10^{-3}$ can be used for alignment and diagnostics).

Another commonly used technique for detection of ions in metastable states is photoionization. In this case, however, photoionization will lead to loss of ions from the storage ring, leading to reduced beam lifetime and loss of statistical sensitivity.

A possible way to detect ions in the $2S$ state is by measuring absorption of a laser beam tuned to a $2S \rightarrow nP$ transition ($n \geq 3$). Detection of small absorption can be carried by arranging a multipass system, or a resonant cavity for the laser beam. The necessary number of passes can be estimated from the condition that the total absorption is ~ 1. This straightforward calculation shows that if one uses an optical cavity with number of passes $\sim 10^4$, in order to avoid loss in statistical sensitivity, it will be necessary to apply a Stark field, so the Stark-induced $1S \rightarrow 2S$ transition amplitude significantly exceeds the $M1$ amplitude.

In the estimates above we assumed $\Delta\gamma/\gamma = 10^{-6}$. This approximately corresponds to the Doppler limit for RHIC (assuming that the $1S \rightarrow 2P$ transition is used for cooling),

$$\left(\frac{\Delta\gamma}{\gamma}\right)_D = \sqrt{\frac{\Gamma_{2P}}{M_{\rm ion}}} \approx 4 \times 10^{-7}, \quad \text{for } Z = 10. \tag{12}$$

In principle, laser cooling rates can be very fast [12]. However, similar to the situation discussed above, the laser power has to be low enough to avoid excessive ion losses due to photoionization from the $2P$ state. On the other hand, the cooling rate has to be high enough to compensate the heating effect of the intrabeam scattering. The cooling rate per turn can be estimated as

$$\frac{1}{(\Delta\gamma/\gamma)^2}\frac{d(\Delta\gamma/\gamma)^2}{dn} = -2\chi_{\rm cooling}\frac{\hbar\omega_{\rm laser}}{Mc^2}\frac{\Gamma_{2P}z_R}{c}\frac{\gamma}{\Delta\gamma}, \tag{13}$$

where

$$\chi_{\rm cooling} = \frac{(E1\,\tilde{E})^2}{\Gamma_{2P}^2}. \tag{14}$$

The heating rate due to intrabeam scattering can be estimated as (a detailed discussion of intrabeam scattering can be found, e.g., in [13])

$$\frac{1}{(\Delta\gamma/\gamma)^2}\frac{d(\Delta\gamma/\gamma)^2}{dn} \approx \frac{N_{\rm ions}r_{\rm ion}^2 C}{\gamma^3\varepsilon_{\rm tr}^{3/2}\sqrt{\beta^*}\,l}\left(\frac{\gamma}{\Delta\gamma}\right)^2. \tag{15}$$

where $r_{\rm ion} = Z^2e^2/Mc^2$ is the classical radius of an ion, C is the storage ring circumference. For RHIC and $\Delta\gamma/\gamma = 10^{-6}$, from Eq. (15), one estimates that without cooling $(\Delta\gamma/\gamma)^2$ would double in about 3000 turns. In steady state conditions, the cooling rate (13) should be equal to the heating rate (15). This gives the minimum required value of the cooling saturation parameter $\chi_{\rm cooling} \approx 10^{-4}$. This corresponds to a significantly lower laser power than for the PNC measurement. Ionization losses due to the cooling process can be estimated as above; they correspond to ion lifetime ≈ 30 h. Therefore, it appears possible to achieve the necessary energy spread.

We have shown that it is feasible to perform sensitive parity-violation measurements in relativistic ions. These experiments become possible due to modern developments in relativistic ion storage rings, lasers, and laser cooling. Of particular importance for these experiments are high ion currents, long ion lifetime in a storage ring, small emittance, and energy spread. High ion energies make it possible to tune visible and near-UV lasers in resonance with transitions of interest. The proposed technique has sensitivity sufficient for testing physics beyond the standard model. Further development in ion injectors will lead to higher stored currents which may allow even more sensitive PNC experiments in the future.

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