# $K^\pi = 0^+$ and $4^+$ Two-Phonon $\gamma$-Vibrational States in $^{166}$Er

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The $(n, n'\gamma)$ reaction has been used to search for two-phonon $\gamma$-vibrational states in $^{166}$Er. Levels at 1943 and 2028 keV have been observed with collective transitions to the $\gamma$ band. The latter, which has $B(E2; 2028 \rightarrow 2_\gamma^+) = 7.4 \pm 2.5$ W.u. (Weisskopf unit) is interpreted as the $I, K^\pi = 4, 4^+$ two-$\gamma$-phonon state. The former, for which the data indicate spin 0, has $B(E2; 1943 \rightarrow 2_\gamma^+) = 21 \pm 6$ W.u. and is interpreted as the $I, K^\pi = 0, 0^+$ two-$\gamma$-phonon state. This is the first observation of the $K^\pi = 0^+$ two-$\gamma$-phonon state in a well-deformed nucleus, and its identification places stringent limits on the nuclear models. [S0031-9007(97)03409-1]

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The existence of two-phonon states in deformed nuclei has been the subject of considerable debate for over 30 years. The recent measurement [1] of enhanced $E2$ transitions from a level at 2055 keV in $^{168}$Er, interpreted as the $K^\pi = 4^+$ two-phonon $\gamma$ vibration ($4_{\gamma\gamma}^+$), has not silenced the controversy which is now centered on the magnitude of the two-$\gamma$-phonon component in the wave function needed to reproduce the enhanced $E2$ rate and the extent to which these states appear in other nuclei [2–7]. Following the discovery of the $4_{\gamma\gamma}^+$ state in $^{168}$Er, attempts [2] were made to locate candidates for these states in other nuclei, mainly by considering branching ratios and energy systematics. For many of these candidates, Burke [3] has argued that data exist which rule out the possibility that the main components in their wave functions are of two-phonon character. This demonstrated the need for absolute $B(E2)$-value measurements and for considering all data before assigning levels as two-phonon states.

Certain models, such as the self-consistent collective-coordinate model (SCCM) [8], the multiphonon method (MPM) [9], the dynamic-deformation model (DDM) [10], etc., predict that states with properties of $4_{\gamma\gamma}^+$ states should be widespread in the well-deformed rare-earth region. The quasiparticle-phonon nuclear model (QPNM) [4], on the other hand, predicts that $4_{\gamma\gamma}^+$ states should exist in a few special cases only, such as $^{164}$Dy, $^{166}$Er, and $^{168}$Er. This limited set of nuclei arises from the behavior of the density of levels in the vicinity of the $4_{\gamma\gamma}^+$ states; these nuclei are predicted as being the only ones for which the density is sufficiently low that the two-$\gamma$-phonon states are not greatly fragmented.

A feature common to many models is that they predict a relatively pure $K^\pi = 0^+$ two-phonon $\gamma$ vibration ($0_{\gamma\gamma}^+$) should not exist. Bohr and Mottelson [11] suggested that for $^{168}$Er these states should lie above the $4_{\gamma\gamma}^+$ excitation, in the vicinity of 2.5 MeV. In a more detailed treatment, Dumitrescu and Hamamoto [12] found that the positions of the two-$\gamma$-phonon states depended on the anharmonicities introduced into the Hamiltonian. In particular, by introducing a $\gamma$-dependent contribution to the moment of inertia, the $0_{\gamma\gamma}^+$ excitation could lie lower than the $4_{\gamma\gamma}^+$ state. As well, the introduction of a $\gamma$-unstable shape could produce nearly degenerate $0_{\gamma\gamma}^+$ and $4_{\gamma\gamma}^+$ states at an energy of $\simeq$2.5 times that of the $\gamma$ vibration ($2_\gamma^+$). The SCCM predicts [8] that the energy of the $0_{\gamma\gamma}^+$ excitation is strongly dependent on the quadrupole-quadrupole strength $\kappa$, whereas the $4_{\gamma\gamma}^+$ state has a weaker dependence. Therefore, for appropriate values of $\kappa$, the two may approach a degenerate condition. The QPNM predicts [4] that while the $4_{\gamma\gamma}^+$ state may exist in a few nuclei, the $0_{\gamma\gamma}^+$ state lies at energies $\gtrsim$2.5 MeV and will be strongly fragmented. Similarly, the MPM predicts [9] that the $0_{\gamma\gamma}^+$ state is very anharmonic, with a ratio $E(0_{\gamma\gamma}^+)/E(2_\gamma^+) \sim 3.5$, while $E(4_{\gamma\gamma}^+)/E(2_\gamma^+) \sim 2.8$. As a consequence of the predicted high excitation energy of the $0_{\gamma\gamma}^+$ state, this excitation may lose its collective character due to mixing with the large level density of other, noncollective excitations [9]. The DDM [10] indicates that the *lowest* lying $0^+$ states in $^{164-168}$Er are two-$\gamma$ phonon in nature. This possibility was also suggested recently by Casten and von Brentano [13] based on $sd$-IBM (interacting boson model) calculations, but was challenged by Burke and Sood [14] and by Günther *et al.* [15]. Therefore, evidence of the existence of the $0_{\gamma\gamma}^+$ state and its location can yield important information, not only on the values of various parameters needed for models, but also as a much more sensitive test of different model approaches than the $4_{\gamma\gamma}^+$ state.

In order to search for possible two-$\gamma$-phonon states, a series of $(n, n'\gamma)$ measurements was performed at the University of Kentucky Van de Graaff facility. Nearly monoenergetic neutrons from the $^3$H$(p, n)^3$He reaction bombarded an 82 g sample of $Er_2O_3$ enriched to 98% in $^{166}$Er. Gamma-ray excitation functions with neutron energies from $E_n = 1.4$ to 3.1 MeV, angular distributions at $E_n = 2.1$ and 2.5 MeV, and $\gamma\gamma$ coincidence measurements with collimated 3.2 MeV neutrons were performed. In the singles measurements, the $\gamma$ rays were detected with an HPGe detector with 57% relative efficiency and a resolution of 2.1 keV FWHM at 1332 keV. The in-beam energy calibrations were continuously monitored through the use of a $^{60}$Co source whose $\gamma$ rays were recorded simultaneously with those produced by the $(n, n'\gamma)$ reaction.

  

In addition to spectroscopic information, the angular distributions can be used to deduce level lifetimes via the Doppler-shift attenuation method (DSAM). The energy of a $\gamma$ ray emitted by a nucleus recoiling in a stopping medium is given by

$$E_\gamma(\theta_\gamma) = E_0[1 + \beta F(\tau)\cos\theta_\gamma], \quad (1)$$

where $E_\gamma(\theta_\gamma)$ is the energy of the $\gamma$ ray observed at an angle $\theta_\gamma$ with respect to the recoil direction (taken to be the direction of the incident neutrons), $E_0$ is the unshifted $\gamma$-ray energy, and $\beta$ is the recoil velocity in units of $c$. The extracted experimental attenuation factor $F(\tau)$ can be compared with the $F(\tau)$ calculated using the formalism of Ref. [16], from which the level lifetime can be determined.

Since a $K^\pi = 4^+$ band head is expected (from the Alaga rules) to decay to both the spin 2 and 3 members of the $\gamma$ band, transitions which had strong coincidences with $\gamma$ rays from the $2^+_\gamma$ and $3^+_\gamma$ levels and differed by 73 keV (the $3^+_\gamma - 2^+_\gamma$ energy difference) were identified. Shown in Fig. 1 are portions of the coincidence spectra with gates set on transitions decaying from the $2^+_\gamma$ and $3^+_\gamma$ levels. The 1119 and 1192 keV transitions originate from the level at 1978 keV, which was suggested [2] to be the $4^+_{\gamma\gamma}$ state. Recent Coulomb excitation experiments [17] suggested that a level at 2028 keV had a large component of the two-phonon $\gamma$ vibration. The present work confirms that this level decays by the 1169 and 1242 keV transitions. Other possible candidates were ruled out due to observed strong transitions to the ground state band, previously known spin/parity assignments, or the apparent noncollective nature of the transitions. The 1157 keV transition will be discussed below.

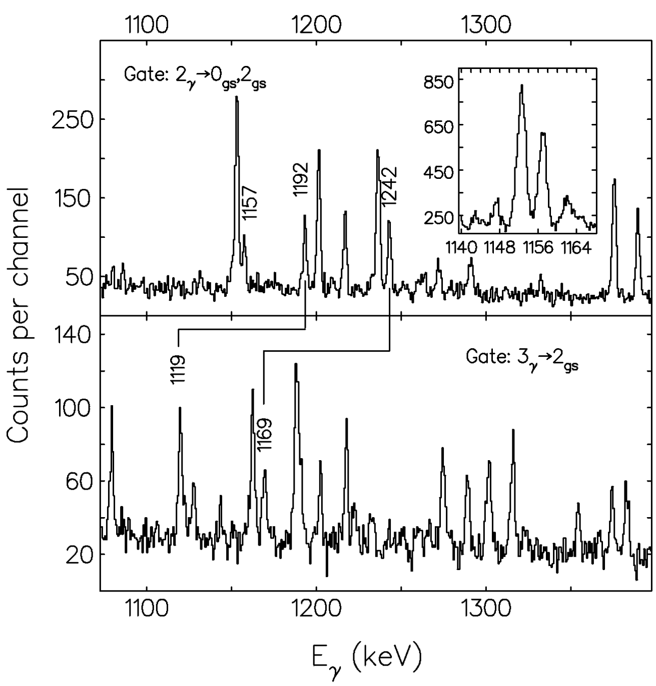


FIG. 1. Selected coincidence spectra from the $^{166}$Er$(n, n'\gamma\gamma)$ reaction performed with 3.2 MeV neutrons. The upper part shows coincidences with the 705 ($2^+_\gamma \to 2^+_{\rm gsb}$) and 786 keV ($2^+_\gamma \to 0^+_{\rm gs}$) transitions, while the lower part displays coincidences with the 779 keV ($3^+_\gamma \to 2^+_{\rm gsb}$) transition. The inset displays the portion of the spectrum around the 1157 keV transition obtained during the angular distribution experiment with 2.1 MeV neutrons. The 1157 keV $\gamma$ ray is well resolved from other transitions.

Fahlander *et al.* [17] had shown, using angular correlations, that the 2028 keV level was consistent with spin 4. Furthermore, using the Alaga rules, a $K = 4$ assignment was adopted. In the present work, an analysis of the angular distributions and the excitation functions also indicates a spin of 4, and the same conclusion as Ref. [17] is reached regarding the $K$ value [18]. Therefore, in two independent experiments, the 2028 keV level has been assigned as having $I, K^\pi = 4, 4^+$.

Fahlander *et al.* [17] observed an 1157 keV transition in their Coulomb excitation work, but their data indicated only that it fed either the $2^+_\gamma$ or $3^+_\gamma$ levels. Furthermore, as no angular distribution for the 1157 keV $\gamma$ ray was obtained, the spin of the issuing level was not determined. If the 1157 keV transition was assumed to feed the $2^+_\gamma$ level, the absence of a transition to the $3^+_\gamma$ state could be used to imply [17] that the originating level had $I^\pi = 0^+$ or $1^-$. It was further suggested [17] that it originated from a spin 0 state, in which case the $B(E2; 0^+ \to 2^+_\gamma)$ value calculated from the yield of the 1157 keV $\gamma$ ray would be $12^{+6}_{-4}$ W.u. (Weisskopf unit).

In the present work, based on the coincidence relations and excitation function thresholds, the 1157 keV $\gamma$ ray is placed definitely as a decaying transition from a level at 1943 keV. No other transitions from this level are observed. The angular distribution of the 1157 keV $\gamma$ ray is isotropic, which is suggestive of spin 0, but not definitive. Shown in Fig. 2 are excitation functions for selected levels with known spins, and those of the 1943 and 2028 keV levels. The experimental excitation functions deviate from the calculated curves for bombarding energies greater than 2.4 MeV; this is probably related to the omission of levels >2 MeV in the calculation, and the choice of the optical potential used. However, as can be seen, the curves are quite distinct for most spins, and the excitation function for the 1943 keV level, together with the isotropic angular distribution of the 1157 keV $\gamma$ ray, lead to the unique assignment of spin 0. The *absence* (upper limit of 4%) of an $E2$ transition to the $2^+$ level of the ground state band is quite remarkable since this transition should be strongly favored based on the $E^5_\gamma$ dependence of the transition rate, and suggests that one should consider this $0^+$ state to be a possible collective excitation based on the $\gamma$ vibration. Shown in Fig. 3 are plots of $\gamma$-ray energy as a function of $\cos\theta$ for the 1242 and 1157 keV $\gamma$ rays. The experimental value of $F(\tau)$ is noted for each transition, and the lifetimes are listed in Table I.

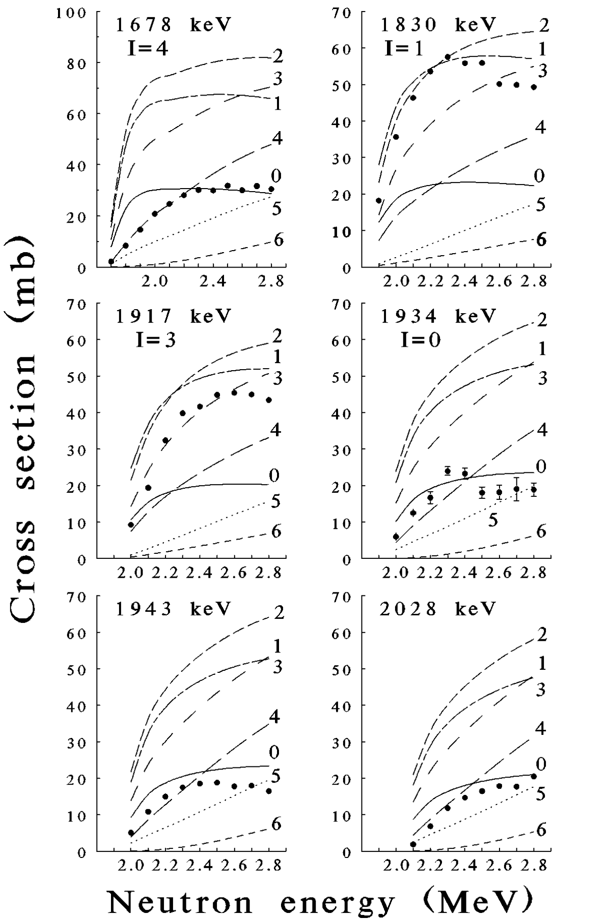


FIG. 2. Excitation functions for levels with known spins; $4^+$ (1678), $1^-$ (1830), $3^-$ (1917), $0^+$ (1934), compared with calculations from the program CINDY [19] for the respective parities. The shape of the excitation function of the 1943 keV level agrees with that calculated for spin 0, and also with that of the 1934 keV state. The shape of the excitation function for the 2028 keV level agrees with that of the 1678 keV state. Ambiguities between spin 0 and 4 can be removed by referring to the angular distribution data.

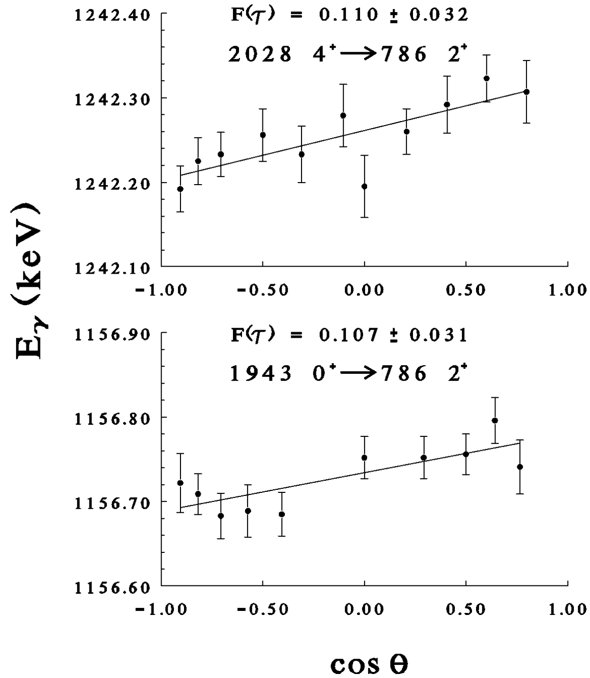


FIG. 3. Measured $\gamma$-ray energy as a function of $\cos\theta$ for transitions from the $4^+_{\gamma\gamma}$ and $0^+_{\gamma\gamma}$ states. Noted are the $F(\tau)$ values determined from linear fits to the data, and the level lifetimes are given in Table I.

An upper limit of $B(E2; 4^+ \rightarrow 2^+_\gamma) < 5$ W.u. for the 1978 keV level is established from the present data, consistent with the results of Fahlander *et al.* [17] of $0.9 \pm 0.4$ W.u. For the 2028 keV level, the observed $B(E2; 4^+ \rightarrow 2^+_\gamma) = 7.4 \pm 2.5$ W.u. value suggests a collective enhancement when compared to the $2^+_\gamma \rightarrow 0^+_{\text{gs}}$ $B(E2)$ value [20] of $5.5 \pm 0.4$ W.u., and is in agreement with the value of $4.9 \pm 1.8$ W.u. determined from Coulomb excitation [17]. The ratio $B(E2; 4^+ \rightarrow 2^+_\gamma)/B(E2; 2^+_\gamma \rightarrow 0^+_{\text{gs}}) = 1.3 \pm 0.4$ is less than the expected harmonic value of 2.8, but is not unlike the value of 1.5 observed [1] in $^{168}$Er. For the 1943 keV level, the $B(E2; 0^+ \rightarrow 2^+_\gamma) = 21 \pm 6$ W.u. also indicates an enhancement, yielding a $B(E2)$ ratio with the $2^+_\gamma \rightarrow 0^+_{\text{gs}}$ of $3.8 \pm 1.3$, in reasonable agreement with the expected harmonic value of 5.0 for a $0^+_{\gamma\gamma}$ state.

In Table II, the properties of two-$\gamma$-phonon states expected in various models are listed with the observed experimental values. (For some models, predictions are available only for $^{168}$Er; however, the similarity of the properties of low-lying states suggests that the values listed are probably appropriate for $^{166}$Er as well.) While all models listed in Table II can satisfactorily describe the $4^+_{\gamma\gamma}$ state, only the harmonic oscillator, the Bohr-Mottelson (BM) approach, and the SCCM give a good description of the $0^+_{\gamma\gamma}$ state. The SCCM prediction could be further improved by a suitable adjustment of the quadrupole-quadrupole strength $\kappa$ which would lower the $0^+_{\gamma\gamma}$ energy. The QPNM and MPM both predict that the relatively pure $0^+_{\gamma\gamma}$ state should not exist due to a high degree of fragmentation, contrary to the present results, while the DDM predicts it to be the first excited $0^+$ level and to have a much more enhanced $E2$ decay rate. Detailed results for all excited $0^+$ states in the $sdg$-IBM calculations for $^{168}$Er, expected to be similar for $^{166}$Er, were not published in Ref. [21]. However, $sd$-IBM calculations indicate that most of the $B(E2; 0^+_i \rightarrow 2^+_\gamma)$ strength is exhausted by the $0^+_2$ state [13], and thus the IBM predictions listed are those

TABLE I. Measured lifetimes, relative $\gamma$-ray intensities, and multipole-mixing ratios for proposed two-$\gamma$-phonon states.

| $E_x$ [keV] | $\tau$ [fs] | $E_\gamma$ [keV] | Placement | $I_{\text{rel}}$ | $\delta$ |
|---|---|---|---|---|---|
| 2028.2 | 320(110) | 1242.3 | $4^+_{\gamma\gamma} \rightarrow 2^+_\gamma$ | 0.47(1) | |
| | | 1168.8 | $4^+_{\gamma\gamma} \rightarrow 3^+_\gamma$ | 0.46(1) | 4.5(1.0) |
| | | 455.7 | $4^+_{\gamma\gamma} \rightarrow 4^-$ | 0.07(1) | |
| 1942.7 | 350(100) | 1156.8 | $0^+_{\gamma\gamma} \rightarrow 2^+_\gamma$ | 1.0 | |

TABLE II. Comparison of the experimental properties with several different model approaches for proposed two-$\gamma$-phonon states.

| | $\frac{E_x(4^+_{\gamma\gamma})}{E_x(2^+_{\gamma\gamma})}$ | $\frac{E_x(0^+_{\gamma\gamma})}{E_x(2^+_{\gamma\gamma})}$ | $\frac{B(E2;4^+_{\gamma\gamma}\to 2^+_{\gamma})}{B(E2;2^+_{\gamma}\to 0^+_{gs})}$ | $\frac{B(E2;0^+_{\gamma\gamma}\to 2^+_{\gamma})}{B(E2;2^+_{\gamma}\to 0^+_{gs})}$ |
|---|---|---|---|---|
| Expt. | 2.58 | 2.47 | 1.3(4) | 3.8(13) |
| Harmonic | 2.0 | 2.0 | 2.78 | 5.0 |
| BM [a] | 2.5 | 2.6 | 2.5 | 4.0 |
| QPNM [b] | 2.65 | | 1.8 | $\ll 1$ |
| DDM | 2.4 [c] | 1.5 [d] | 1.3 [c] | 7.3 [d] |
| IBM [e] | 2.5 | 1.5 | 1.4 | 1.8 |
| SCCM [f] | 2.57 | 3.10 | 1.9 | 1.9 |
| MPM [g] | 2.82 | 3.68 | 0.57 | 0.07 |

[a] The energy ratios are taken from Ref. [12], while the $B(E2)$-value ratios are those for $^{168}$Er from Ref. [11] for the corresponding ratio.
[b] The $0^+_{\gamma\gamma}$ state is severely fragmented and hence does not exist in this model [4].
[c] Values for $^{168}$Er [10].
[d] The lowest excited $0^+$ state is interpreted as the two-$\gamma$-phonon state in this model for $^{164,166,168}$Er [10].
[e] From Ref. [21] calculated for $^{168}$Er. The properties for the $0^+_2$ are listed.
[f] From Ref. [8].
[g] From Ref. [9].

for the $0^+_2$ state in the $sdg$ calculation. While an enhanced $B(E2)$ ratio is predicted in both the $sd$ and $sdg$ calculations, the predicted energy is too low. In the $sd$-IBM this is essentially unavoidable [13,22]. Given the increased number of parameters in the $sdg$ calculations, it may be possible that this situation could be remedied somewhat. As well, in both the $sd$ and $sdg$ calculations, the $0^+_2$ state receives a significant two-neutron-transfer population [23], contrary to the experimental results for the $0^+_{\gamma\gamma}$ state, the $0^+_5$ level, which was not observed in two-neutron-transfer reactions [24].

In summary, the $^{166}$Er$(n, n'\gamma)$ reaction has been used to search for candidates for two-$\gamma$-phonon states. Levels at 2028 and 1943 keV are determined to have $I^\pi = 4^+$ and $0^+$, respectively. The measured $B(E2)$ values from these levels to the $\gamma$-band head are $B(E2; 4^+ \to 2^+_\gamma) = 7.4 \pm 2.5$ W.u. and $B(E2; 0^+ \to 2^+_\gamma) = 21 \pm 6$ W.u., and thus provide evidence for collective enhancements. The level at 2028 keV is assigned as the $K^\pi = 4^+$ two-$\gamma$-phonon state, and its properties are very similar to those of the corresponding state in $^{168}$Er. The 1943 keV level is assigned as the $K^\pi = 0^+$ two-$\gamma$-phonon state. This is the first observation of the $0^+_{\gamma\gamma}$ state in a well-deformed nucleus. The existence of such a state contradicts predictions of the multiphonon method and the quasiparticle-phonon nuclear model, and its energy and $B(E2)$ value disagree with the dynamic deformation model. The IBM predicts that only the $0^+_2$ state will exhibit collective transitions to the $\gamma$ band, and that it should be significantly populated in two-neutron-transfer reactions, in disagreement with the experimental properties of the $0^+_{\gamma\gamma}$ level. There is good agreement with predictions of the harmonic oscillator, self-consistent collective-coordinate model, and a Bohr-Mottelson approach.

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