Comment on "Enhancement of the Tunneling Density of States in Tomonaga-Luttinger Liquids"

In their recent Letter [1], Oreg and Finkel'stein (OF) calculated the electron density of states (DOS) for tunneling into a repulsive Luttinger liquid (LL) close to the location of an impurity. They found that the DOS $\rho(\omega) \sim \omega^{1/2g-1}$ is not only enhanced compared to that of a pure system $\rho_0 \sim \omega^{(g+1/g)/2-1}$, but also diverging at low energy if $g > 1/2$, g being the standard LL parameter ($g < 1$ for repulsive interactions). Such a behavior of the DOS would have important experimental consequences.

In this Comment we intend to show that OF's calculation suffers from a subtle flaw which, being corrected, results in the DOS ,

$$\rho(\omega) \sim \omega^{1/g-1}, \tag{1}$$

that is not only vanishing at $\omega \to 0$ but, in fact, suppressed in comparison with the DOS of a pure LL.

We bosonize the Fermi field as follows:

$$\psi = \sum_{a=R,L} \frac{\eta_a}{\sqrt{2\pi\alpha}} e^{i/2(\Theta/\sqrt{g}+\epsilon_a\sqrt{g}\Phi)}, \tag{2}$$

where α is a short distance cutoff, $R(L)$ refers to the right (left) moving component of the Fermi field, $\eta_R = \tau_x$, $\eta_L = -i\tau_y$, and $\epsilon_{R(L)} = \pm$. Here Φ and Θ are free Bose fields satisfying $[\Phi(x), \Theta(y)] = 2\pi i\,\mathrm{sgn}(x-y)$. The Pauli matrices $\tau_{x,y}$ stand to ensure the correct anticommutation relations between the right and left moving fields. Although these operators are often absorbed into a suitable definition of the Bose field, the equivalent representation (2) is more convenient for our purposes.

In terms of the Bose fields, the Hamiltonian of the system takes the form

$$H = H_0 + \frac{V_{bs}}{\pi\alpha}\tau_z \cos[\sqrt{g}\,\Phi(0)], \tag{3}$$

where H_0 is the free field Hamiltonian and the second term describes the impurity backscattering. Notice that τ_z is a conserved quantity, so that one can set $\tau_z = 1$ ($H = H_\uparrow$) or $\tau_z = -1$ ($H = H_\downarrow$). Particular care is required when calculating correlation functions involving $\tau_{x,y}$ which cause transitions between the degenerate ground states $|\uparrow(\downarrow)\rangle$, thus leading to a kind of orthogonality catastrophe.

The retarded local electron Green function is

$$G^{(R)}(t) = \frac{i\theta(t)}{2\pi\alpha}\left\{\left(\frac{\alpha}{\alpha+it}\right)^{1/2g}\langle\uparrow|\tau_+(t)\sin A(t)\tau_-(0)\sin A(0)|\uparrow\rangle + \left(\frac{\alpha}{\alpha-it}\right)^{1/2g}\langle\uparrow|\tau_+(0)\cos A(0)\tau_-(t)\cos A(t)|\uparrow\rangle\right\},$$

where $A = \sqrt{g}\,\Phi/2$ and the power law factors arise from the correlation of the Θ field, which is decoupled from the impurity. As correctly noticed by OF, the Φ field at the impurity site develops a finite average value, the fluctuations around which are massive. So the asymptotic behavior of the correlation functions can be obtained by simply replacing Φ by its average value. For $V_{bs} > 0$, $\langle\uparrow|\Phi|\uparrow\rangle = \pi/\sqrt{g}$, and it is the first term of $G^{(R)}$ which asymptotically dominates, while for $V_{bs} < 0$ it is the second term. By neglecting phase factors, we find that, in both cases,

$$t^{1/2g}G^{(R)} \propto \langle\uparrow|\tau_+(t)\tau_-(0)|\uparrow\rangle = \langle\uparrow|e^{iH_\uparrow t}e^{-iH_\downarrow t}|\uparrow\rangle,$$

which makes evident the analogy to the x-ray edge problem. The above correlator can be written as $\langle\uparrow|U(t)U^\dagger(0)|\uparrow\rangle$, where U is such that $UH_\uparrow U^\dagger = H_\downarrow$. Since the action of U is to shift $\Phi \to \Phi + \pi/\sqrt{g}$, it is of the form $U = \exp(i\pi J/2)$, where $J = N_R - N_L$ is the total electron current. For $V_{bs} = 0$, J is conserved, but it acquires its own dynamics when $V_{bs} \neq 0$. In particular, $\langle J(t)J(0)\rangle = (2/\pi^2 g)\ln t$ [2]. We therefore conclude that, at large times, $G^{(R)}(t) \sim (1/t)^{1/g}$, leading to the DOS Eq. (1). This result is in agreement with the original analysis of Kane and Fisher [3].

In conclusion, our results show that the phase factors arising from the anticommutativity of the right and left Fermi fields are not innocuous. Ignoring them, one finds OF's results. Hence, the neglect of those phase factors is the likely origin of the OF overestimation of the DOS.

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