Dirt Softens Soap: Anomalous Elasticity of Disordered Smectics

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We show that a smectic in a disordered medium (e.g., aerogel) exhibits anomalous elasticity, with the compression modulus $B(\mathbf{k})$ vanishing and the bend modulus $K(\mathbf{k})$ diverging as $\mathbf{k} \to \mathbf{0}$. In addition, the effective disorder develops long ranged correlations. These divergences are *much* stronger than those driven by thermal fluctuations in pure smectics, and are controlled by a zero temperature glassy fixed point, which we study in an $\epsilon = 5 - d$ expansion. We discuss the experimental implications of these theoretical predictions. [S0031-9007(97)03385-1]

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The effects of quenched disorder on the properties of condensed matter systems continue to be a fascinating area of active research, which includes the study of disordered superconductors [1], charge density waves [2], Josephson junction arrays [3], and Helium in aerogel [4], to name a few. Some of this attention has focused [5,6] on liquid crystals in the random environment of an aerogel. While a complete picture of aerogel-confined liquid crystals is still being developed [6], in this Letter we show that a smectic phase of these systems possesses strong anomalous elasticity, when subjected to an arbitrarily weak amount of quenched disorder. This result has important experimental consequences.

The anomalous elasticity of *pure* smectics was predicted some time ago [7], and is characterized by bulk compressional and tilt moduli $B(\mathbf{k})$ and $K(\mathbf{k})$ which, respectively, vanish and diverge at long wavelengths ($\mathbf{k} \to \mathbf{0}$). This is a general property of all one-dimensional crystals in which the direction of the 1D ordering wave vector is chosen spontaneously. As a consequence of this *spontaneous* breaking of rotational symmetry (a property of smectics but not of charge density waves), in such systems, compression can be relieved by smoothing out fluctuations (wrinkles in the smectic layers), so the effective layer compressional modulus $B(\mathbf{k})$ vanishes at long wavelengths. Similarly, in the presence of fluctuations, a bending of smectic layers necessarily leads to a compression, which implies that the effective tilt modulus $K(\mathbf{k})$ diverges at long wavelengths. Unfortunately, this *thermally* driven behavior in *pure* smectics is difficult to observe experimentally because the effect is very weak (logarithmic) in 3D.

The main ingredients necessary for anomalous elasticity, namely *spontaneously* broken rotational invariance and fluctuations, both exist even at zero temperature in quenched disordered smectics. In this Letter we demonstrate the existence of anomalous elasticity in quenched-disordered smectics which is significantly stronger than the marginal anomalous elasticity of thermal smectics, and is described by a zero-temperature fixed point that is perturbatively accessible in $d = 5 - \epsilon$ dimensions. The elastic anomaly is much stronger in quenched disordered smectics because layer fluctuations are much larger, even at $T = 0$, than in a pure smectic at $T > 0$.

One experimental signature of these divergences is in the smectic correlation length for smectics in aerogel, which has a different, universal dependence on the bare smectic elastic constants and the aerogel density than predicted by harmonic theory. Detailed predictions for these lengths can be found at the end of this Letter.

Our model of a quenched disordered smectic starts with de Gennes' theory [8] for the smectic density field ψ and the nematic director $\hat{\mathbf{n}}$, and includes a disorder field that couples to the nematic director via

$$\delta H_d = -\int d^d r (\mathbf{g}(\mathbf{r}) \cdot \mathbf{n})^2, \tag{1}$$

where $\mathbf{g}(\mathbf{r})$ is a quenched random vector along the locally preferred nematic alignment. Such preferences can arise because, e.g., the nematogens may align with the local, randomly oriented aerogel strands. Below the nematic–smectic-A transition ψ can be written as $|\psi| e^{iq_0 u(\mathbf{r})}$, with $u(\mathbf{r})$ describing the local displacement of the smectic layers from perfect periodic order. Furthermore, far below the nematic transition (i.e., inside the smectic phase) we can take $\hat{\mathbf{n}} \approx \hat{\mathbf{z}} + \delta\mathbf{n}$, where $\hat{\mathbf{z}}$ is the mean normal to the smectic layers, and $\perp$ denotes directions orthogonal to $\hat{\mathbf{z}}$. Integrating out the "massive" $|\psi|$ and $\delta\mathbf{n}$ fields, which has the effect of replacing $\delta\mathbf{n} \to \nabla_\perp u$ (i.e., the Higg's mechanism), and keeping only the most relevant terms, we obtain the elastic Hamiltonian that defines our model,

$$H = \int_r \left[\frac{K}{2} (\nabla_\perp^2 u)^2 + \frac{B}{2} \left(\partial_z u - \frac{1}{2} (\nabla_\perp u)^2 \right)^2 + \mathbf{h}(\mathbf{r}) \cdot \nabla_\perp u \right], \tag{2}$$

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where $\mathbf{h}(\mathbf{r}) \equiv g_z(\mathbf{r})\mathbf{g}(\mathbf{r})$ is quenched random tilt disorder that for simplicity we take to be Gaussian, zero mean, and completely characterized by

$$\overline{h_i(\mathbf{r})h_j(\mathbf{r}')} = \Delta\delta^d(\mathbf{r} - \mathbf{r}')\delta_{ij}\,. \tag{3}$$

The use of short-ranged correlations, even though the *density* of the (fractal) aerogel has long-ranged correlations, is justified, since the *orientations* of its constituent silica strands, beyond their microscopic persistence length, are certainly short-range correlated [5,6]. We will focus here on the behavior of the smectic in the (large) window of length scales between the intrinsic orientational correlation length of the aerogel and the distance between disorder-induced smectic dislocations.

The anharmonic terms included in Eq. (2) are required by the underlying global rotational invariance of the smectic phase [7], hidden by the *spontaneous* choice of the layers to stack along the $\hat{\mathbf{z}}$ direction.

To compute self-averaging quantities, e.g., the disorder averaged free energy, it is convenient (but not necessary) to employ the replica "trick" [9] that relies on the identity $\overline{\log Z} = \lim_{n\to 0} \frac{\overline{Z^n}-1}{n}$. This allows us to work with a translationally invariant field theory at the expense of introducing n replica fields (with the $n \to 0$ limit to be taken at the end of the calculation). After replicating and integrating over the disorder $\mathbf{h}(\mathbf{r})$ utilizing Eq. (3), we obtain

$$H[u_\alpha] = \frac{1}{2}\int_{\mathbf{r}} \sum_{\alpha=1}^{n}\left[K(\nabla_\perp^2 u_\alpha)^2 + B\left(\partial_z u_\alpha - \frac{1}{2}(\nabla_\perp u_\alpha)^2\right)^2\right] - \frac{\Delta}{2T}\int_r \sum_{\alpha,\beta=1}^{n} \nabla_\perp u_\alpha \cdot \nabla_\perp u_\beta\,, \tag{4}$$

from which the noninteracting propagator $G_{\alpha\beta}(\mathbf{q}) \equiv V^{-1}\langle u_\alpha(\mathbf{q})u_\beta(-\mathbf{q})\rangle_0$ can be easily obtained,

$$G_{\alpha\beta}(q) = TG(\mathbf{q})\delta_{\alpha\beta} + \Delta q_\perp^2 G(\mathbf{q})^2, \tag{5}$$

with $G(\mathbf{q}) = 1/(Kq_\perp^4 + Bq_z^2)$. The fluctuations associated with the disorder [the term in Eq. (5) proportional to Δ] are much larger as $\mathbf{q} \to \mathbf{0}$ than those associated with thermal fluctuations (the term proportional to T).

We first attempt to assess the effects of the anharmonicities, disorder, and thermal fluctuations by performing a simple perturbation expansion in the nonlinearities of $H[u_\alpha]$. The lowest order correction δB to the bare elastic compressional modulus B comes from a part of the diagram in Fig. 1.

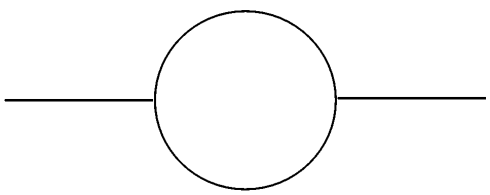

FIG. 1. Feynman graph that renormalizes the elastic moduli K, B, and the disorder variance Δ.

A simple analysis gives

$$\delta B = -\frac{B^2}{2}\int_{\mathbf{q}}^{>} [TG(\mathbf{q})^2 + 2\Delta q_\perp^2 G(\mathbf{q})^3]q_\perp^4\,, \tag{6}$$

$$\approx -B^2\Delta\int_{-\infty}^{\infty}\frac{dq_z}{2\pi}\int^{>}\frac{d^{d-1}q_\perp}{(2\pi)^{d-1}}\,\frac{q_\perp^6}{(Kq_\perp^4 + Bq_z^2)^3}\,, \tag{7}$$

$$\approx -\frac{3}{16}\,\frac{C_{d-1}}{5-d}\,\Delta\left(\frac{B}{K^5}\right)^{1/2} L^{5-d}, \tag{8}$$

where the constant $C_d = 2\pi^{d/2}/[(2\pi)^d\Gamma(d/2)]$, we have dropped the thermal contribution that is subdominant to the disorder part, shown only the dominant contribution for $d < 5$, and introduced a long wavelength cutoff L (defined by restricting the wave vector integral to $q_\perp > 1/L$). The divergence of this correction as $L \to \infty$ signals the breakdown of conventional harmonic elastic theory on length scales longer than $\xi_{NL}^\perp$, which we define as the value of L at which $|\delta B(\xi_{NL}^\perp)| = B$. This definition gives

$$\xi_{NL}^\perp = \left(\frac{16(5-d)K^{5/2}}{3C_{d-1}B^{1/2}\Delta}\right)^{1/(5-d)}, \tag{9}$$

which for physical 3D smectics is given by $\xi_{NL}^\perp = (64\pi K^{5/2}/3B^{1/2}\Delta)^{1/2}$. We can also obtain a nonlinear crossover length in the z direction $\xi_{NL}^z = (\xi_{NL}^\perp)^2/\lambda \sim K^2/\Delta$, where $\lambda \equiv (K/B)^{1/2}$, by imposing the infrared cutoff in the z direction.

To understand the physics beyond this crossover scale, i.e., to make sense of the apparent infrared divergences found in Eq. (8), we employ the standard momentum shell renormalization group (RG) transformation. We separate the displacement field into high and low wave vector components: $u_\alpha(\mathbf{r}) = u_\alpha^<(\mathbf{r}) + u_\alpha^>(\mathbf{r})$, where $u_\alpha^>(\mathbf{q})$ has support in the wave vector range $\Lambda e^{-\ell} < q_\perp < \Lambda$ and Λ is an ultraviolet cutoff of order $1/\xi_{NL}^\perp$, integrate out the high wave vector part $u_\alpha^>(\mathbf{r})$, and rescale the lengths and long wavelength part of the fields with $r_\perp = r_\perp' e^\ell$, $z = z'e^{\omega\ell}$, and $u_\alpha^<(\mathbf{r}) = e^{\chi\ell}u_\alpha'(\mathbf{r}')$, so as to restore the uv cutoff back to Λ. The underlying rotational invariance insures that the graphical corrections preserve the rotationally invariant operator $[\partial_z u_\alpha - \frac{1}{2}(\nabla_\perp u_\alpha)^2]$, renormalizing it as a whole. It is therefore convenient (but not necessary) to choose the dimensional rescaling that also preserves this operator; the appropriate choice is $\chi = 2 - \omega$. This rescaling then leads to the zeroth order RG flows of the effective couplings $K(\ell) = Ke^{(d-1-\omega)\ell}$, $B(\ell) = Be^{(d+3-3\omega)\ell}$, and $(\Delta/T)(\ell) = (\Delta/T)e^{(d+1-\omega)\ell}$. From these dimensional couplings one can construct two dimensionless couplings $\tilde{g}_1 \equiv (B/K^3)^{1/2}$ and $\tilde{g}_2 \equiv \Delta(B/K^5)^{1/2}$, whose flow is given by $\tilde{g}_1(\ell) = \tilde{g}_1 e^{(3-d)\ell}$ and $\tilde{g}_2(\ell) = \tilde{g}_2 e^{(5-d)\ell}$. $\tilde{g}_1$ is just the coupling that becomes relevant in $d < 3$ and was discovered in Ref. [7] to lead to anomalous elasticity in pure smectics. It is, however, only marginally irrelevant in $d = 3$ and therefore only leads to a weak anomalous elasticity in

physical 3D smectics. In contrast, the upper critical dimension d_{uc} below which $\tilde{g}_2$ becomes relevant is $d_{uc} = 5$, and leads to much stronger anomalous elasticity that should be experimentally observable in disordered 3D smectics. These observations imply that temperature is a strongly irrelevant variable near the disorder dominated fixed point. We will therefore set $T = 0$ in all subsequent calculations.

The integration over the high wave vector components of u_α can only be accomplished perturbatively in nonlinearities of $H[u]$. This perturbation theory can be represented graphically; the graph giving the leading order corrections δB, δK, and $\delta\Delta$ (with the part diagonal in the replica indices α, β renormalizing K and B, and the part independent of α, β correcting Δ) is shown in Fig. 1 [10]. Evaluating it, and performing the rescalings described above, we obtain the following RG flow equations:

$$\frac{dB(\ell)}{d\ell} = (d + 3 - 3\omega - \frac{3}{16}g_2)B, \tag{10}$$

$$\frac{dK(\ell)}{d\ell} = (d - 1 - \omega + \frac{1}{32}g_2)K, \tag{11}$$

$$\frac{d(\Delta/T)(\ell)}{d\ell} = (d + 1 - \omega + \frac{1}{64}g_2)(\Delta/T), \tag{12}$$

where we have defined a dimensionless coupling constant $g_2 \equiv \Delta(B/K^5)^{1/2}C_{d-1}\Lambda^{d-5}$, which obeys

$$\frac{dg_2(\ell)}{d\ell} = \epsilon g_2 - \frac{5}{32}g_2^2, \tag{13}$$

with $\epsilon \equiv 5 - d$. As required the flow of g_2 is independent of the arbitrary choice of the anisotropy rescaling exponent ω. The RG flow Eq. (13) shows that the Gaussian $g_2^* = 0$ fixed point becomes unstable for $d < 5$, and the low temperature phase is controlled by a stable, nontrivial, glassy $T = 0$ fixed point at $g_2^* = 32\epsilon/5$.

The existence of this nontrivial fixed point leads to the anomalous elasticity, which we can calculate using the following matching approach. For this purpose it is convenient to use our RG results to evaluate the connected disordered averaged two-point $u(\mathbf{k})$ correlation function $G(\mathbf{k}) \propto \overline{\langle|u(\mathbf{k})|^2\rangle} - \overline{\langle u(\mathbf{k})\rangle\langle u(-\mathbf{k})\rangle}$. The power of the renormalization group is that it establishes a connection between a correlation function at a small wave vector (which is impossible to calculate in perturbation theory due to the infrared divergences) to the same correlation function at large wave vectors, which can be easily calculated in a controlled perturbation theory. This relation for $G(\mathbf{k})$ is

$$G(\mathbf{k}_\perp, k_z, K, B, g_2) = e^{(3+d-\omega)\ell}G(\mathbf{k}_\perp e^{\ell}, k_z e^{\omega\ell}, K(\ell), B(\ell), g_2(\ell)), \tag{14}$$

where the prefactor on the right hand side comes from the dimensional rescaling using the exact Ward identity $\chi = 2 - \omega$, and we have traded in the disorder variable Δ for the dimensionless coupling g_2. To establish the anomalous behavior of K, we look at $k_z = 0$. We then choose the rescaling variable ℓ^* such that $k_\perp e^{\ell^*} = \Lambda$. We also choose $k_\perp$ sufficiently small such that $g_2(\ell^*)$ has reached our nontrivial fixed point g_2^*. Eliminating ℓ^* in favor of $k_\perp$, we then obtain

$$G(\mathbf{k}_\perp, 0, K, B, g_2) = \left(\frac{\Lambda}{k_\perp}\right)^{3+d-\omega} G(\Lambda, 0, K(\ell^*), B(\ell^*), g_2^*). \tag{15}$$

Since the right hand side is evaluated at the Brillouin zone boundary, it can be calculated perturbatively in the fixed point coupling g_2^*. To lowest order we obtain

$$G(\mathbf{k}_\perp, 0, K, B, g_2) \approx \frac{(\Lambda/k_\perp)^{3+d-\omega}}{\Lambda^4 K(\Lambda/k_\perp)^{(d-1-\omega+g_2^*/32)}}, \tag{16}$$

$$\equiv \frac{1}{K(k_\perp)k_\perp^4}, \tag{17}$$

where we integrated Eq. (11) to obtain $K(\ell^*)$, and defined the anomalous tilt modulus which diverges at long length scales

$$K(k_\perp) = K(k_\perp/\Lambda)^{-\eta_K}, \tag{18}$$

with an anomalous exponent

$$\eta_K = \tfrac{1}{32}g_2^* = \tfrac{1}{5}\epsilon, \tag{19}$$

$$\eta_K = \tfrac{2}{5}, \quad \text{for } d = 3. \tag{20}$$

Similar calculations for the other coupling constants and other directions of $\mathbf{k}$ show that, in general,

$$K(\mathbf{k}) = K(k_\perp\xi_{NL}^\perp)^{-\eta_K} f_K(k_z\xi_{NL}^z/(k_\perp\xi_{NL}^\perp)^\zeta), \tag{21}$$

$$B(\mathbf{k}) = B(k_\perp\xi_{NL}^\perp)^{\eta_B} f_B(k_z\xi_{NL}^z/(k_\perp\xi_{NL}^\perp)^\zeta), \tag{22}$$

$$\Delta(\mathbf{k}) = \Delta(k_\perp\xi_{NL}^\perp)^{-\eta_\Delta} f_\Delta(k_z\xi_{NL}^z/(k_\perp\xi_{NL}^\perp)^\zeta), \tag{23}$$

with the anisotropy exponent $\zeta \equiv 2 - (\eta_B + \eta_K)/2$, which would be 2 in the absence of anharmonic effects, $\eta_B = 3g_2^*/16 = 6\epsilon/5 = 12/5$ in $d = 3$, and $\eta_\Delta = g_2^*/64 = \epsilon/10 = 1/5$ in $d = 3$.

Of course, we do not completely trust the extrapolation of these small ϵ results down to $\epsilon = 2$ ($d = 3$). However, since by definition $dg_2/d\ell = 0$ at the nontrivial fixed point, this condition implies an *exact* relation between the anomalous exponents

$$5 - d + \eta_\Delta = \frac{\eta_B}{2} + \frac{5}{2}\eta_K, \tag{24}$$

which is obviously satisfied by the anomalous exponents, computed here to first order in ϵ. This Ward identity between the anomalous exponents can be equally easily obtained from a self-consistent integral equation for the $u - u$ correlations functions, using renormalized wave vector dependent elastic moduli and disorder variance [6].

At length scales beyond $\xi_{NL}^{\perp}$ and ξ_{NL}^{z}, the elasticity and fluctuations of the disordered smectic are controlled by our new glassy fixed point. One of the important consequences can be seen in the layer fluctuations that can be observed in x-ray scattering experiments. For instance, layer displacement fluctuations along z are described by

$$C(z) \equiv \overline{\langle [u(0_\perp, z) - u(0_\perp, 0)]^2 \rangle}, \\ = \int \frac{d^d k}{(2\pi)^d} \frac{2[1 - \cos(k_z z)]\Delta(\mathbf{k}) k_\perp^2}{[K(\mathbf{k}) k_\perp^4 + B(\mathbf{k}) k_z^2]^2} . \quad (25)$$

One can then naturally define the x-ray translational correlation length ξ_z^X as the length along z at which $C(z = \xi_z^X) \equiv a^2$, where a is the smectic layer spacing. A simple calculation, using Eqs. (21)–(23) leads in 3D to

$$\xi_z^X = (a/\lambda)^{2/\gamma} K^2/\Delta = (a/\lambda)^{2/\gamma} \xi_{NL}^z , \quad (26)$$

where $\gamma \equiv (\eta_B + \eta_K)/\zeta$. Note that this x-ray correlation length is finite even as $T \to 0$. This result is consistent with the experimental observation [5] that the x-ray correlation length for smectics in aerogel saturates at some finite value at low temperatures. Note also that this length should be different for different smectics in the same aerogel, since B, K, and Δ will change from smectic to smectic. Since we expect $\Delta \propto \rho_A$, the aerogel density [6], the aerogel density dependence of ξ_z^X could test the prediction of Eq. (26). Likewise, the *temperature* dependence of ξ_z^X could be used to determine γ, since the *bulk* $K(T)$ and $B(T)$ that implicitly appear in Eq. (26) have temperature dependence that can be extracted from measurements on bulk materials.

Note also that this correlation length is longer than the nonlinear crossover length for $\lambda < a$ (i.e., for large B). For $\lambda \gg a$ (small B), $C(z)$ reaches a^2 before z reaches ξ_{NL}^z, and hence anharmonic effects are unimportant. In this case, the correlation length ξ_z^X can be determined in a harmonic theory [which amounts to evaluating the integral in Eq. (25) with $K(\mathbf{k})$, $B(\mathbf{k})$, and $\Delta(\mathbf{k})$ replaced by their constant (bare) values K, B, and Δ]. This gives $\xi_z^X = a^2 BK/\Delta = (a/\lambda)^2 \xi_{NL}^z$, which is, reassuringly, much less than ξ_{NL}^z in the limit $a \ll \lambda$ in which we have asserted it applies.

Although our entire discussion so far has focused only on the effects of *orientational* disorder, we have shown [6] that *translational* disorder (i.e., random pinning of the positions of the layers), is *less* important, at long wavelengths (in $d < 5$) than the orientational disorder. Thus, the results described herein are directly applicable to real smectics, where both kinds of disorder are present.

We have also ignored dislocations so far. Further detailed analysis [6] shows that in a harmonic theory dislocations unbind for arbitrarily weak disorder. However, their effects are felt only on length scales greater than $\xi_{\rm disl} = (a^2/\lambda) e^{t_0 \lambda/\Delta}$, where t_0 is a parameter of order BK, which is much longer, in the weak disorder ($\Delta \to 0$) limit, than the translational correlation length ξ_z^X found here. Including anomalous elasticity, we find [6] a shorter $\xi_{\rm disl}$ that nonetheless remains much longer than ξ_z^X. Hence our predictions for ξ_z^X and the anomalous elasticity studied here remain valid.

In summary, we have found that the addition of disorder to a smectic liquid crystal phase leads to anomalous elasticity that is much stronger than in the pure system. This elasticity is controlled by a zero temperature glassy fixed point, perturbative in $\epsilon = 5 - d$. We expect that in contrast to marginally weak anomalous elasticity of pure smectics, the strong anomaly in the elasticity of disordered smectics, described here, should be readily observable.

Note added.—After this work was completed we learned of earlier work [11] on the model Eq. (4) in the context of spin glasses. However, this earlier work misses the essential physics of the nontrivial renormalization of disorder, and therefore makes *quantitatively and qualitatively wrong* predictions of nearly all results; e.g., the anomalous exponents, disorder correlations, scaling exponent relations, etc., correct versions of which are given in our Eqs. (12)–(24).

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