Neutron Electric Dipole Moment in the Standard Model: Complete Three-Loop Calculation of the Valence Quark Contributions

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(Received 18 April 1997)

We present a complete three-loop calculation of the electric dipole moment of the u and d quarks in the standard model. For the d quark, more relevant for the experimentally important neutron electric dipole moment, we find cancellations which lead to an order of magnitude suppression compared with previous estimates. [S0031-9007(97)03356-5]

PACS numbers: 11.30.Er, 12.15.Ji, 13.40.Em

The electric dipole moment (EDM) of the neutron, d_n, has been the subject of experimental searches for almost a half century [1]. Discovery of a nonzero d_n would be direct evidence of violation of both time reversal symmetry and parity. If the symmetry under simultaneous charge conjugation, space inversion, and time reversal (CPT) is exact, T violation is equivalent to CP violation (see also a discussion in [2]). Most theories, which explain CP violation observed in neutral kaon decays or address the question of matter-antimatter asymmetry in the universe, predict the existence of d_n at some level. The current experimental upper bound [3–5]

$$|d_n| < 1.1 \times 10^{-25} e \text{ cm} \tag{1}$$

stringently constrains physics both within the standard model (possible nonperturbative effects, such as the so-called θ term [6]) and beyond it, such as supersymmetry and supergravity [7], fourth generation of fermions [8], CP violation in the Higgs sector [9], etc. (for detailed reviews see [10,11]). It also severely constrains the W boson EDM [12].

The next generation of experiments is going to search for d_n with an accuracy improved by several orders of magnitude. For example, a new technique based on storing polarized ultracold neutrons and a polarized gas of ^{3}He in superfluid ^{4}He is expected to permit a measurement of d_n with the sensitivity of

$$\sigma(d_n) = 4 \times 10^{-29} e \text{ cm} \tag{2}$$

in a one year run [13].

In the electroweak sector of the standard model a complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix violates CP and, via loop effects, induces an EDM for all non-self-conjugate particles with spin. In the case of the neutron, an EDM can be generated by electroweak interactions in various ways. Penguin diagrams with a W exchange between constituent quarks offer one possibility. Such effects were estimated to give contributions to d_n which could perhaps be as large as $10^{-30} e$ cm [14]. This estimate is controversial and involves a large uncertainty; see, e.g., a discussion in [15]. A more conservative estimate of the long distance standard model effects is $2 \times 10^{-32} e$ cm [16].

Another possibility, and the main focus of our work, is d_n induced by an EDM of the up and down quarks ($d_{u,d}$) inside the neutron. Using SU(6) wave functions, one finds

$$d_n = \tfrac{4}{3}\, d_d - \tfrac{1}{3}\, d_u\,. \tag{3}$$

Quark EDM cannot be generated in the standard model at the one-loop level because the relevant amplitudes do not change the quark flavor and each CKM matrix element is accompanied by its complex conjugate; no T-violating complex phase can arise. At the two-loop level individual diagrams have complex phases (see Fig. 1) and contribute to the EDM [17–19]. However, the sum over all quark flavors in the intermediate states leads to the vanishing of the EDM at two loops [20] (a similar effect for leptons was found in Ref. [21]). This interesting and seemingly accidental cancellation was also analyzed in [22,23].

The fact that the quark EDM appears only at the three-loop level in the standard model greatly complicates theoretical estimates. The largest effect is due to the exchange of two W bosons and one gluon; this was first discussed by Shabalin [24] in a model with quarks carrying integral charges 0 and 1. In that model the charged quark EDM was found to be suppressed by at least six inverse powers of the W mass, if one assumes quarks to be light. The case of quarks with physical, fractional values of electric charges was examined in [22]. It was found that the strong cancellation found earlier was

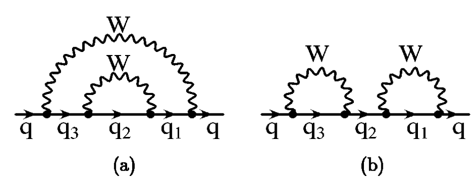

FIG. 1. Two types of flavor structure of diagrams contributing to quark EDM. Gluon lines are not indicated. Here and in the following figures it is understood that an external electric field can interact with any charged particle in the diagram.

an artifact of the assumed pattern of charges and quark EDM was found at the level of (quark masses)$^4/M_W^4$ with further enhancements by logs of quark mass ratios; in fact, in this order the EDM of the quark is proportional to the electric charge of its isospin partner—so that the EDM of a charge 1 quark in Shabalin's model is suppressed.

The analysis of Ref. [22], the most complete study to date, was carried out in the framework of the leading logarithmic approximation within the effective Fermi theory under an assumption that all quarks are lighter than the W boson. Since it turns out that the top quark outweighs the W boson, this approach is no longer valid. In fact, the large mass of the top quark significantly enhances some electroweak processes (e.g., the decays $Z \to b\overline{b}$ and $b \to s\gamma$, as well as $B^0 - \overline{B}^0$ mixing). In view of the anticipated improvement of the experimental accuracy, a detailed evaluation of the standard model contributions to the quark EDM is clearly warranted. Here, we report the complete results of such an analysis.

The quark matrix element of the electromagnetic current, J_μ^{em}, can be written as (with $q = p' - p$)

$$\begin{aligned} \langle p'|J_\mu^{\text{em}}|p\rangle &= \overline{u}(p')\Gamma_\mu u(p)\,, \\ \Gamma_\mu &= F_1(q^2)\gamma_\mu + iF_2(q^2)\sigma_{\mu\nu}q^\nu \\ &\quad - F_3(q^2)\gamma_5\sigma_{\mu\nu}q^\nu \\ &\quad + F_A(q^2)(\gamma_\mu q^2 - 2m_q q_\mu)\gamma_5\,. \end{aligned} \tag{4}$$

The form factors at $q^2 = 0$ give the electric charge, anomalous magnetic moment, electric dipole moment, and anapole moment in units of $e = |e|$ [e.g., for a down quark $F_1(0) = -1/3$], so that $d_q = eF_3(0)$.

The three-loop calculation of the light quark EDM is facilitated by the clear hierarchy of masses $m_{u,d} \ll m_s \ll m_c \ll m_b \ll M_W \ll m_t$. On the other hand, as will be seen shortly, the relevant mass ratios are not large enough for their logarithms to be the dominant effect. Therefore, in addition to the logarithmic terms, we also need the nonlogarithmic constants. A systematic expansion of Feynman diagrams in terms of mass ratios and their logs is possible in the framework of asymptotic expansions (for a recent review see [25]). The problem is slightly complicated by the many mass scales present in each diagram. The details of our procedures will be described elsewhere. Here we only remark that the calculation technically resembles the two-loop electroweak corrections to the muon anomalous magnetic moment [26], apart from the presence of the third loop.

The electric dipole moment in the standard model has an important feature which simplifies its calculation. Namely, if the source of CP violation is in the complex phase of the CKM matrix, the EDM vanishes if any two up-type or down-type quarks have equal masses. This circumstance has been taken advantage of in the Fermi theory calculation in Ref. [22]. Under the assumption that all quarks are lighter than the W boson, it was possible to replace the W propagators by $1/M_W^2$; effects of large virtual momenta which could "feel" the exact structure of these propagators are, to first approximation, independent of quark masses, and therefore their sum gives no contribution to the EDM. The resulting effective theory diagrams are shown in Fig. 2.

The situation is actually more complicated since the top quark is heavier than the W boson. It turns out that there are six topologies of diagrams with the top quark which contribute to the EDM: three for the down quark (Fig. 3) and three different ones for the up quark (Fig. 4). The diagrams in Figs. 3 and 4 have to be understood in the following way: The W propagator, connected to light quark lines only, can be contracted to a point and replaced by $1/M_W^2$; the other W propagator, connected to the top quark and indicated explicitly in the figures, is treated differently. Namely, the entire top-W loop is replaced by an effective operator insertion. Technically, this amounts to performing the integral over its obvious internal momentum, after having expanded top and W propagators in remaining momenta and M_W. We note that contributions of internal momentum in the t-W loop of the order much less than m_t are suppressed; this is obvious in the unitary gauge which we use in this calculation.

Contributions of diagrams containing the top quark are finite and independent of m_t for both up- and down-type quarks (throughout this paper we adopt the approximation in which we neglect terms suppressed by inverse powers of quark masses). In particular, we have found no terms enhanced by m_t^2, such as m_t^2/M_W^6.

The question which now arises is whether the effective Fermi theory is sufficient for the calculation of the remaining diagrams, without the top quark. This turns out not to be the case. In diagrams with an external down quark we have to include the contributions depicted in Fig. 3, with the top quark replaced by up and charm quarks. Similarly, for an external up quark, diagrams such as shown in Fig. 4 contribute, with $t \to c$. In these diagrams the internal momenta of the order of M_W^2 give contributions to the EDM which are no longer canceled by the corresponding top diagrams. They have to be added to the (divergent) effective Fermi theory results. In the sum, divergences cancel and logarithms of M_W are combined

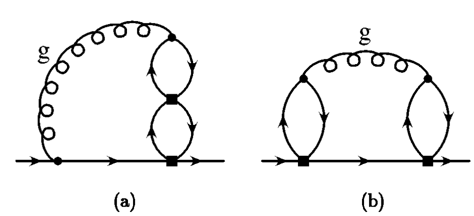

FIG. 2. Effective Fermi theory diagrams in a model with all quarks lighter than the W boson. (a) Results from the diagram in Fig. 1(a) and (b) from Fig. 1(b).

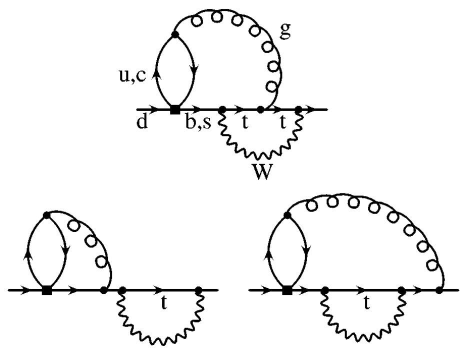

FIG. 3. Heavy top quark contributions to d_d.

with those of the mass of the second heaviest quark, m_b. In this way we arrive at our final formulas for the down quark,

$$\frac{d_d}{e} = \frac{m_d m_c^2 \alpha_s G_F^2 \tilde{\delta}}{108\pi^5}\left[\left(L_{bc}^2 - 2L_{bc} + \frac{\pi^2}{3}\right)L_{Wb} + \frac{5}{8}L_{bc}^2 - \left(\frac{335}{36} + \frac{2}{3}\pi^2\right)L_{bc} - \frac{1231}{108} + \frac{7}{8}\pi^2 + 8\zeta_3\right] + \mathcal{O}(m^2/M^2), \tag{5}$$

and for the up quark,

$$\frac{d_u}{e} = \frac{m_u m_s^2 \alpha_s G_F^2 \tilde{\delta}}{216\pi^5}\left[\left(-L_{bs}^2 + 2L_{bs} + 2 - \frac{2\pi^2}{3}\right)L_{Wb} - L_{bc}L_{cs}^2 + 2L_{bc}L_{cs} - \frac{5}{8}L_{bs}^2 - \left(\frac{259}{36} + \frac{\pi^2}{3}\right)L_{bs} + \left(\frac{140}{9} + \pi^2\right)L_{cs} - \frac{121}{108} + \frac{41}{36}\pi^2 - 4\zeta_3\right] + \mathcal{O}(m^2/M^2), \tag{6}$$

where $\mathcal{O}(m^2/M^2)$ denotes terms with an additional suppression by quark or W boson masses; $L_{ab} \equiv \ln(m_a^2/m_b^2)$, ζ_3 is the Riemann zeta function ($\zeta_3 = 1.202\ldots$), and $\tilde{\delta}$ denotes the CP violating invariant

$$\tilde{\delta} = s_1^2 s_2 s_3 c_1 c_2 c_3 \delta, \tag{7}$$

where, for the CKM matrix, we use the convention of [22,27]. From our expressions it is not obvious that the quark EDM vanishes if any two up- or down-type quarks have degenerate masses. This is because we have assumed a hierarchy of masses, and retained only the leading terms in the expansions in mass ratios. Such expansion, while delivering a simple and compact formula, obscures the symmetry of the full result.

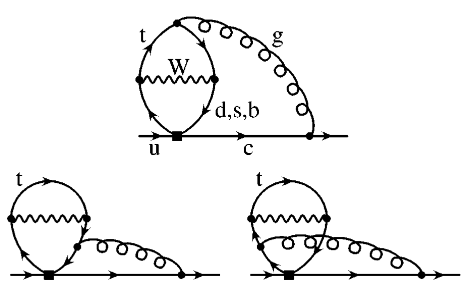

FIG. 4. Heavy top quark contributions to d_u.

It is instructive to compare our findings with the results obtained in the Fermi theory [22]. In that reference the terms with three powers of logarithms were calculated and if the top quark mass is replaced by the mass of the W boson they coincide with our $L_{bc}^2 L_{Wb}$ (for d_d) and $-L_{bs}^2 L_{Wb} - L_{bc}L_{cs}^2$ (for d_u). However, it turns out that the terms with the highest power of logarithms are not the dominant components of the complete result. In fact, the large negative coefficient of the single logarithms L_{bc} leads to more than a complete cancellation of the triple log contribution to d_d. Partial cancellation is also found in d_u. For the numerical estimates we use the following values of parameters: $\tilde{\delta} = 5 \times 10^{-5}$, $\alpha_s = 0.2$, $m_s = 0.2$ GeV, $m_c = 1.5$ GeV, $m_b = 4.5$ GeV, $M_W = 80$ GeV. We find

$$d_d = -0.7 \times 10^{-32}\,\frac{m_d}{\text{GeV}}\,e\ \text{cm}, \qquad d_u = -0.3 \times 10^{-32}\,\frac{m_u}{\text{GeV}}\,e\ \text{cm}. \tag{8}$$

Previous estimates were based only on the terms with three powers of logs; from those we find

$$d_d(\text{triple log approx.}) = +5 \times 10^{-32}\,\frac{m_d}{\text{GeV}}\,e\ \text{cm}, \qquad d_u(\text{triple log approx.}) = -0.4 \times 10^{-32}\,\frac{m_u}{\text{GeV}}\,e\ \text{cm}. \tag{9}$$

We see that the "non-leading-log" terms suppress the quark EDM, especially strongly for the d quark, where they even change the sign of the effect. For the experimentally interesting neutron EDM, the d quark EDM is the more important quantity. The suppression factor we found decreases the part of the standard model contribution generated by the quark EDM and renders it definitely unobservable for the experiments in the near future.

Inserting current quark masses for $m_{u,d}$ in Eq. (8), we obtain the following numerical values:

$$d_d = -0.7 \times 10^{-34} e\ \text{cm} \quad \text{for } m_d = 10\ \text{MeV}, \qquad d_u = -0.15 \times 10^{-34} e\ \text{cm} \quad \text{for } m_u = 5\ \text{MeV}. \tag{10}$$

We should stress here that we have neglected terms with more than one power of light quark masses $m_{u,d}$. They tend to result in an additional suppression, especially for d_u, where m_s^2 should be replaced by $m_s^2 - m_d^2$ in most terms.

Regarding the accuracy of our result, it is clear that the issue of what light quark masses should be used is most important and requires further study. This is connected with the issue of the scale of the running α_s which is difficult to address since our calculation is of the leading order in QCD. For the triple-logarithmic term it is known from [22] that the characteristic gluon virtuality is between c and b quark masses (for d_d). For the nonleading terms this scale could be different. Also, since d_d results from large cancellations among terms which are 5 to 10 times larger than the final value, a precise prediction would require taking into account corrections of the order $M_W^2/m_t^2 \approx 20\%$ and $m_c^2/m_b^2 \approx 10\%$ which can change the result by a factor of 2 or so. It is, however, rather certain that the non-leading-log terms we presented in this paper significantly reduce the value of d_d compared with previous estimates. If the next generation of experiments detects nonzero d_n it will have to be attributed to a different mechanism than the standard model light quark EDM.

We are very grateful to Professor I. Khriplovich, Professor F. Klinkhamer, Professor W. Marciano, and Professor N. Uraltsev for advice and helpful comments. We thank Professor J. H. Kühn for his interest in and support for this work. This research was supported by Grant No. BMBF 057KA92P and by “Graduiertenkolleg Elementarteilchenphysik” at the University of Karlsruhe.

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