Accurate Four-Body Response Function with Full Final State Interaction: Application to Electron Scattering off ^{4}He

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The longitudinal (e, e') response function of ^{4}He is calculated precisely with full final state interaction. The explicit calculation of the four-body continuum states is avoided by the method of integral transforms. Precision tests of the response show the high level of accuracy. Nonrelativistic nuclear dynamics are used. The agreement with experimental data is very good over a large energy range for all considered momentum transfers ($q = 300$, 400, 500 MeV/c). Only at higher q the theoretical response overestimates the experimental one beyond the quasielastic peak. [S0031-9007(96)02200-4]

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A new method for the calculation of the inelastic response of an N-body system to an external probe is proposed in Ref. [1]. It allows an exact calculation without the knowledge of the N-body scattering state. The high level of accuracy of the method has been shown for the longitudinal electron scattering responses of the nuclear two- and three-body systems [1,2]. The real superiority of the approach, however, becomes evident when applied to a four-body system. In fact, a solution of the four-body medium energy continuum state problem is presently out of reach, nonetheless four-body response functions can be reliably calculated as pointed out in the following. In this Letter we consider the important longitudinal electron scattering response function R_L of ^{4}He which is calculated for the transfer momenta $q = 300$, 400, and 500 MeV/c. For $q = 500$ MeV/c it is the first accurate calculation with the final state nuclear interaction fully taken into account. Our results are obtained within the framework of the nonrelativistic nuclear dynamics and using the single-particle form of the electromagnetic operator. Such studies allow establishing the limits of validity of this conventional framework for the lightest tightly bound nucleus. Particularly interesting is the higher q region. For more than a decade there has been a lot of discussion for complex nuclei regarding this region. An accurate calculation for ^{4}He will help to shed some light in this range of q values.

The idea of Ref. [1] is to calculate the response in an indirect way. First the Lorentz transform (LT)

$$\Phi(\sigma = -\sigma_R + i\sigma_I, q) = \int d\omega \, \frac{R(\omega, q)}{(\omega - \sigma_R)^2 + \sigma_I^2} \tag{1}$$

of the response function

$$R(\omega, q) = \sum_n |\langle n|\Theta(\mathbf{q})|0\rangle|^2 \delta(\omega - E_n + E_0) \tag{2}$$

is calculated, where $|0\rangle$ is the ground state of the system, E_0 is the ground state energy, $\Theta(\mathbf{q})$ is the excitation operator, and $\sigma_R > 0$, $\sigma_I \neq 0$. The solution of the following equation:

$$(H - E_0 - \sigma_R + i\sigma_I)|\Psi\rangle = \Theta|0\rangle, \tag{3}$$

leads directly to the LT,

$$\Phi(\sigma, q) = \langle\Psi|\Psi\rangle. \tag{4}$$

In a second step $R(\omega, q)$ is obtained via the inversion of the transform. The solution of Eq. (3) is unique. Indeed, the homogenous equation has only the trivial solution because the Hamiltonian H has only real eigenvalues. Since Ψ has to fall off exponentially one can use similar techniques as for the solution of the ground state problem. Thus the complicated boundary condition of a four-body scattering state has not to be considered at all.

In the past other integral transforms were proposed, namely, Stieltjes [3] and Laplace transforms [4,5]. The Laplace transforms of the ^{4}He longitudinal response were obtained with a realistic force for $q = 300$ and 400 MeV/c via a Green function Monte Carlo calculation (GFMC) [4]. Also the Laplace transforms of the transverse response and the effects of two-body operators on the transforms in both longitudinal and transversal cases were considered via a GFMC [5]. Good agreement with the transforms of the experimental data is found. There is, however, a fundamental problem in obtaining response functions themselves from these transforms. Unlike the LT they sample contributions over a large energy range. This results in big problems for the inversion [6]. Nevertheless the longitudinal $R(\omega, q)$ of ^{4}He has been obtained by an inversion of the Laplace transform for $q = 400$ MeV/c [4]. The result is rather similar to ours in Fig. 4. We are not able to fully interpret this agreement since the statistical errors of a GFMC lead to an uncertainty in the inversion of the Laplace transform. Unfortunately, the inversion error is not estimated in Ref. [4], which in general can be sizable [6]. On the contrary, for the LT inversion problems are much less

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important [1,2]. Moreover, the numerical effort for the calculation of the LT seems to be much smaller than for the Laplace transform. However, a fair comparison can only be made when both calculations are performed for the same potential model.

Our Hamiltonian includes central even potentials

$$V(ij) = V_{31}(r_{ij})P_{\sigma}^{+}(ij)P_{\tau}^{-}(ij) + V_{13}(r_{ij})P_{\sigma}^{-}(ij)P_{\tau}^{+}(ij) \tag{5}$$

plus Coulomb force providing realistic descriptions of the S-wave phase shifts up to the pion threshold. We construct the V_{31} and V_{13} potentials by modifying the complete NN interaction of Ref. [7]. The disregarded tensor force is effectively simulated via a dispersive correction ($V \rightarrow V - V_{\text{tensor}}^2/\text{const}$). The potentials obtained lead to almost the same phase shifts as in Ref. [7]. A full description of the potential will be published elsewhere [8]. It describes the static properties of ^{4}He rather well leading to a binding energy of 30.5 MeV and an rms radius of 1.41 fm. Also the description of the elastic form factor is rather realistic up to its first minimum. The present ansatz for the potential will lead to results quite similar to those for more general nuclear forces. The three-nucleon studies undertaken so far testify to this opinion [9]. Although more intensive the calculations with a completely realistic nuclear force are also quite feasible within our approach.

In the following we describe the techniques we use for solving the dynamic equation (3). We seek for the solution in the form of an expansion over the correlated hyperspherical basis first used in Ref. [10]. The expansion converges quickly in few-nucleon bound state problems [10,11]. Our basis functions are of the form

$$JR_N(\rho)[Y_{KLM}^{[f]\mu}(\Omega)\theta_{S=0,T}^{[\tilde{f}]\tilde{\mu}}]^a. \tag{6}$$

Here ρ is the hyperradius, $\rho = (\xi_1^2 + \xi_2^2 + \xi_3^2)^{1/2}$, $\vec{\xi}_i$ are the normalized Jacobi vectors, and Ω denotes collectively eight hyperangular variables. The quantities $Y_{KLM}^{[f]\mu}$ are hyperspherical harmonics (HH) with hyperangular K and orbital L, M momentum quantum numbers. These HH are components μ of irreducible representations $[f]$ of the four-particle permutation group S(4). The spin-isospin functions θ (see, e.g., [12]) enter Eq. (6) with the same spin and isospin values $S = 0, T = 0$, and $T = 1$ as in the expansion of the right-hand side of Eq. (3). They belong to the conjugate representation $[\tilde{f}]$ of S(4). The square brackets mean coupling to the function antisymmetric with respect to permutations of both spatial and spin-isospin particle coordinates. R_N are the hyperradial functions, and J is the Jastrow correlation factor.

The system of equations for the expansion coefficients is obtained by projecting Eq. (3) onto the subset of functions (6) with K up to some $K_{\max}$ and N up to some $N_{\max}$. This system is split with respect to L, M, and T values. Since $L(^4\text{He}) = 0$ in our model the L quantum number coincides with the multipole order of the transition operator. The response, as well as Eq. (4), is independent of a $\mathbf{q}$ direction that can be chosen along the z axis. Only the $M = 0$ value gives a nonzero contribution in this case. The matrix elements are calculated with a Monte Carlo integration.

The HH entering Eq. (6) are constructed by applying the convenient form, see, e.g., [13], of the Young operators to the simple Zernike-Brinkman–type HH. The multiplicities of various $[f]$ representations at given K and L values are obtained as traces of the Young operators calculated in the Zernike-Brinkman basis [13,14].

The hyperradial functions of the form [13,15] $R_N(\rho) \sim L_N^8(\rho/b)\exp(-\rho/2b)$ are used. Here L_N^{α} are Laguerre polynomials, and b is a scale parameter which is kept the same for all the σ values considered and is chosen to enable sufficiently fast overall convergence. The results are rather insensitive to the b values. The rate of the hyperradial convergence in our case is lower than in the bound state calculations (e.g., [13,15]), and better R_N can perhaps be found.

The two-body correlation function $f(r)$ entering the Jastrow factor is taken to be spin independent and is chosen in a conventional way. At $r \leq r_0$ it is a solution to the Schrödinger equation with the potential taken as the half-sum of the triplet and singlet NN forces. The r_0 point is chosen from the condition $f'(r_0) = 0$. At $r > r_0$ $f(r) = f(r_0)$. The kinetic energy matrix elements with the Jastrow factor are cast to a convenient form [10].

We calculate the LT of $R_L(\omega, q)$ with $\sigma_I = 20$ MeV. The quantities in Eqs. (2) and (3) pertain to the center of mass system, and

$$\Theta(\mathbf{q}) = \sum_k \left[\frac{1 + \tau_3(k)}{2} + \frac{G_E^n(q^2)}{G_E^p(q^2)} \frac{1 - \tau_3(k)}{2} \right] \times e^{i\mathbf{q}(\mathbf{r}_k - \mathbf{R}_{\text{c.m.}})},$$

where $G_E^{p,n}$ are nucleon Sachs form factors. In order to reach convergence we choose a sufficiently large $N_{\max}$ for the hyperradial functions ($N_{\max} = 20$, 25, and 30 for $q = 300$, 400, and 500 MeV/c, respectively). The multipole transitions of the charge operator are taken into account up to a maximal order $L_{\max}$. From the evaluation of the various multipole contributions to the Coulomb sum rule we find that the following $L_{\max}$ values lead to an exhaustion of the sum rule by more than 99%: $L_{\max} = 4$, 5, and 6 for $q = 300$, 400, and 500 MeV/c, respectively. These $L_{\max}$ values are adopted at solving Eq. (3). The maximal hyperangular order $K_{\max}$ is taken equal to 7; only in the case of $L_{\max} = 6$ the value 8 is used. This is sufficient to completely exhaust the various multipole strengths for $q = 300$ MeV/c. Even for $q = 500$ MeV/c one misses only a small fraction of the strength of the less important multipoles with $L \geq 4$ (see also discussion below).

The results for the LT are shown in Fig. 1. Unlike Stieltjes and Laplace transforms it is already obvious directly from the LT that the response is governed by the quasielastic peak. The inversion is performed with the same sets of basis functions used in Refs. [1,6]. Contrary to the nuclear two- and three-body systems, we

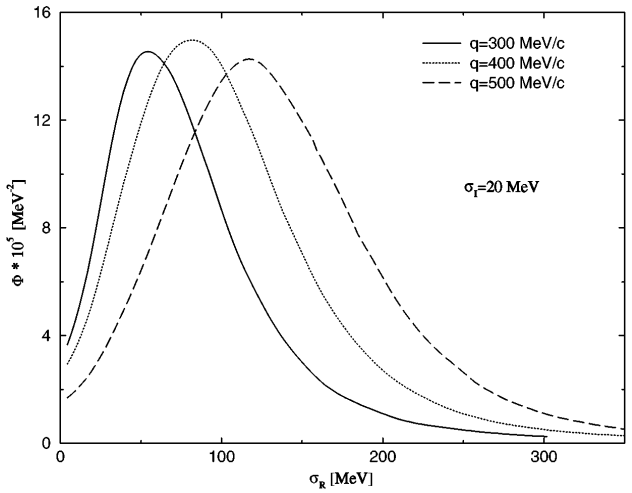

FIG. 1. LT at $q = 300$, 400, and 500 MeV/c.

cannot, of course, compare the $R(\omega, q)$ obtained from the inversion with a direct calculation of the response according to Eq. (2). Nonetheless it is possible to test the precision of the response function results. A first test is the separate inversion of all the various multipoles. It serves as a very important sum rule check, since for a given multipole one can compare the sum rule from the evaluation as ground state expectation value with that obtained from an explicit integration of the response. This check leads to very good results with relative errors of about 1% for most of the transitions (average errors: 1.1%, 1.0%, and 2.0% for $q = 300$, 400, and 500 MeV/c, respectively). Somewhat larger errors are found only for $q = 500$ MeV/c, where the less important higher multipoles ($L \geq 4$) are slightly underestimated by about 3%–4%. As mentioned above $K_{\max}$ should be chosen somewhat larger for a complete exhaustion of the strength of these multipoles. Nonetheless we may say that the sum rule results show the good accuracy of our method. In Fig. 2 the isoscalar and isovector parts of the response function obtained from the separate inversion are shown for $q = 500$ MeV/c. One sees that almost all multipoles have the typical structure due to the quasielastic peak. The only exception is the isoscalar Coulomb monopole which exhibits a peak close to threshold. For the two lower momentum transfers this $C0$ peak is even more pronounced. For $q = 300$ MeV/c its height reaches already one third of the quasielastic peak height. The isovector strength is twice as large as the isoscalar one.

Another very important check for the precision of the method is obtained by the inversion of the total LT. The resulting $R(\omega, q)$ should not differ from that obtained from the separate inversion discussed above. Before discussing these results we should mention that we encounter at low energy for $q = 400$ and 500 MeV/c similar inversion problems for the full LT as described in Ref. [1]. We solve this problem in a similar way as in Ref. [1], i.e., by separate inversions for the sum of isoscalar $C0$ and $C1$ and for the sum of all remaining multipoles; nevertheless in the following it will be called total inversion. The total response functions resulting from separate and total inversions are shown in Fig. 3 for the three considered momentum transfers. From the good agreement of the various curves it is evident that the inversion is very unproblematic. Differences between the two inversion methods are only found at lower energies; however, they are quite unimportant. We consider the inversion of the total $\Phi(\sigma, q)$ as the more accurate result, since we obtain a better fit to the calculated LT in the low-energy region. The total Coulomb sum rule is reproduced very precisely by the inversion of the total LT. We find relative errors of 0.2%, 0.4%, and 1.6% for $q = 300$, 400, and 500 MeV/c, respectively. The reason for the somewhat larger error at $q = 500$ MeV/c has been already discussed above.

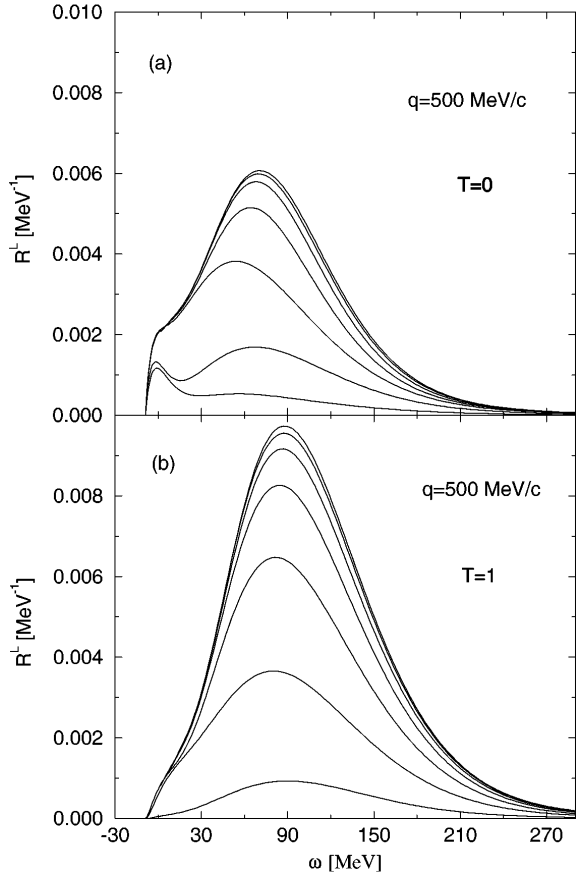

FIG. 2. Separate inversions of the various isoscalar (a) and isovector (b) multipoles of the LT ($q = 500$ MeV/c). The various curves correspond to successive addition of multipole contributions from $C0$ to $C6$.

After having demonstrated the precision of the method we compare our results with experimental data. To this end we have to consider that the response function of Eq. (1) is defined for point particles. In order to compare with experiment we have to multiply $R(\omega_{\rm lab}, q)$ with the square of the proton charge form factor $G_E^p(q^2 - \omega_{\rm lab}^2)$, where $\omega_{\rm lab} = \omega + q^2/2M(^4\mathrm{He})$. We take the dipole fit to G_E^p with the usual relativistic correction [16]. In Fig. 4

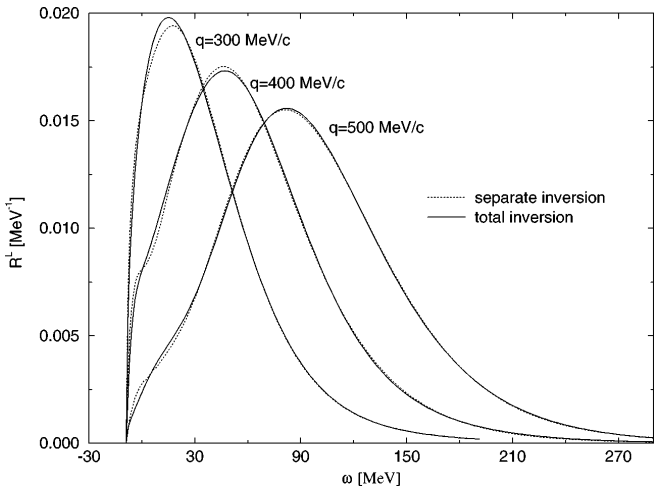

FIG. 3. Response functions from total (full curves) and separate inversions (dotted curves).

we show our results in comparison with experimental data [17,18]. It is readily evident that for the lower q value of 300 MeV/c the agreement between theory and experiment is very good. The low-energy wings of the responses at $q = 400$ and 500 MeV/c are also in a very good agreement with experiment. In particular, the rather complicated threshold structure of the experimental R_L at $q = 400$ and 500 MeV/c is described extremely well. Beyond the quasielastic peak the theoretical result overestimates the experimental one somewhat at $q =$ 400 MeV/c and in a more pronounced way at $q =$ 500 MeV/c. If the experimental results are correct, the theoretical formulation should include subnuclear and/or relativistic effects in order to remove the discrepancy.

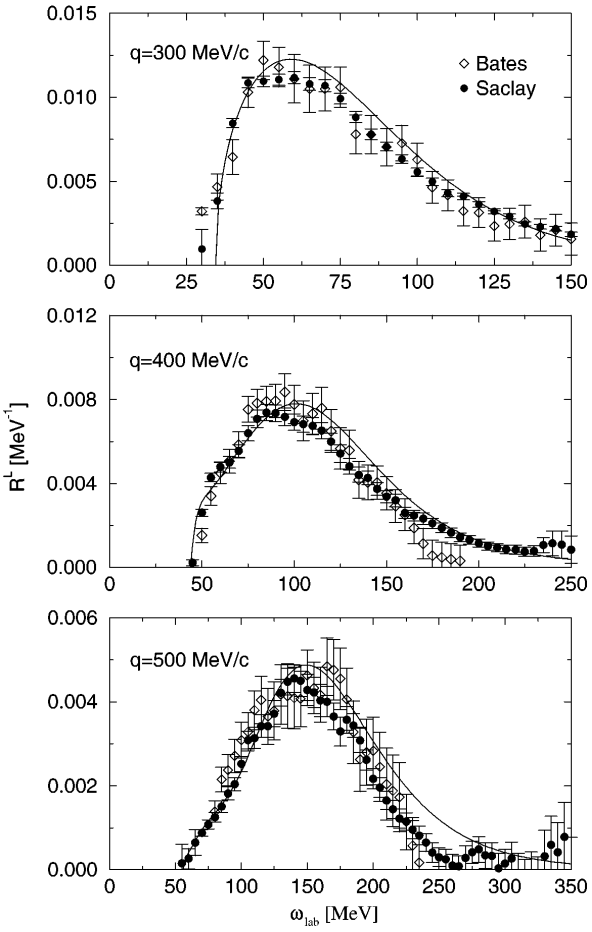

FIG. 4. Response functions from total inversions with inclusion of proton charge form factor (see text) in comparison to experimental data.

In conclusion, we may say that we have successfully applied the method of Ref. [1] to a four-body system response to an external probe with full final state interaction. This enabled us to calculate the accurate longitudinal response function of ^{4}He. We have shown that the results are very precise. We obtain an excellent agreement with experiment at the momentum transfer of 300 MeV/c as well as for the low-energy wings at $q = 400$ and 500 MeV/c. At the latter q values the theoretical results overestimate the experimental ones beyond the quasielastic peak. The calculation of the transverse response with the present potential model is in progress [8].

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