Measurement of the Electromagnetic Coupling at Large Momentum Transfer

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We report the first purely electroweak measurement of the strengthening of the electromagnetic coupling, α_{QED} with increasing momentum transfer Q^2, by comparing the process $e^+e^- \to \mu^+\mu^-$ with the process $e^+e^- \to e^+e^-\mu^+\mu^-$. The data were accumulated at an average center-of-mass energy $\langle\sqrt{s}\rangle = 57.77$ GeV using the TOPAZ detector at the TRISTAN e^+e^- collider. We measure $\alpha_{\mathrm{QED}}^{-1}(Q^2)$ to change from its known value of $\alpha_{\mathrm{QED}}^{-1} \cong 137.0$ at $Q^2 = 0$ to $128.5 \pm 1.8(\mathrm{stat}) \pm 0.7(\mathrm{syst})$ at $Q^2 = (57.77\ \mathrm{GeV}/c)^2$. This result agrees with electroweak predictions. [S0031-9007(96)02091-1]

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The theory of electromagnetism predicts that the coupling, α_{QED}, will strengthen with rising momentum transfer Q^2, while the strong coupling α_{strong} is predicted, and has been measured [1], to decrease with rising Q^2. The possibility that these two couplings may at some energy become equal has lead to speculation that new physics exists in that regime [2]. The strengthening of the electromagnetic force with rising Q^2 can be understood as a result of the "bare" charge causing a "polarization" of the vacuum. All charges are surrounded by clouds of virtual photons, which spend part of their existence dissociated into fermion-antifermion pairs. The virtual fermions with charges opposite to the bare charge will be, on average, closer to the bare charge than those virtual particles of like sign. Thus, at large distances, we observe a reduced bare charge due to this screening effect. As we probe closer we penetrate into the cloud of virtual particles, decreasing the screening effect and observing more of the bare charge and thus a strengthening of the coupling.

When comparing theoretical calculations with experimental results in the large Q^2 regime, it is found that the loop or vacuum graphs of the photon propagator, shown schematically in Fig. 1(a), can be effectively "absorbed" or eliminated by a redefinition of the QED coupling, making it Q^2 dependent. Calculations then rely on an evolution of the QED coupling from $Q^2 = 0$ where it is precisely measured $\alpha_{\mathrm{QED}}^{-1}(0) = 137.035\,989\,5 \pm 0.000\,006\,1$ [3] to its value $\alpha_{\mathrm{QED}}^{-1}(Q^2)$ at the Q^2 value of interest. This procedure of eliminating the vacuum graphs as well as the use of the Ward identity [4] greatly simplify theoretical calculations.

Because of the importance of the running coupling to physics, it should be observed experimentally. However, the variation of α_{QED} is only logarithmic with Q^2, requiring high energy experiments for direct observation. The TRISTAN e^+e^- collider at KEK is unique in its ability to measure this variation because of its large

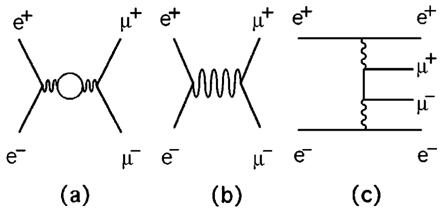

FIG. 1. Sample diagrams for (a) a loop or vacuum polarization graph, (b) the dominant diagram for $e^+e^- \to \mu^+\mu^-$, and (c) the multiperipheral process, the dominant process for "antitagged" $e^+e^- \to e^+e^-\mu^+\mu^-$ events.

center-of-mass energy dominated by electromagnetic photon exchange.

The Q^2 variation of α_{QED} has been studied before by Banerjee and Ganguli [5] by combining results from all LEP experiments. They found good agreement with the standard model. However, this is an estimation based on Z^0 parameters measured at the Z^0 peak, where the photon contribution is overwhelmed. Therefore, their work illustrates the consistency of the parametrization of the standard model and is not a direct measurement of $\alpha_{\text{QED}}(M_Z^2)$.

A more direct measurement of the coupling has been done by the TOPAZ group [6] using TOPAZ hadronic data normalized by low angle Bhabha scattering data. From this result, the coupling was measured to be $\alpha_{\text{QED}}^{-1}[Q^2 = (57.77\ \text{GeV}/c)^2] = 128.6^{+0.9}_{-0.8}(\text{stat})^{+2.7}_{-2.5}(\text{syst})$, where the uncertainty in the integrated luminosity dominated the systematic error. This result was found by assuming that quantum chromodynamic, QCD radiative corrections are well understood to high order, α_{strong}^3 in the strong coupling.

In this experiment we use an integrated luminosity of $268.0 \pm 4.0\ \text{pb}^{-1}$ [7] to measure the variation of α_{QED} as a function of Q^2 by taking the ratio of the measured number of $e^+e^- \to \mu^+\mu^-$ events, produced at average $Q^2 = (57.77\ \text{GeV}/c)^2$ [8], to the measured number of antitagged $e^+e^- \to e^+e^-\mu^+\mu^-$ events (where the final state electron and positron escape down the beam pipe, unobserved) produced with a median Q^2 of $7 \times 10^{-5}\ (\text{GeV}/c)^2$. These processes are shown schematically in Figs. 1(b) and 1(c), respectively. The cross section of the single photon process is proportional to $\alpha_{\text{QED}}^2[Q^2 = (57.77\ \text{GeV}/c)^2]$, while the two photon process is proportional to $\alpha_{\text{QED}}^4(Q^2 \cong 0)$.

Our method has several advantages: First, the systematic error is greatly reduced because we are using two similar data samples taken at the same time using the same detector subsystems, as well as the same data reduction software. Most notably, systematic error from an independent luminosity subsystem measurement is avoided, and detection efficiency errors generally cancel due to the similar event signatures for both processes within the detector. Second, this approach is less model dependent as these processes are purely electroweak in nature and require no assumptions about QCD. Third, at $\sqrt{s} = 57.77$ GeV, the Z^0 makes only a 6% contribution to the number of $e^+e^- \to \mu^+\mu^-$events produced so that single photon exchange dominates the interaction. Finally, this method avoids the need to make assumptions about the Q^2 evolution of the coupling in the normalizing sample. For example, in the TOPAZ experiment, the lowest angle Bhabha scattering data have an average Q^2 of about $5\ (\text{GeV}/c)^2$ with a corresponding coupling $\alpha_{\text{QED}}^{-1}[Q^2 = 5\ (\text{GeV}/c)^2] \sim 134$.

The data were analyzed with the TOPAZ detector, which is described in detail in Ref. [9]. The number of observed $e^+e^- \to \mu^+\mu^-(\gamma)$ events is predicted to be

$$N_{\mu^+\mu^-} = \mathcal{L}\, \alpha_{\text{QED}}^2(Q^2) g[1 + a\alpha_{\text{QED}}(0)],$$

where $N_{\mu^+\mu^-}$ is the number of single photon produced muons selected from the data, $\mathcal{L}$ is the integrated luminosity, g is the Born cross section with the coupling removed multiplied by the detection efficiency and acceptance [10], Q^2 is the square of the momentum transfer for single photon muon pair production, $(57.77\ \text{GeV}/c)^2$, and α_{QED} is the electromagnetic coupling at that momentum transfer. The constant a is a radiative correction to the Born term for events which satisfy the selection criteria. It contains only so-called external photonic corrections [11] for real ($Q^2 = 0$) photons. The radiative correction is small $[a\alpha_{\text{QED}}(0) = -0.1353 \pm 0.0017]$ because our event selection criteria, especially the requirement that the angle between the two muon momenta be greater than 170°, favor nonradiative events.

We estimate g and a separately, using the MINAMITATEYA Monte Carlo generator [12] and a complete detector simulation. The calculation of g is exact, and independent of the value of α_{QED}. For the calculation of a, the generator performs a complete electroweak calculation valid to order α_{QED}^3. While the calculation of a allowed α_{QED} to vary with Q^2, fixing α_{QED} in this calculation would have changed the correction, $a\alpha_{\text{QED}}(0)$ by less than 0.001.

The number of experimentally observed events for $e^+e^- \to e^+e^-\mu^+\mu^-$ can likewise be compared to theoretical predictions. Thus, we write

$$N_{e^+e^-\mu^+\mu^-} = \mathcal{L}\, \alpha_{\text{QED}}^4(0) f(1 + b\alpha_{\text{QED}}(0)),$$

where $N_{e^+e^-\mu^+\mu^-}$ is the number of two photon produced muons selected from the data; f is the Born cross section for the process $e^+e^- \to e^+e^-\mu^+\mu^-$ with the coupling removed and the effects of detector efficiency and acceptance included [10]. Finally, b is a radiative correction to the Born term for events which satisfy the selection criteria.

In this case, f was calculated with full detector simulation using the tree level Monte Carlo program of Kuroda [13], but is independent of the value of α_{QED}. This simulation produced excellent agreement with the experimental data [14].

The constant b was calculated as the ratio of two separate Monte Carlo generators [15,16] from Berends *et al.* The first calculates all the nonradiative or tree level diagrams. It was found that only the multiperipheral diagrams, Fig. 1(c), made a significant contribution given our selection criteria [17]. The second generates radiative corrections using the tree level multiperipheral diagrams and a subset [18] of the radiative graphs shown to make a significant contribution [19] to this process up to order α_{QED}^5.

The radiative calculation uses a fixed value of $\alpha_{\text{QED}}(0)$, but includes vacuum polarization graphs up to the appropriate order α_{QED}^5. The fact that the process $e^+e^- \to e^+e^-\mu^+\mu^-$ has a nonzero Q^2 value is taken into account as a correction to $\alpha_{\mathrm{QED}}(0)$ by these vacuum polarization graphs.

Comparing these Monte Carlo calculations, the radiative term $b\alpha_{\mathrm{QED}}(0)$ is found to be -0.007 ± 0.007. The smallness of this correction results from our demand that both the final state electron and positron not be observed in the detector.

Both the single photon generator and the two photon generator have theoretical errors, estimated by the authors [12,15,16]. The theoretical calculations used a Z mass of 91.189 GeV$/c^2$, a top quark mass of 176 GeV$/c^2$, and the Higgs mass was set to 300 GeV$/c^2$.

The ratio of the measured number of single photon events to two photon events can then be written as

$$\frac{N_{\mu^+\mu^-}}{N_{e^+e^-\mu^+\mu^-}} = \frac{\alpha^2_{\mathrm{QED}}(Q^2)}{\alpha^4_{\mathrm{QED}}(0)} \frac{g}{f} \frac{(1 + a\alpha_{\mathrm{QED}}(0))}{(1 + b\alpha_{\mathrm{QED}}(0))}.$$

The experimentally determined value of the electromagnetic coupling, as a function of Q, is then

$$\frac{\alpha^2_{\mathrm{QED}}(Q^2)}{\alpha^4_{\mathrm{QED}}(0)} = \frac{N_{\mu^+\mu^-}}{N_{e^+e^-\mu^+\mu^-}} \frac{f}{g} \frac{(1 + b\alpha_{\mathrm{QED}}(0))}{(1 + a\alpha_{\mathrm{QED}}(0))}.$$

The selection of $e^+e^- \to \mu^+\mu^-$ candidates is described in Refs. [20,21]. For this analysis, however, we made the following modifications which reduce tracking related systematic errors: (1) muons are identified by their penetration through an absorber into the muon chamber system [14] rather than by their energy deposition in the lead-glass calorimeter, and (2) the angular region of interest has been restricted to $0.1 \leq |\cos\theta_\mu| \leq 0.66$. With these modifications, the muon tagging procedure and geometric acceptance for $e^+e^- \to \mu^+\mu^-$ candidates are the same as those for $e^+e^- \to e^+e^-\mu^+\mu^-$ candidates. Other kinematic cuts on the sample were (3) the momentum transverse to the beam, of each muon, had to be larger than $E_{\mathrm{beam}}/3$, and (4) the angle between the two muons had to be greater than 170°.

With these criteria, the measured number of $e^+e^- \to \mu^+\mu^-$ events after background subtraction is $2775 \pm 52.7(\mathrm{stat}) \pm 8.7(\mathrm{syst})$. The selection contained backgrounds of $0.8\% \pm 0.07\%$ from τ pair production, $0.5\% \pm 0.3\%$ from cosmic ray muons, and $0.42\% \pm 0.01\%$ from $e^+e^- \to e^+e^-\mu^+\mu^-$ events.

The selection of $e^+e^- \to e^+e^-\mu^+\mu^-$ candidate events is described fully in Refs. [14,22]. The kinematic requirements on the sample were (1) the momentum transverse to the beam of each muon had to satisfy $1.9 \leq p_t \leq 10$ GeV$/c$, (2) the angle of the muon momenta with respect to the beam direction had to satisfy $0.1 \leq |\cos\theta_\mu| \leq 0.66$, and, finally, (3) the antitagging condition demanded that no electromagnetic energy of magnitude greater than $0.32E_{\mathrm{beam}}$ be deposited in the calorimetry system, which covers the angular region $3.2° \leq \theta \leq 176.8°$.

With these criteria, the measured number of antitagged $e^+e^- \to e^+e^-\mu^+\mu^-$ events, after background subtraction, is $2347 \pm 49(\mathrm{stat}) \pm 4.6(\mathrm{syst})$. The selection contained backgrounds of $1.2\% \pm 0.2\%$ due to cosmic ray events and $0.04\% \pm 0.02\%$ due to τ pair production. Other possible backgrounds, such as μ pair production, hadronic events, two photon τ pair production, two photon π production, as well as others were studied using Monte Carlo calculations and each was estimated to contribute less than 0.5 events background.

Combining the presented values yields the measurement of the electromagnetic coupling at $Q = 57.77$ GeV$/c$,

$$\alpha^{-1}_{\mathrm{QED}}((57.77\ \mathrm{GeV}/c)^2) = 128.5 \pm 1.8(\mathrm{stat}) \pm 0.7(\mathrm{syst}).$$

By taking the ratio of the two measurements, many systematic errors are eliminated. Those contributions to the systematic error that do not cancel, in terms of $\delta\alpha^{-1}_{\mathrm{QED}}/\alpha^{-1}_{\mathrm{QED}}$, are (a) 0.2% from differences in the time-of-flight hit efficiency as a function of muon momentum, affecting the trigger efficiency, (b) 0.2% and 0.1%, respectively, from uncertainty in the factors g and the $e^+e^- \to \mu^+\mu^-$ radiative correction, and (c) 0.3% and 0.4%, respectively, from uncertainty in the factors f and the $e^+e^- \to e^+e^-\mu^+\mu^-$ radiative corrections. Our measurement is limited by statistics.

Our measurement is in agreement with the theoretical prediction [23] of

$$\alpha^{-1}_{\mathrm{QED}}((57.77\ \mathrm{GeV}/c)^2) = 129.6 \pm 0.1.$$

The measurement is shown in Fig. 2 together with the theoretical prediction. Our experimental result differs from the value $\alpha_{\mathrm{QED}}(0)^{-1} = 137.0$ by 4.5 standard deviations. The probability of a fluctuation causing a deviation this large is about 7×10^{-6}.

Our previous publication [6] presented a measurement of the electromagnetic coupling using a technique uncorrelated to the present measurement. Combining our present measurement with our previous result we find

$$\alpha^{-1}_{\mathrm{QED}}((57.77\ \mathrm{GeV}/c)^2) = 128.6 \pm 1.6.$$

The combined TOPAZ result differs from $\alpha_{\mathrm{QED}}(0)$ by 5.3 standard deviations.

In conclusion, we have measured the electromagnetic coupling α_{QED} at a momentum transfer, $Q^2 = (57.77\ \mathrm{GeV}/c)^2$, using data taken with the TOPAZ detector at the TRISTAN e^+e^- collider at KEK, Japan's National Laboratory for High Energy Physics. Our measured value differs significantly from $\sim 1/137$ its known value at low energy. We find good and significant agreement with the prediction of the rise in strength of the electromagnetic coupling with increasing momentum transfer. The strong force has already been measured to decrease in strength with increasing momentum transfer [1]. Our measurement of the increase in strength of α_{QED} for large Q^2 supports the idea that the electromagnetic coupling

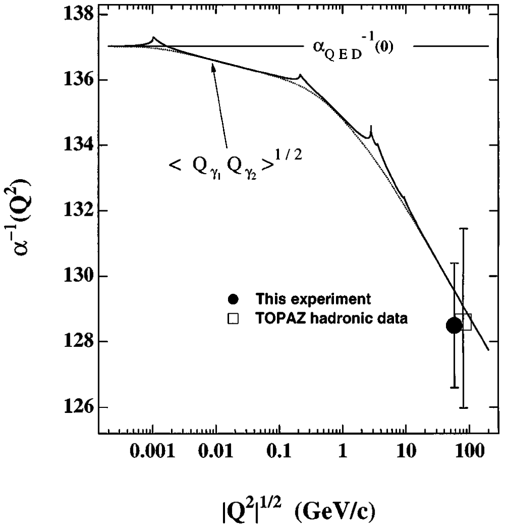

FIG. 2. The measured and theoretical electromagnetic coupling as a function of momentum transfer Q. The solid and dotted lines correspond to positive and negative Q^2 predictions, respectively. As we probe closer to the bare charge, its effective strength increases. $\langle Q_{\gamma_1} Q_{\gamma_2} \rangle^{1/2}$ denotes the square root of the median value for the product of the photon momentum transfers in the antitagged $e^+e^- \to e^+e^-\mu^+\mu^-$ sample. The hadronic data point has been shifted for display.

and the strong coupling are approaching each other in the limit of high momentum transfers.

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[7] The luminosity was measured using Bhabha scattering in a low angle calorimetry system, the FCL. See S. Noguchi *et al.* (Ref. [9]). It should be noted, however, that the luminosity is not used in the current measurement.

[8] We report our result at a $|Q|$ equal to the center-of-mass energy of the beams, which is the estimated median $|Q|$ value of our sample. Because of the emission of initial state radiation, the average $|Q|$ value of the sample is estimated to be 57.3 GeV$/c$. The difference represents a shift in the coupling $\delta\alpha^{-1}/\alpha^{-1} \approx 10^{-4}$.

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