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Transition from Phase Locking to the Interference of Independent Bose Condensates: Theory versus Experiment

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The macroscopic interference of two Bose condensates released from a double minimum potential has been demonstrated recently [M. R. Andrews *et al.,* Science **275**, 637 (1997)]. In this Letter we show the excellent agreement between those experiments and theoretical predictions based on the nonlinear Schrödinger equation. In addition, the transition from interference of coupled condensates, comparable with the Josephson effect in superconductors, to the interference of independent Bose condensates is studied. [S0031-9007(97)03260-2]

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The recent experimental breakthrough to Bose-Einstein condensation with small numbers of atoms collected in magnetic traps [1–4] has now culminated in the demonstration of the macroscopic coherence of Bose condensates [5]. The interference of two condensates released from a divided magnetic trap has provided compelling evidence for the intrinsic coherence of the many-atom ground state in the trap. This observation is closely related to the Josephson effect in superconductors. Quantum interference of many particles suggests a *classical* interpretation of the matter wave, resembling the classical limit of electromagnetic fields. Such a description using a macroscopic wave function with a definite, stable phase implies broken gauge symmetry. Hence the production of a coherent particle beam also amounts to the first demonstration of a pulsed "atom laser"—a milestone for the further development of atom optics, atom interferometry, and atom lithography.

The experiment reported in [5] illuminates two aspects of broken gauge symmetry. First, it proves the coherence properties of a single Bose condensate by the interference with a second one. In this Letter we will confirm the coherence of the condensate wave function by a detailed comparison between the measured condensate dynamics and its theoretical description in terms of a nonlinear Schrödinger equation [6]. Second, it enables one to study the transition from independent to coupled Bose condensates, since in the current experimental setup the separation between the condensates can be varied. For large separation the interference of initially independent condensates is studied; for small separation the initial condensates merge, and the situation can be regarded as an analog of a Josephson junction.

The broken gauge symmetry of a classical field implies that an interference pattern created by independent Bose condensates must depend on an arbitrary relative phase between the condensates. This phase varies randomly between different experimental runs to ensure the gauge symmetry of the ensemble, but not during a single run. Such a view has been confirmed by a detailed analysis of correlation functions [7,8]. The postulated variation of the interference pattern between different runs would reveal the spontaneous aspect of broken symmetry.

In contrast, if the external potential and the repulsive self-interaction lead to a non-negligible overlap between the two condensates, tunneling occurs and the condensates are no longer independent. Instead the system possesses a nondegenerate ground state, which for small coupling can be approximated by a well-defined coherent superposition of the two previously independent condensate wave functions. Since only this combined ground state is macroscopically occupied, the spatial phase of the interference pattern becomes locked.

The recent experimental data [5] show that a phase locking indeed occurs for small separations between the condensates. For larger separations, the experiment does not yet allow one to determine whether the spatial phase of the pattern varies randomly from shot to shot as expected for independent condensates. In this Letter we show a strong qualitative change in the shape of the interference pattern occurring when the chemical potential μ equals the barrier height V_0. For $\mu < V_0$, we find an interference pattern of slowly varying amplitude, nearly insensitive to a spatial phase shift. In contrast, for $\mu \geq V_0$, a pronounced central peak occurs, locking the position of the interference pattern.

In the recently used Ioffe trap [5] 5×10^6 sodium atoms are collected in a cigar-shaped trap volume which is divided into two halves along the longitudinal (x) direction by a sheet of far-blue detuned laser light. The longitudinal and transverse frequencies are $\omega_x = 2\pi \times 19$ Hz and $\omega_{y,z} = 2\pi \times 250$ Hz, respectively. A laser beam of wavelength 514 nm is focused into a light sheet of 12 μm by 97 μm with a variable power of typically several mW. The height of the potential barrier V_0 is directly proportional to the laser power and can be varied accordingly. The condensate expansion is started by switching off both the magnetic and the laser field and eventually leads to an overlap of the two clouds and to an expanding interference pattern.

The dynamics is determined by a numerical integration of the time-dependent Gross-Pitaevskii equation (GPE) [6,8–10]

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \left(-\frac{\hbar^2 \nabla^2}{2m} + U|\psi(\mathbf{r}, t)|^2\right)\psi(\mathbf{r}, t), \quad (1)$$

where U is the strength of the atom-atom interaction, $U = \frac{4\pi\hbar^2 a}{m}$, and ψ is normalized to the condensate number $\int |\psi|^2 d\mathbf{r} = N_0$. For the scattering length we use a value of $a = 3$ nm [11]. The validity of the GPE is limited to a time scale where the perturbing effects of collisions on the condensate phase are negligible. The joint experimental and theoretical investigations of the interference prove that this *coherence time* is much longer than the interaction time. Previous experimental and theoretical results concerning the Bose gas dynamics could be explained by a hydrodynamical treatment of the GPE. However, in the present work, true wavelike features of the dynamics, e.g., the interference, are also included and directly compared with the experiment for the first time.

The initial ground state of the double well potential is calculated numerically by an imaginary time propagation accounting for the kinetic energy. In the case of coupled condensates this method unambiguously yields the nondegenerate ground state. However, for well separated condensates the ground state is virtually degenerate and can be written as a coherent superposition

$$\psi(\mathbf{r}) = \psi_1(\mathbf{r}) + \psi_2(\mathbf{r})e^{i\phi}$$

of the two independent condensate wave functions $\psi_{1,2}(\mathbf{r})$ with an (*a priori*) arbitrary relative phase ϕ. Also in this case the imaginary time propagation yields the symmetric combination with $\phi = 0$ as initial state.

We now turn to the comparison of the theoretical results with the experimental data. First we study the dependence of the fringe period on the potential barrier, i.e., the laser power. We recall that in the case of noninteracting atoms the expected interference term superimposes an equidistant density modulation upon the two spreading density distributions, reaching 100% visibility in the central region [10]. Two initial Gaussian wave packets of width σ_0 separated by a distance $2d$ produce a spatial modulation term

$$\cos\left(\frac{2mdx}{\hbar(t + t_0^2/t)} + \phi\right),$$

where $t_0 = m\sigma_0^2/\hbar$ [12]. For independent condensates the spatial phase ϕ is the random parameter that ensures the gauge symmetry of the ensemble. For well separated condensates the t_0^2/t term can be neglected and the distance x_f between adjacent interference maxima thus varies asymptotically as $x_\mathrm{f} = \pi\hbar t/(md)$. This linear time dependence is approached very rapidly also for interacting atoms, provided that the initial condensates are initially well separated, i.e., by more than σ_0. For an interacting atom cloud, the interference pattern is modified mainly because of the initial acceleration of the atoms. However, our numerical results show that the acceleration decreases due to the decreasing density, and an asymptotic behavior of the central fringe spacing according to $x_\mathrm{f} = \tilde{v}_\mathrm{f} t + x_0$ is reached. The shift $x_0 > 0$ depends on the initial distribution and can be found only by the integration of the GPE. It has not been allowed for in Fig. 3(a) of Ref. [5]. However, the numerical data for the asymptotic expansion velocity are well extrapolated by $\tilde{v}_\mathrm{f} = \pi\hbar/m\tilde{d}$ [10], where $\tilde{d}$ depends on the shifted center of gravity $\langle|x|\rangle = \int_0^\infty x\rho(\mathbf{r})$ of the initial condensates

$$\tilde{d} = \sqrt{\langle|x|\rangle d}\,. \quad (2)$$

Figure 1 gives the separation of the first and second fringe as a function of the laser power.The agreement between experimental (crosses) and theoretical extrapolation (solid line) is best at large laser power, i.e., for large light shifts and large initial separations of the condensates. The expansion velocity of the fringes, as well as the actual fringe spacing at a given time, are found to vary inversely proportionally to the length $\tilde{d}$. For small light shifts, however, the potential barrier no longer provides a complete separation of the condensates. As a result the width of the central fringe increases. It is no longer inversely proportional to $\tilde{d}$, but still in excellent agreement with the integration of the GPE (triangles).

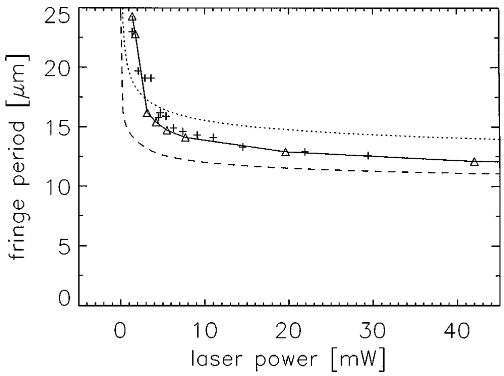

FIG. 1. Separation of the fringe minima next to the center as a function of the laser power, which is proportional to the height of the potential barrier between the condensates. (+) mark experimental points, (△) calculated points. The upper line gives the prediction $x_{\mathrm{f}} = \pi \hbar t / m d$, the lower one $x_{\mathrm{f}} = \pi \hbar t / m \tilde{d}$, for $t = 40$ ms. The theoretical points are connected by straight lines to guide the eye.

The buildup and broadening of the central peak can be traced back to an initial wave function which does not show complete separation. Figure 2(a) gives the initial condition, showing nearly separated condensates, for a barrier created with a laser power of $P = 1.4$ mW. Figure 2(b) shows the calculated interference pattern at time $t = 40$ ms. Figure 3 shows the experimental patterns for the same laser power, at time $t = 0$ ms (a), and $t = 40$ ms (b). In both Figs. 2 and 3 the central fringe clearly dominates at $t = 40$ ms, and the initial condition does not show a clear separation. Thus, we are close to the point at which the condensates become coupled due to their incomplete initial separation. The observed patterns [see also Fig. 2(a) in Ref. [5]) are only consistent with calculated patterns with a fixed phase $\phi = 0$. The theory thus allows us to pin down a definite phase of the initial wave function. The transition is reached at a laser power around 3.6 mW in the

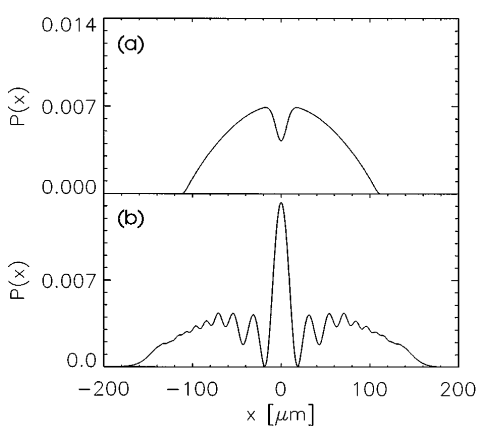

FIG. 2. Theoretical density patterns for a laser power $P = 1.4$ mW. (a) Cut through the initial distribution; (b) cut taken at time $t = 40$ ms.

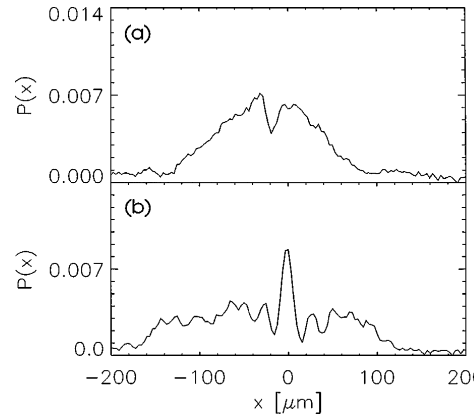

FIG. 3. Experimental density patterns for the same value of the laser power $P = 1.4$ mW. (a) Cut through the initial distribution; (b) cut taken at time $t = 40$ ms.

recent experiment, and happens quite sharply. Of course, uncertainties with regard to the atom number and the laser beam profile translate into an uncertainty of the above transition.

We finally compare the measured fringe patterns and the theoretical predictions with respect to the finite optical resolution in the experiment. Figure 4(a) displays a theoretical interference pattern at time $t = 40$ ms, for given experimental parameters, whereas Fig. 4(b) shows the experimental data for the same parameters and should be directly compared with Fig. 4(a). We note three features. First, the agreement concerning the fringe spacing in the central region is excellent. No marked central peak can be singled out. Second, the experimental contrast of the interference is reduced in the central region and vanishes rapidly outside the center. Third, a narrowing of the fringe spacing occurs in the wings of the theoretical interference

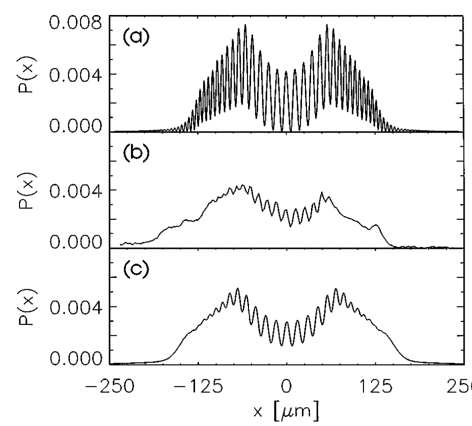

FIG. 4. Interference patterns in a horizontal plane for two independent Bose-Einstein condensates expanding during free fall at $t = 40$ ms. (a) Result from numerical integration of the time-dependent GPE, (b) experimental data for a laser power $P = 5.5$ mW, and (c) the result from (a) convoluted with an optical transfer function with contrast of 40%.

pattern. We relate the last two observations by considering the combined role of interactions and of imperfect optical resolution. Andrews *et al.* have already roughly estimated that the actual visibility of the central interference fringes should be close to 100%. The resolution of the imaging system was determined in Ref. [5] by recording the interference from a standard optical test pattern, giving a contrast of 40% at the spatial period of the condensate. To account for this, we have to consider the convolution of the density pattern with a Gaussian kernel

$$K(x) = \exp(-x^2/2\gamma^2)/\sqrt{\pi}\,\gamma\,,$$

where γ measures the smallest resolved distance. By convolution with $K(x)$, an ideal sinusoidal pattern with spatial period $2\pi/k$ acquires a reduced visibility of $\mathcal{V} = \exp(-k^2\gamma^2/2)$. From the experimentally determined value $\mathcal{V} = 40\%$ at $k = 2\pi/(15\ \mu\mathrm{m})$ we can therefore deduce $\gamma = 3.2\ \mu$m, and now convolute our theoretical data with the kernel $K(x)$ using this value of γ. The resulting theoretical pattern is given in Fig. 4(c). The dramatic improvement in matching the experimental data is strong evidence for a true visibility close to 100%. The narrower fringes at the wings of the pattern are not resolved anymore after convolution with $K(x)$. The theoretical patterns clearly show that the reduction of visibility in the wings of the distribution is not due to a loss of coherence of the overlapped condensates. The effect of fringe narrowing itself is solely due to the repulsive mean field of the condensates and vanishes at small nonlinearity.

In principle, a reduction of the visibility can be deduced from the decay of the correlation function. Such a decay follows from the particle number dispersion in an interacting Bose gas [13,14]. The resulting decay of the first order correlation function takes the form $g^{(1)}(\tau) = \exp[-\tau^2/(2\tau_c^2)]$. For Poissonian atom number fluctuations we estimate τ_c for the data of the MIT experiment with $\mu = h \times 5.1$ kHz and $N_0 = 5 \times 10^6$ to be on the order of $\tau_c = \frac{5\hbar\sqrt{N_0}}{\sqrt{2}\mu} \simeq 0.24s$. In the current experimental situation, this correlation time is larger than the full expansion time and much larger than the exposure time to record an interference pattern. Inserting the interaction time of 40 ms, the phase diffusion should induce no more than a 2% decay of the correlation, which translates directly into the visibility. This is an upper bound for the effects of the particle number dispersion.

The detailed comparison between theory and experiment has not only yielded perfect agreement for independent as well as for coupled condensates. It has also lead to conclusions that could not be obtained from the experiment alone. Most importantly, it has been proven that the phase is maintained during the entire interference experiment and over a spatial extension of up to 400 μm. Maintenance of coherence can therefore be expected also in possible applications of atomic Bose-Einstein condensation.

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[1] M.H. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, Science **269**, 198 (1995).
[2] K.B. Davis, M.-O. Mewes, M.R. Andrews, N.J. van Druten, D.S. Durfee, D.M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[3] M.-O. Mewes, M.R. Andrews, N.J. van Druten, D.S. Durfee, D.M. Kurn, and W. Ketterle, Phys. Rev. Lett. **77**, 416 (1996); M.-O. Mewes *et al.*, Phys. Rev. Lett. **77**, 988 (1996).
[4] D.S. Jin, J.R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, Phys. Rev. Lett. **77**, 420 (1996).
[5] M.R. Andrews, C.G. Townsend, H.-J. Miesner, D.S. Durfee, D.M. Kurn, and W. Ketterle, Science **275**, 637 (1997).
[6] E.P. Gross, Nuovo Cimento **20**, 454 (1961); L.P. Pitaevskii, Sov. Phys. JETP **13**, 451 (1961).
[7] J. Javanainen and S.M. Yoo, Phys. Rev. Lett. **76**, 161 (1996).
[8] M. Naraschewski, H. Wallis, A. Schenzle, J.I. Cirac, and P. Zoller, Phys. Rev. A **54**, 2185 (1996).
[9] W. Hoston and L. You, Phys. Rev. A **53**, 4254 (1996).
[10] H. Wallis, A. Röhrl, M. Naraschewski, and A. Schenzle, Phys. Rev. A **55**, 2109 (1997).
[11] The spectroscopic measurement of E. Tiesinga *et al.*, J. Res. Natl. Inst. Stand. Technol. **101**, 505 (1996), yielded $a = 2.75 \pm 0.3$ nm, whereas the measurements of [3] and the calculations of Y. Castin and R. Dum, Phys. Rev. Lett. **77**, 5315 (1996), are consistent with $a = 3.4 \pm 1.6$ nm. The accuracy of the integration method has been shown in [10].
[12] For strong nonlinearity (high-atom number) the approximation of the initial wave functions by Gaussians becomes insufficient.
[13] E.M. Wright, D.F. Walls, and J.C. Garrison, Phys. Rev. Lett. **77**, 2158 (1996).
[14] M. Lewenstein and L. You, Phys. Rev. Lett. **77**, 3489 (1996).