## **Precision Higgs Boson Mass Determination at Lepton Colliders**

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We demonstrate that a measurement at future colliders of the Bjorken process  $e^+e^-$ ,  $\mu^+\mu^- \rightarrow ZH$ in the threshold region can yield a precise determination of the Higgs boson mass. With an integrated luminosity of 100 fb<sup>-1</sup> it is possible to measure the standard model Higgs mass to within 45 MeV (60 MeV) at  $\mu^+\mu^-$  ( $e^+e^-$ ) collider for  $m_H = 100$  GeV. [S0031-9007(97)03221-3]

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One of the triumphs of the LEP program was the measurement of the Z-boson mass to 2 MeV. Expectations are also quite good for the measurement of the W-boson mass  $(M_W)$  and the top quark mass  $(m_t)$  in the future, perhaps achieving precision of order 10 MeV for  $M_W$  and 2 GeV for  $m_t$  at the Tevatron and the LHC [1]. Precise values for  $M_W$  and  $m_t$  can also be obtained at lepton colliders by measuring the  $\ell^+\ell^- \rightarrow WW$  and  $\ell^+\ell^- \rightarrow t\bar{t}$  ( $\ell = e \text{ or } \mu$ ) threshold cross sections, as studied in [2]. These measurements will allow an indirect prediction for the Higgs boson mass  $(m_H)$  and will test the consistency of the standard model (SM) at the two-loop level once  $m_H$  is known [3].

In this Letter we point out that, analogously, a very accurate determination of  $m_H$  is obtained by measuring the threshold cross section for the Higgs-strahlung process [4]  $\ell^+\ell^- \rightarrow ZH$ . With integrated luminosity  $L = 100 \text{ fb}^{-1}$ , a  $1\sigma$  precision of order 45 (60) MeV is possible for a  $m_H = 100 \text{ GeV SM Higgs at}$  a  $\mu^+\mu^-$  ( $e^+e^-$ ) collider. This error in  $m_H$  is smaller than that achievable via final state mass reconstruction for a typical detector, and would then be the most accurate determination of  $m_H$  at an  $e^+e^-$  collider.

The SM Higgs boson is easily discovered in the *ZH* production mode by running the machine well above threshold, e.g., at  $\sqrt{s} = 500$  GeV. For  $m_H \leq 2M_W$  the dominant Higgs boson decay is to  $b\bar{b}$  and most backgrounds can be eliminated by *b* tagging. At the next linear  $e^+e^-$  collider (NLC) the accuracy for  $m_H$  via reconstruction using final state momenta is strongly dependent on the detector performance and signal statistics:  $\Delta m_H \approx R_{\text{event}} (\text{GeV})/\sqrt{N}$ , where  $R_{\text{event}}$  is the single-event resolution and *N* is the number of signal events. At an SLD-type detector, the single-event resolution for reconstruction of the Higgs mass is about 4 GeV for most *ZH* final states [5]. For the SM Higgs boson, when running at  $\sqrt{s} = 500$  GeV the accuracy expected for the  $m_H$  determination is then [5]

$$\Delta m_H \simeq 180 \text{ MeV} \left(\frac{50 \text{ fb}^{-1}}{L}\right)^{1/2}$$
. (1)

which takes into account effective branching ratios and

appropriate efficiencies. The accuracy obtained for a "super"-LC detector, assuming substantial luminosity at a lower energy, would be more competitive with that we shall obtain via the threshold technique [6,7]. However, in the not unlikely case that the detector is of the SLD-type, the best means for measuring  $m_H$  will be to first determine  $m_H$  to within a few hundred MeV in  $\sqrt{s} = 500$  GeV running, which will also yield a precise measurement of  $\sigma(ZH)$ , and then reconfigure the collider for maximal luminosity in the threshold energy region  $\sqrt{s} \approx M_Z + m_H$ .

In Fig. 1 we show the cross section for the Bjorken process  $\ell^+\ell^- \rightarrow ZH$  for Higgs masses from 50 to 150 GeV. Since the threshold behavior is *S* wave, the rise in the cross section in the threshold region is rapid, as can be seen for the case of  $m_H = 100$  GeV in the inset figure, the cross section being a few tenths of a pb. Also shown in the inset figure are the effects of initial state radiation (ISR) and beam energy smearing assuming a



FIG. 1. The cross section vs  $\sqrt{s}$  for the process  $\ell^+\ell^- \rightarrow Z^*H \rightarrow f\bar{f}H$  for a range of Higgs masses. The inset shows the detailed structure for  $m_H = 100$  GeV in the threshold region. Also shown in the inset are the effects of initial state radiation (ISR) and beam energy smearing assuming a Gaussian spread  $R_e = 1\%$  for  $e^+e^-$  and  $R_\mu = 0.1\%$  for  $\mu^+\mu^-$ .

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Gaussian spread  $R_e = 1\%$  for  $e^+e^-$  and  $R_\mu = 0.1\%$  for  $\mu^+\mu^-$ . These effects reduce the sharp rise of the cross section to some extent. At LEP II, the few hundred pb<sup>-1</sup> of luminosity that might be devoted to such a threshold would yield just a handful of events. However, much higher luminosity is possible at threshold at the NLC [8] or a muon collider [9,10].

In the ideal case that the normalization of the measured ZH cross section as a function of  $\sqrt{s}$  can be precisely predicted, including efficiencies and systematic effects, sensitivity to the SM Higgs boson mass is maximized by a single measurement of the cross section at  $\sqrt{s} = M_Z$  +  $m_H$  + 0.5 GeV, just above the real particle threshold. With a  $\sim \pm 180$  MeV measurement of  $m_H$  from initial running [see Eq. (1)],  $\sqrt{s}$  can be set quite close to this optimal point. As an example of the precision that might be achieved, suppose  $m_H = 100$  GeV and backgrounds are neglected. The ZH cross section is 120 fb and is rising at a rate of 0.05 fb/MeV. With L = 50 fb<sup>-1</sup> and including an overall (b tagging, geometric, and event identification) efficiency of 40%, this yields  $2.4 \times 10^3$ events, or a measurement of the cross section to about 2%. From the slope of the cross section one concludes that a  $m_H$  measurement with accuracy of roughly 50 MeV is possible.

In practice, there will be systematic errors associated with experimental efficiencies as well as for theoretical predictions of the ZH cross section and H branching ratio(s) that will be very difficult to reduce below the 1% level. The ratio of the cross section measured at  $\sqrt{s}$  well above threshold in the initial H discovery to that measured right at threshold is thus the key to determining  $m_H$ . The theoretical uncertainties will cancel in the ratio. Given the high luminosity that should be available for measurements both well above threshold and right at threshold, changes in b tagging, geometrical efficiencies, and jet misidentification as a function of  $\sqrt{s}$  may be understood at the <1% level, provided the final-focus reconfiguration required to optimize luminosity at the lower threshold  $\sqrt{s}$  does not impel detector changes that would lead to significant changes in the experimental systematic effects.

For a more precise estimate of the accuracy with which  $m_H$  can be measured, we employ *b* tagging and cuts in order to reduce the background to a very low level. Specifically, we require: (1) tagging of both *b*'s in the event (for which an overall 50% efficiency will be assumed); (2)  $|M_{b\bar{b}} - m_H| < 5$  GeV; (3) 80  $< M_{\text{recoil}} < 105$  GeV (i.e., broadly consistent with  $M_Z$ ), where  $M_{\text{recoil}} \equiv [p_{\text{recoil}}^2]^{1/2}$  with  $p_{\text{recoil}} = p_{\ell^+} + p_{\ell^-} - p_b - p_b^-$ ; and (4)  $|\cos \theta_{b,\bar{b},\text{recoil}}| < 0.9$ , where  $\theta$  is the polar angle with respect to the beam direction. [Note that the restriction on  $M_{\text{recoil}}$  means that constructive interference of *ZH* diagrams with *WW* (*ZZ*) fusion diagrams in the  $\nu_\ell \bar{\nu}_\ell H$  ( $\ell^+ \ell^- H$ ) channels [11] will be small.] With these cuts the only significant background will be that



FIG. 2. The  $\ell^+\ell^- \to ZH \to Zb\bar{b}$  signal and the irreducible  $\ell^+\ell^- \to Zb\bar{b}$  background vs  $\sqrt{s}$ , including *b* tagging and cut requirements (1)–(4); see text.

from ZZ production, where at least one of the Z's decays to  $b\bar{b}$ . In Fig. 2, we compare the cross section versus  $\sqrt{s}$  for the  $\ell^+\ell^- \rightarrow Zb\bar{b}$  background to that for the  $\ell^+\ell^- \rightarrow ZH$  (with  $H \rightarrow b\bar{b}$ ) signal, where the signal is computed for  $m_H = \sqrt{s} - M_Z - 0.5$  GeV. The background is very much smaller than the signal unless  $m_H$ is close to  $M_Z$ . Initial state radiation and beam energy spread are not included in this figure.

The expected precision for the SM Higgs mass is given in Fig. 3 for an integrated luminosity of 100 fb<sup>-1</sup>. The precision degrades as  $m_H$  increases because the signal cross section is smaller (see Fig. 1). The background from the Z peak reduces the precision for  $m_H \approx M_Z$ . Effects of ISR and beam energy smearing are included here, assuming a Gaussian spread  $R_e = 1\%$  and  $R_{\mu} =$ 0.1%. These effects yield a reduction in sensitivity to the SM Higgs mass of 15% at a muon collider and 35% at an  $e^+e^-$  collider. A precision of the SM Higgs mass determination to within 45 MeV (60 MeV) for  $m_H =$ 100 GeV may be available at a muon  $(e^+e^-)$  collider.

The analysis can be generalized beyond the SM, with the ZZH coupling  $(g_{ZZH})$  and the total Higgs width  $(\Gamma_H)$  taken to be free parameters. In order to simultaneously determine  $m_H$ ,  $g_{ZZH}$ , and  $\Gamma_H$ , measurements could be made at the three c.m. energies  $\sqrt{s} = m_H +$  $M_Z + 20$  GeV,  $\sqrt{s} = m_H + M_Z + 0.5$  GeV, and  $\sqrt{s} =$  $m_H + M_Z - 2$  GeV. The  $e^+e^- \rightarrow ZH \rightarrow Zb\bar{b}$  rates at the first two of these energies would simultaneously determine  $m_H$  and  $\sigma(ZH)B(H \rightarrow b\bar{b})$ , where  $\sigma(ZH) \propto g_{ZZH}^2$ . The inclusive (recoil spectrum) ZH event rate directly measures  $\sigma(ZH)$ , and  $B(H \rightarrow b\bar{b})$  can then be computed from  $\sigma$  and  $\sigma B$ . The subthreshold measurement will be sensitive to the width, if  $\Gamma_H$  is of order 100 MeV or larger.



FIG. 3. The precision  $\Delta m_H$  attainable from a 100 fb<sup>-1</sup> measurement of the  $Zb\bar{b}$  cross section at  $\sqrt{s} = M_Z + m_H + 0.5$  GeV as a function of  $m_H$ , including *b* tagging and cuts (1)–(4). Initial state radiation and beam energy spread are included. Solid curves are results for a  $\mu^+\mu^-$  collider and dashed curves for an  $e^+e^-$  collider. A precise measurement of the cross section well above threshold is presumed to be available.

The solid curves in Fig. 4 show the statistical precision that can be obtained for  $m_H = 100$  GeV in a threeparameter fit to  $m_H$ ,  $g^2_{ZZH}B(H \rightarrow b\bar{b})$ , and  $\Gamma_H$ , including initial state radiation and beam energy spread at a muon collider, using an integrated luminosity of 100/3 fb<sup>-1</sup> at each of the above three values of  $\sqrt{s}$  in the threshold region. The crosses in the center of the ellipses indicate the input values. With a three-parameter fit, the attainable error in  $m_H$  is about 110 MeV at the  $1\sigma$  level. The second panel in Fig. 4 shows that there is significant sensitivity to the Higgs width  $\Gamma_H$  if it is of order 100 MeV. If  $\Gamma_H$  is very narrow (~3 MeV is predicted in the SM), then  $\Delta m_H \sim \pm 80$  MeV is possible (see the dashed ellipse) from a fit to  $m_H$  and  $\sigma(ZH)B(H \rightarrow b\bar{b})$  by devoting 50 fb<sup>-1</sup> at each of the two c.m. energies,  $\sqrt{s} =$  $m_H + M_Z + 20$  GeV and  $\sqrt{s} = m_H + M_Z + 0.5$  GeV.

Since  $m_H$  is determined by the ratio of the cross sections at the two energies, systematic uncertainties will cancel almost completely for such closely spaced energies, and the error in  $m_H$  will be almost entirely statistical. The measurement of  $\sigma(ZH)B(H \rightarrow b\bar{b})$  would be at the  $\pm 2\%$  statistical level (which is better than the precision that can be reached with L = 200 fb<sup>-1</sup> accumulated at  $\sqrt{s} = 500$  GeV [12]); at this level of statistical error, the systematic uncertainties on  $\sigma B$  from *b* tagging, geometrical cuts, and event-identification efficiencies will probably dominate. We note that electroweak radiative corrections to the cross section are estimated to be less than 1% for  $m_H \sim 100$  GeV [13].

Two comments are particularly relevant. First, a  $\pm 60$  (45) MeV uncertainty on  $m_H$  achievable at an  $e^+e^-$  ( $\mu^+\mu^-$ ) collider would allow almost immediate centering on the *s*-channel Higgs resonance peak at a muon collider (thereby avoiding expending luminosity on



FIG. 4. Solid curves show the  $\Delta \chi^2 = 1$  contours for determining the Higgs mass versus  $g_{ZZH}^2 B(H \rightarrow b\bar{b})$ , or versus  $\Gamma_H$ , by devoting 100/3 fb<sup>-1</sup> to each of the c.m. energies  $\sqrt{s} = M_Z + m_H + 0.5$  GeV.  $\sqrt{s} = M_Z + m_H + 20$  GeV and  $\sqrt{s} = M_Z + m_H - 2$  GeV at a muon collider; *b* tagging and cuts (1)–(4) are imposed and initial state radiation and beam energy smearing are included. A Higgs mass  $m_H = 100$  GeV is assumed. The dashed curve shows the  $\Delta \chi^2 = 1$  contour that results when  $\Gamma_H$  is negligibly small and 50 fb<sup>-1</sup> is devoted to each of the c.m. energies  $\sqrt{s} = m_H + M_Z + 0.5$  GeV and  $\sqrt{s} = m_H + M_Z + 20$  GeV.

a scan location of the peak). For  $m_H \leq 2M_W$ , a fine scan over the *s*-channel Higgs resonance at a muon collider would then yield an extraordinarily precise determination of  $m_H$  along with a determination of the total *H* width that is far more accurate [9] than achievable by other means [12] in this mass region. Second, an accurate determination of  $m_H$  could prove to be of considerable value in testing the consistency of the radiative corrections to the Higgs mass.

In conclusion, we have shown that with sufficient luminosity it is possible to determine the Higgs boson mass to a high level of precision by measuring the  $\ell^+\ell^- \rightarrow ZH \rightarrow Zb\bar{b}$  cross section just above threshold and normalizing to a second measurement either well above threshold or near the ZH cross section peak. A third measurement below the threshold may be sensitive to  $\Gamma_H$  if it is sufficiently larger than the SM value. Such measurements would simultaneously determine  $m_H$ ,  $\sigma(ZH)B(H \rightarrow b\bar{b})$  and  $\Gamma_H$  at a level of accuracy that could distinguish a standard model Higgs boson from its many possible (e.g., supersymmetric) extensions.

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