Comment on "Onsager Reciprocity Relations without Microscopic Reversibility"

In a recent paper Gabrielli, Jona-Lasinio, and Landim [1] point out, among other things, that the Onsager reciprocity relations may hold, even though the dynamics is *not* invariant under time reversal. This is an important observation since the standard derivation of the Onsager relations requires time reversal symmetry [2], and their validity in more general contexts could be very useful, cf. [3]. To demonstrate their claim, Gabrielli, Jona-Lasinio, and Landim construct a model of a two-component lattice gas (in one dimension) which violates detailed balance, but for which the diffusion matrix is still computable. The corresponding Onsager matrix is then explicitly found to be symmetric.

It is the purpose of this Comment to point out that the model considered, while of considerable interest, is of a very special type [1]. It is an example of a class of binary (or general multicomponent) systems having a particular "mirror-type" symmetry between the two components. For these systems, Onsager reciprocity is valid essentially independent of the microscopic dynamics. While this is interesting by itself, it casts doubt about the extension of the results of [1] to more general systems.

To describe the above class of mixtures we can start with a one-component system of particles (without momentum conservation) whose macroscopic density profile is governed by the nonlinear drift-diffusion equation [3]

$$\frac{\partial}{\partial t}\rho(\mathbf{r},t) + \nabla\cdot[\mathbf{j}(\rho) - D(\rho)\nabla(\mathbf{r},t)] = 0\,. \qquad (1)$$

Examples are stochastic lattice gases with or without external driving, such as the asymmetric zero-range process of [1], and classical particles subject to a constant external electric field and inelastic, impurity or phonon, scattering.

We now color the particles, say, blue ($i = 1$) and red ($i = 2$), independently with probability p_i. This internal degree of freedom is simply carried along by the dynamics. (Physical examples which approximate such idealizations could be different isotopes or different spin states which are unchanged by interactions.) A little thought shows that if we call ρ_i the density of component with color i then the two-component drift-diffusion equation will have the general form

$$\frac{\partial}{\partial t}\rho_i + \nabla\cdot\left[\mathbf{j}_i(\rho_1,\rho_2) - \sum_{j=1}^{2} D_{ij}(\rho_1,\rho_2)\nabla\rho_j\right] = 0\,, \quad i = 1,2\,, \qquad (2)$$

with $\mathbf{j}_i = (\rho_i/\rho)\mathbf{j}$, $\rho \equiv \rho_1 + \rho_2$, and D_{ij} to be determined.

To obtain D_{ij} we note that a color-blind person, who only sees the total density profile ρ, could sum Eq. (2) to obtain Eq. (1). Also $D_{12} \to 0$ as $\rho_1 \to 0$, $D_{21} \to 0$ as $\rho_2 \to 0$. Thus D is necessarily of the general form $D_{ij} = b\delta_{ij} + c\rho_i$ with b, c depending only on ρ. $b(\rho)$ is the self-diffusion coefficient of a single tagged particle in the system with uniform density ρ [4] and $b + \rho c = D$; cf. [5], where this form is obtained from an explicit computation.

We now want to write Eq. (2) in the Onsager force-flux form. This requires that we know the entropy of the two component system $s(\rho_1,\rho_2)$. Let the uncolored system have an "entropy" $s(\rho)$, cf. [3] for a definition in terms of steady state fluctuations, then, since the coloring is done independently, the two-component system will have the entropy

$$s(\rho_1,\rho_2) = -\frac{\rho_1}{\rho}\ln\frac{\rho_1}{\rho} - \frac{\rho_2}{\rho}\ln\frac{\rho_2}{\rho} + s(\rho)\,. \qquad (3)$$

Thus $\partial^2 s/\partial\rho_i\partial\rho_j = (\chi^{-1})_{ij} = -(\rho_i)^{-1}\delta_{ij} - a$, with a depending only on the total density ρ, and $\chi_{ij} = -(1 + a\rho)^{-1}(\rho_i\delta_{ij} + a\rho_1\rho_2(-1)^{i+j})$. One now readily checks that the Onsager matrix L [1–3], which is defined by $L_{ij} = (D\chi)_{ij}$, is indeed symmetric. Our analysis readily extends to the case where D is symmetric d by d matrix.

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