One-Dimensional Spin-Liquid without Magnon Excitations

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(Received 2 December 1996)

It is shown that a sufficiently strong four-spin interaction in the spin-$1/2$ spin ladder can cause dimerization. Such interaction can be generated either by phonons or (in the doped state) by the conventional Coulomb repulsion between the holes. The dimerized phases are thermodynamically undistinguishable from the Haldane phase, but have dramatically different correlation functions: the dynamical magnetic susceptibility, instead of displaying a sharp single magnon peak near $q = \pi$, shows only a two-particle threshold separated from the ground state by a gap. [S0031-9007(97)03059-7]

PACS numbers: 75.10.Jm, 75.40.Gb

The qualitative difference between the universal properties of one-dimensional Heisenberg antiferromagnets with half-integer and integer spin, originally predicted by Haldane [1], is now well understood. Typical examples are the $S = 1/2$ and $S = 1$ chains. The spin $S = 1/2$ chain has a quasiordered singlet ground state with algebraically decaying spin correlations. Its spectrum is gapless, and the elementary excitations (spinons), carrying spin $1/2$, in all physical states with integer total spin appear only in pairs [2]. Deconfinement of the spinons implies that conventional $S = 1$ magnons fail to be stable quasiparticles: the spectral density of the staggered magnetization $\mathbf{n}(x) \sim (-1)^n \mathbf{S}_n$ shows a purely incoherent background.

An alternative picture of a disordered spin liquid with a Haldane gap is effectively realized when two $S = 1/2$ Heisenberg chains are put together to form a spin ladder [3]. In the standard model of spin ladders the interchain interaction is the Heisenberg exchange $J_\perp$; at $J_\perp \neq 0$ the spinons confine to form triplet (magnon) and singlet excitations with gaps m_t and m_s ($m_t, m_s \sim J_\perp$). The triplet excitations contribute a coherent δ peak to the dynamical spin susceptibility $\chi''(q,\omega)$ near $q = \pi$ and $\omega = m_t$ [4]. In this respect the spin liquid behaves like a conventional magnet with the only difference that here the magnons are "optical"; that is, they have a spectral gap.

In this Letter we discuss an example of a disordered spin liquid with a gapful spectrum but *without* coherent magnon excitations. Such a state can be realized in an extended model of the spin $S = 1/2$ Heisenberg ladder which, apart from the direct transverse exchange $J_\perp$, also includes a four-spin interaction

$$\mathcal{H} = J_\parallel \sum_{j=1,2} \sum_n \mathbf{S}_j(n) \cdot \mathbf{S}_j(n+1) + \sum_n \{J_\perp \mathbf{S}_1(n) \cdot \mathbf{S}_2(n) + V[\mathbf{S}_1(n) \cdot \mathbf{S}_1(n+1)] \times [\mathbf{S}_2(n) \cdot \mathbf{S}_2(n+1)]\}. \quad (1)$$

It will be assumed that $|J_\perp|, |V| \ll J_\parallel$. The new term in (1) represents an interchain coupling of the spin-dimerization fields $\epsilon_j(n) = (-1)^n \mathbf{S}_j(n) \cdot \mathbf{S}_j(n+1)$ ($j = 1, 2$) which can be either (i) effectively mediated by spin-phonon interaction (see below) or (ii) in the doped phase generated by the conventional Coulomb repulsion between the holes moving in the spin correlated background [5].

In a single $S = 1/2$ Heisenberg chain described in the continuum limit in terms of a massless scalar field $\Phi_s(x)$ [6], the dimerization operator $\epsilon(x) \sim \cos\sqrt{2\pi}\,\Phi_s(x)$ is a strongly fluctuating field with the same scaling dimension $1/2$ as that of the three components of the staggered magnetization $(-1)^n \mathbf{S}_n \sim \mathbf{n}(x) \sim (\cos\sqrt{2\pi}\,\Theta_s(x), \sin\sqrt{2\pi}\,\Theta_s(x), -\sin\sqrt{2\pi}\,\Phi_s(x))$, where Θ_s is the field dual to Φ_s. Therefore the interchain coupling term $V\epsilon_1\epsilon_2$, being as relevant as the transverse exchange $J_\perp \mathbf{n}_1 \cdot \mathbf{n}_2$, may significantly affect the low-energy properties of spin ladders. Indeed, while for small $|V|$ the generalized model (1) occurs in the same Haldane spin liquid phase as that of the conventional spin ladder ($V = 0$), at large enough $|V|$ the model (1) displays *non-Haldane* spin liquid phases characterized by a *spontaneous* dimerization. The thermodynamic properties of these phases, determined by the absolute values of the gaps, are indistiguishable from those for the Haldane phase. However, the correlation functions of the two systems differ drastically: in a non-Haldane spin liquid we are going to discuss the spectral function of $\mathbf{n}$ is entirely exhausted by an incoherent background. This picture should be opposed to the dimerized (spin-Peierls) $S = 1/2$ Heisenberg chain with alternating exchange $J_{n,n+1} = J[1 + \delta(-1)^n]$ where, due to the presence of the kink-antikink bound states in the spin excitation spectrum, triplet magnons constitute well defined, coherent quasiparticles.

It has been shown earlier [4,5] that, in the continuum limit, the Hamiltonian (1) decouples into four massive real fermionic fields, or equivalently, four noncritical 2D Ising models exhibiting the underlying SU(2) $\times Z_2$ symmetry: $H = \sum_{a=1,2,3} H_{m_t}[\xi_a] + H_{m_s}[\xi_0]$, where

$$H_m[\xi] = -\frac{iv_s}{2}(\xi_R \partial_x \xi_R - \xi_L \partial_x \xi_L) - im\xi_R \xi_L . \quad (2)$$

The triplet and singlet masses are given as follows:

$$m_t = \pi a_0 (J_\perp |a_s|^2 - V|a_c|^2), \qquad m_s = -\pi a_0 (3J_\perp |a_s|^2 + V|a_c|^2), \quad (3)$$

where a_c and a_s are nonuniversal constants. The thermodynamic properties are determined only by $|m_t|, |m_s|$, but the symmetry of the ground state and the behavior of the dynamical sisceptibility $\chi(q,\omega)$ crucially depend on the relative signs of the masses m_t and m_s as well. This follows from the fact that the representation for the staggered spin density and dimerization operators in terms of the Ising order and disorder variables [4] depends on the sign of $m_t m_s$. It will be assumed below that $V < 0$, while the sign of $J_\perp$ can be arbitrary. We shall show that for different signs of m_t and m_s, as long as the triplet branch of the spectrum remains the lowest ($|m_t| < |m_s|$), the two-chain ladder is in the Haldane-liquid phase with coherent $S = 1$ and $S = 0$ single-magnon excitations. If m_t and m_s have the same sign, the ground state represents a dimerized, spin-disordered phase in which coherent magnon modes are absent.

Transitions from the Haldane to dimerized phases take place when either the triplet excitations become gapless, with the singlet mode still having a finite gap, or vice versa. The transition at $m_t = 0$ belongs to the universality class of the critical, exactly integrable, $S = 1$ spin chain [7]; the corresponding non-Haldane phase with $|m_t| < |m_s|$ represents the dimerized state of the $S = 1$ chain with spontaneously broken translational symmetry and doubly degenerate ground state [8]. The critical point $m_s = 0$ is of the Ising type; it is associated with a transition to another dimerized phase ($|m_t| > |m_s|$), not related to the $S = 1$ chain.

We start our discussion by considering the standard $J_\parallel$-$J_\perp$ two-chain ladder, with spin-phonon coupling included via a substitution $J_\parallel \rightarrow J_{\parallel,j}(n, n+1) = J_\parallel + \lambda[u_j(n) - u_j(n+1)]$. Only the staggered part of the lattice displacement field along the chains, $u_j(n)$, couples to the spin-dimerization operator $\epsilon_j(n)$. We assume that $\omega_0 \gg J_\perp$, where ω_0 is the phonon frequency at $2k_F$, as is the case for most ladder systems known. In this limit, the $2k_F$ phonons can be treated in terms of a quantum Gaussian random field whose main effect is mediation of an instantaneous effective coupling between the spin-dimerization fields of each chain. We replace the staggered parts of $u_j(n)$ by real scalar fields $\Delta_j(x)$, $u_j(n) = (-1)^n (a/2\lambda)\Delta_j(x)$, and ignore the kinetic energy of vibrations. Then integrating over the displacement fields Δ_j we get the effective interaction

$$\Delta S_{\rm eff}[\epsilon] = -\frac{g_0^2(1+\gamma/4)}{2+\gamma}\int dx\, d\tau (\epsilon_1^2 + \epsilon_2^2) - \frac{g_0^2 \gamma}{2(2+\gamma)}\int dx\, d\tau\, \epsilon_1 \epsilon_2 , \quad (4)$$

where $g_0 = \lambda/\sqrt{K_\parallel a}$ is the spin-phonon coupling constant, and $\gamma = K_\perp/K_\parallel$ is the ratio of the longitudinal and transverse spring constants. The interchain coupling term $\sim\epsilon_1\epsilon_2$ in (4) is a relevant perturbation with dimension 1, as opposed to terms $\sim\epsilon_j^2$ which are only marginal and therefore can be neglected. This explains the origin of the extra V-term in the extended spin ladder model (1) and fixes the value of the constant $V \sim -g_0^2\gamma/(2+\gamma) < 0$.

Let us consider various phases of the model (1). We assume that $V < 0$, while the sign of $J_\perp$ can be arbitrary. If $J_\perp < 0$, the mass $m_s > 0$, while the sign of m_t depends on the strength of $|V|$ such that for small $|V|$ the signs of the two masses are opposite. All results obtained in Ref. [4] are applicable to this case, and we present them briefly for completeness. The total ($\mathbf{n}^+ = \mathbf{n}_1 + \mathbf{n}_2$) and relative ($\mathbf{n}^- = \mathbf{n}_1 - \mathbf{n}_2$) staggered magnetizations are given by [4]

$$\mathbf{n}^+ \sim (\sigma_1\mu_2\sigma_3\sigma_0,\ \mu_1\sigma_2\sigma_3\sigma_0,\ \sigma_1\sigma_2\mu_3\sigma_0), \quad (5)$$

$$\mathbf{n}^- \sim (\mu_1\sigma_2\mu_3\mu_0,\ \sigma_1\mu_2\mu_3\mu_0,\ \mu_1\mu_2\sigma_3\mu_0), \quad (6)$$

where σ_a and μ_a ($a = 1,2,3$) are order and disorder parameters of three, SU(2) degenerate noncritical Ising models corresponding to the massive triplet of the Majorana fields ξ_a, while σ_0, μ_0 refer to the fourth, "singlet" Ising model (ξ_0). The Ising representation for total and relative dimerization fields, $\epsilon_\pm = \epsilon_1 \pm \epsilon_2$, can be similarly found to be

$$\epsilon_+ \sim \mu_1\mu_2\mu_3\sigma_0, \qquad \epsilon_- \sim \sigma_1\sigma_2\sigma_3\mu_0 . \quad (7)$$

The triplet mass determines the deviation of the Ising models from criticality: $m_t \sim (T - T_c)/T_c < 0$; this corresponds to the ordered Ising phase with $\mu_j = 0, \langle\sigma_j\rangle \neq 0$ ($j = 0,1,2,3$), in which the two-point correlation functions are given by [9]

$$\langle\sigma(\mathbf{r})\sigma(\mathbf{0})\rangle = G_\sigma(mr) \simeq A\left[1 + \frac{1}{8\pi(mr)^2}e^{-2|m|r} + O(e^{-4|m|r})\right], \quad (8)$$

$$\langle\mu(\mathbf{r})\mu(\mathbf{0})\rangle = G_\mu(mr) \simeq \frac{A}{\pi}K_0(|m|r) + O(e^{-3|m|r}), \quad (9)$$

where A is a nonuniversal parameter, $K_0(x)$ is the MacDonald function, and $\mathbf{r} = (x, v_s\tau)$.

In the region of parameters where $|m_t| \ll |m_s|$, the lowest (triplet) part of the spin ladder spectrum describes universal properties of the $S = 1$ spin chain with the conventional and biquadratic exchange, $H = J\sum_n[\mathbf{S}_n \cdot \mathbf{S}_{n+1} - \beta(\mathbf{S}_n \cdot \mathbf{S}_{n+1})^2]$, near the critical point $\beta = 1$ [7]. This correspondence is valid for any sign of m_t. The present case $m_t < 0$ describes the Haldane massive phase ($\beta < 1$); the leading asymptotics for the spin correlation functions obtained from (5)–(9)

$$\langle\mathbf{n}^+(\mathbf{r})\mathbf{n}^+(\mathbf{0})\rangle \sim K_0(|m_t|r)[1 + O(e^{-2|m_t|r})], \qquad \langle\mathbf{n}^-(\mathbf{r})\mathbf{n}^-(\mathbf{0})\rangle \sim \frac{1}{r^{3/2}}e^{-(2|m_t|+m_s)r} \quad (10)$$

reveal the role of $\mathbf{n}^+$ which determines the staggered magnetization of the effective $S = 1$ chain. Since $K_0(|m|r)$ is the real-space propagator of a free massive boson, $\langle \mathbf{n}^+(\mathbf{r})\mathbf{n}^+(\mathbf{0})\rangle$ contributes a δ peak to the imaginary part of the dynamical spin susceptibility corresponding to a massive triplet magnon. At $q \sim 0$, $\chi''(\omega, q)$ is determined by the correlations of slow components of the total and relative magnetization, giving rise to thresholds at $2|m_t|$ and $|m_t| + m_s$, respectively. Similarly, one finds

$$\begin{aligned} \langle \epsilon_-(\mathbf{r})\epsilon_-(\mathbf{0})\rangle &\sim K_0(|m_s|r)[1 + O(e^{-2m_t r})], \\ \langle \epsilon_+(\mathbf{r})\epsilon_+(\mathbf{0})\rangle &\sim \frac{1}{r^{3/2}}\, e^{-3m_t r}, \end{aligned} \tag{11}$$

implying that the singlet magnon is also a coherently propagating particle—an elementary excitation of the relative dimerization. Excitations of the total dimerization have a $3|m_t|$ threshold. On increasing $|V|$, $|m_t|$ decreases while m_s increases, the inequality $|m_t| < m_s$ thus becoming stronger which makes the low-energy effective picture of the gapful Haldane phase of the $S = 1$ spin chain only better. At $m_t = 0, m_s = 4|J_\perp|$ the triplet of the Majorana fields becomes massless, while the singlet Majorana fermion remains massive. At this point the correlation functions of the relative staggered magnetization and dimerization field follow power laws: $\langle \mathbf{n}^-(\mathbf{r})\mathbf{n}^-(\mathbf{0})\rangle \sim \langle \epsilon_-(\mathbf{r})\epsilon_-(\mathbf{0})\rangle \sim r^{-3/4}$. This critical point belongs to the universality class of the level $k = 2$ SU(2)-symmetric Wess-Zumino-Novikov-Witten model with the central charge $c = 3/2$ [7].

Further increase of $|V|$ makes m_t positive. The change of sign of $m_t m_s$ amounts to the duality transformation in the singlet (ξ_0) Ising system, implying that in formulas (6)–(8) the order (σ_0) and disorder (μ_0) parameters must be interchanged. Moreover, since we are now in the disordered Ising phase ($m_t \sim T - T_c > 0$), $\langle \mu_0 \rangle \neq 0, \langle \sigma_0 \rangle = 0$, the right-hand sides of formulas (8) and (9) should also be interchanged. As a result, the spin and dimerization correlation functions are now given by different expressions:

$$\begin{aligned} \langle \mathbf{n}^+(\mathbf{r}) \cdot \mathbf{n}^+(\mathbf{0})\rangle &\sim K_0^2(m_t r)\,, \\ \langle \mathbf{n}^-(\mathbf{r}) \cdot \mathbf{n}^-(\mathbf{0})\rangle &\sim K_0(m_t r)K_0(m_s r)\,, \end{aligned} \tag{12}$$

$$\begin{aligned} \langle \epsilon_+(\mathbf{r})\epsilon_+(\mathbf{0})\rangle &\sim C\left[1 + O\left(\frac{e^{-2m_{t,s}}}{r^2}\right)\right], \\ \langle \epsilon_-(\mathbf{r})\epsilon_-(\mathbf{0})\rangle &\sim K_0^3(m_t r)K_0(m_s r)\,, \end{aligned} \tag{13}$$

where C is a constant.

From (13) we conclude that the new phase is characterized by long-range dimerization ordering along each chain, with zero relative phase: $\langle \epsilon_1 \rangle = \langle \epsilon_2 \rangle = (1/2)\langle \epsilon_+ \rangle$. In the decoupling limit, $J_\perp = V = 0$, each Heisenberg chain possesses a Z_2 symmetry: this is the symmetry with respect to the interchange of even and odd sublattices generated by one lattice spacing translation. The interchain coupling lowers this $Z_2 \times Z_2$ down to Z_2, the symmetry under *simultaneous* translations by a_0 on the both chains. Thus, the onset of dimerization is associated with spontaneous breakdown of the residual translational Z_2 symmetry, taking place when the interaction $|V|$ exceeds a critical value $|V_c| \sim J_\perp$. This phase coincides with the dimerized phase of the generalized $S = 1$ spin chain with a biquadratic exchange [8].

In the dimerized phase the spin correlations undergo dramatic changes. From (12) we obtain

$$\begin{aligned} \operatorname{Im}\chi_+(\omega, \pi - q) &\sim \frac{\theta(\omega^2 - q^2 - 4m_t^2)}{m_t\sqrt{\omega^2 - q^2 - 4m_t^2}}\,, \\ \operatorname{Im}\chi_-(\omega, \pi - q) &\sim \frac{\theta[\omega^2 - q^2 - (m_t + m_s)^2]}{\sqrt{m_t m_s}\sqrt{\omega^2 - q^2 - (m_t + m_s)^2}}\,, \end{aligned} \tag{14}$$

where $\pm$ signs refer to the case where the wave vector in the direction perpendicular to the chains is equal to 0 and π, respectively. We observe the disappearance of coherent magnon poles in the dimerized spin fluid; instead we find two-magnon thresholds at $\omega = 2m_t$ and $\omega = m_t + m_s$, similar to the structure of $\chi''(\omega, q)$ at small wave vectors in the Haldane fluid phase. The fact that two massive magnons, each with momentum $q \sim \pi$, combine to form a two-particle threshold, still at $q \sim \pi$ rather than $2\pi \equiv 0$ is related to the fact that, in the dimerized phase with $2a_0$ periodicity, the new umklapp is just π.

To get a better understanding of the fact that in the dimerized phase the spectral weight of the spin excitations is entirely incoherent, it is instructive to consider the limiting case $J_\perp = 0$. Since ϵ_j ($j = 1, 2$) are scalars in spin space, the model (1) displays the SU(2) $\times$ SU(2) $\approx$ SO(4) symmetry with respect to independent spin rotations on each chain. Thus, the spectrum is described by O(4) quadruplet of massive Majorana fermions or equivalently, two noninteracting massive Dirac fermions—quantum solitons of two decoupled sine-Gordon models with the coupling constant $\beta^2 = 4\pi$:

$$H = H_+ + H_- = \sum_{s=\pm}\left\{\frac{v_s}{2}[\Pi_s^2 + (\partial_x \Phi_s)^2] - \frac{m}{\pi a_0}\cos\sqrt{4\pi}\,\Phi_s\right\}. \tag{15}$$

The massive Dirac fermions are in fact quantum domain walls (kinks) connecting two Z_2-degenerate dimerized vacua of the two-chain system with $\langle \epsilon_+ \rangle = \pm|\epsilon_0|$. This can be shown by the following simple consideration. A dimerization kink assumes local changes $\epsilon_1 \to -\epsilon_1$, $\epsilon_2 \to -\epsilon_2$. This is equivalent to simultaneous translations by a_0 on each chain, under which $\Phi_j \to \Phi_j \pm \sqrt{\pi/2}$. This, in turn, reduces to one of two possibilities: $\Phi_+ \to \Phi_+ + \sqrt{\pi}$, $\Phi_- \to \Phi_-$, or $\Phi_+ \to \Phi_+$, $\Phi_- \to \Phi_- + \sqrt{\pi}$. But $\sqrt{\pi}$ is just the period of the cosine potentials in (15). Therefore, a single kink of the relative dimerization is nothing but a quantum sine-Gordon soliton (Dirac fermion) carrying a unit topological charge, either in the (+) or (−) channel. Now, in all physical excitations defined in the sector with zero total topological charge, the solitons can appear only in pairs. Since two or more massive solitons (Dirac fermions) cannot propagate coherently, and since there are no soliton-antisoliton bound states at $\beta^2 = 4\pi$ (*free fermions*), one has to conclude that there will be no particlelike δ function peaks in the spectral function of *any* local physical quantity of the system.

Now let us consider the case $J_\perp > 0$, when $m_t > 0$ while m_s may change its sign. If $m_s < 0$, the definitions (5)–(7) still hold, but since $m_t \sim T - T_c > 0$, we are in the disordered Ising phase. This case can be mapped onto the $J_\perp < 0$ one by interchanging all order and disorder parameters. As long as $|m_s| > m_t$, the triplet branch of the spectrum describes the Haldane phase with the relative magnetization $\mathbf{n}^-$, forming effectively the staggered component of the spin density in the $S = 1$ spin chain: $\langle \mathbf{n}^-(\mathbf{r}) \cdot \mathbf{n}^-(\mathbf{0})\rangle \sim K_0(|m_t|r)$.

On increasing $|V|$ $|m_s|$ decreases, and the inequality $|m_s| > m_t$ will eventually be replaced by the opposite one, $|m_s| < m_t$. One might argue that as long as $|x| \gg \xi_s$ (the maximal correlation length $\xi_s \sim v_s/|m_s|$), the asymptotic r dependence of the $\mathbf{n}^-$ correlator remains intact and the coherent magnon peak, though with a reduced amplitude, should still exist. However, this is not so because the large-distance ($|\mathbf{r}| \gg \xi_s$) asymptotics of the $\mathbf{n}^-$ correlator determines its Fourier transform $\langle \mathbf{n}^- \cdot \mathbf{n}^-\rangle_{q,\omega}$ at $|\pi - q|, |\omega| \ll m_s$, so that energies $|\omega| \sim m_t$ are not accessible. At these energies $\chi''(q\omega)$ is mainly contributed by the asymptotics of the correlators $\langle \mathbf{n}^\pm(\mathbf{r}) \cdot \mathbf{n}^\pm(\mathbf{0})\rangle$ in the range $\xi_t \geq |x| \gg \xi_s$ ($\xi_t \sim v_s/m_t$), where μ_0 cannot be replaced by a constant:

$$\langle \mathbf{n}^-(\mathbf{r})\mathbf{n}^-(\mathbf{0})\rangle \sim \frac{1}{r^{1/4}} K_0(m_t r), \qquad \langle \mathbf{n}^+(\mathbf{r})\mathbf{n}^+(\mathbf{0})\rangle \sim \frac{1}{r^{1/4}} K_0^2(m_t r). \tag{16}$$

Thus, even before reaching the critical point $m_s = 0$, the δ peak in dynamical susceptibility disappears, and the effective picture of the Haldane liquid breaks down when, due to softening of the singlet mode, $|m_s|$ becomes comparable with m_t.

At $m_s = 0$ ($|V| = 3J_\perp$) the singlet excitations become gapless, and the intermediate asymptotics (16) are now exact. The dynamical spin susceptibility $\chi''_\mp(q\omega)$ displays thresholds at $\omega = m_t$ and $\omega = 2m_t$, respectively. Near the thresholds

$$\mathrm{Im}\,\chi_+(\omega, \pi - q) \sim \frac{\theta(\omega^2 - q^2 - 4m_t^2)}{(\omega^2 - q^2 - 4m_t^2)^{3/8}}, \qquad \mathrm{Im}\,\chi_-(\omega, \pi - q) \sim \frac{\theta(\omega^2 - q^2 - m_t^2)}{(\omega^2 - q^2 - m_t^2)^{1/8}}, \tag{17}$$

The correlation function of the total dimerization follows a power law $\langle \epsilon^+(\mathbf{r})\epsilon^+(\mathbf{0})\rangle \sim r^{-1/4}$. Thus the critical point $m_s = 0$ belongs to the Ising universality class (with central charge $c = 1/2$) and signals a transition to a spontaneously dimerized phase with $\langle \epsilon^+\rangle \neq 0$. In the dimerized phase ($m_s > 0$) the singlet mass gap is always the smallest indicating that the spin ladder is not in the regime of an effective $S = 1$ spin chain.

The Haldane liquid has a nonlocal topological (string) order parameter [10] whose nonzero value is associated with the breakdown of a hidden discrete ($Z_2 \times Z_2$) symmetry. In the two-chain realization of the Haldane $S = 1$ phase the string order parameter is expressed in terms of the Ising order and disorder variables [4]:

$$\lim_{|x-y|\to\infty} \langle O_{\mathrm{string}}(x,y)\rangle \sim \langle\sigma_1\rangle^2\langle\sigma_2\rangle^2 + \langle\mu_1\rangle^2\langle\mu_2\rangle^2 \tag{18}$$

(notice that the singlet mode does not appear in this expression). Since the Ising systems are noncritical in both phases, it follows from (18) that the string order parameter will be nonzero in the dimerized phases as well, vanishing only at the critical point $m_t = 0$.

Thus we have demonstrated that the Haldane spin liquid is not the only possible phase of a disordered magnet. The distinctive features of another dimerized phase, which can be tested by inelastic neutron scattering and NMR experiments is the absence of coherent single-magnon modes and the π periodicity of the spin excitation spectrum, as opposed to the undimerized phase where the ratio of the energy gaps at $q \sim 0$ and $q \sim \pi$ is 2. The Haldane and dimerized phases are separated by critical points, either of the Ising type or belonging to the universality class of the critical $S = 1$ quantum spin chain.

We are grateful to Yu Lu, Ilya Krive, and Alexander Gogolin for valuable discussions. One of us (A. A. N.) acknowledges hospitality and support from the International Centre for Theoretical Physics.

[1] F. D. M. Haldane, Phys. Lett. **93A**, 464 (1983).
[2] L. D. Faddeev and L. A. Takhtajan, Phys. Lett. **85A**, 375 (1981).
[3] See, for a review, E. Dagotto and T. M. Rice, Science **271**, 618 (1996), and references therein.
[4] D. G. Shelton, A. A. Nersesyan, and A. M. Tsvelik, Phys. Rev. B **53**, 8521 (1996).
[5] D. G. Shelton and A. M. Tsvelik, Phys. Rev. B **53**, 14 036 (1996).
[6] I. Affleck, Nucl. Phys. **B265**, 409 (1986).
[7] P. Kulish, N. Reshetikhin, and E. Sklyanin, Lett. Math. Phys. **5**, 393 (1981); L. Takhtajan, Phys. Lett. **90A**, 479 (1982); H. Babujan, Phys. Lett. **93A**, 464 (1982).
[8] I. Affleck, J. Phys. C **1**, 3047 (1989).
[9] T. T. Wu, B. McCoy, C. A. Tracy, and E. Barouch, Phys. Rev. B **13**, 316 (1976).
[10] M. den Nijs and K. Rommelse, Phys. Rev. B **40**, 4709 (1989); S. M. Girvin and D. P. Arovas, Phys. Scr. **T27**, 156 (1989).