Nuclear Thermometers from Isotope Yield Ratios

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The apparent temperatures measured with double ratios of fragment isotope yields display fluctuations that can be attributed to the sequential decay of heavier particle unstable nuclei. Empirical correction factors which minimize these fluctuations have been obtained for 18 isotope thermometers. These factors appear to be common to the three reactions studied suggesting their application to different reactions at different incident energies. [S0031-9007(97)03214-6]

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Understanding the bulk phases and phase transitions of strongly interacting matter is one of the major objectives for investigations of nucleus-nucleus collisions. The focus of such investigations at excitation energies of the order of $E^*/A \approx 3$–15 MeV is upon the transition between liquid and gaseous phases of nuclear matter [1,2], which calculations predict to be characterized by the copious emission of intermediate mass fragments $3 \leq Z \leq 30$ [2–4]. This emission is likely to occur within a narrow temperature range $4 \leq T \leq 7$ MeV [3,4], bounded at low temperatures by fragment emission barriers and at high temperature by nuclear vaporization [2]. Experimental measurements support features of this interpretation [5–19], but precise determination of the relevant densities and temperatures corresponding to the relevant experimental observations is needed. The latter requires the development of reliable "thermometers" [5].

Temperatures can be extracted from energy spectra by assuming kinetic equilibrium, but are often problematic due to nonthermal collective contributions to the spectra. Temperatures have been extracted from excited state populations [6–12] or isotope ratios [13–19] by assuming chemical equilibrium. Both of these latter measurements are insensitive to collective effects but are sensitive to the secondary decay of particle unstable nuclei after the system disintegrates [7–12,15]. Correction for secondary decay is relatively straightforward for excited state population measurements; however, such measurements require high resolution and detection efficiency and are hard to achieve with high statistical precision [7–12]. Statistically precise measurements of isotope ratios are easier to achieve, but the resulting temperatures appear to depend upon the specific ratios of isotopes examined [15–19]. Understanding the origin of these discrepancies and learning how to control them are important scientific objectives.

If chemical and thermal equilibrium are achieved, one may obtain temperature information from a double isotope ratio defined by [13]

$$R = \frac{Y(A_i, Z_i)/Y(A_i + \Delta A, Z_i + \Delta Z)}{Y(A_j, Z_j)/Y(A_j + \Delta A, Z_j + \Delta Z)}, \qquad (1)$$

where $Y(A_i, Z_i)$ is the total yield of the emitted fragment with mass and charge number A_i and Z_i. ΔA and ΔZ are chosen to be the same for both the numerator and the denominator to cancel out the effects of proton and neutron chemical potentials [13]. To minimize the influence of Coulomb barriers, the emphasis of previous and present investigations has been upon isotopes with $\Delta Z = 0$ and $\Delta A = 1$. In the following, results from relaxing these constraints upon ΔA and ΔZ will also be briefly discussed.

If the ground state yields in Eq. (1) are consistent with thermal equilibrium at breakup, they may be related to the corresponding temperature (T_o) [13] by

$$T_o = \frac{B}{\ln(aR_o)}, \qquad (2)$$

where R_o is the ground state fragment yield ratio, B is a binding energy parameter, and a is the statistical factor that depends on statistical weights of the ground state nuclear spins. In particular,

$$B = BE(A_i, Z_i) - BE(A_i + \Delta A, Z_i + \Delta Z) - BE(A_j, Z_j) + BE(A_j + \Delta A, Z_j + \Delta Z), \qquad (3)$$

$$a = \frac{[2S(A_j, Z_j) + 1]/[2S(A_j, +\Delta A, Z_j + \Delta Z) + 1]}{[2S(A_i, Z_i) + 1]/[2S(A_i, +\Delta A, Z_i + \Delta Z) + 1]} \times \left[\frac{A_j/(A_j + \Delta A_j)}{A_i/(A_i + \Delta A_i)}\right]^{\eta}. \qquad (4)$$

Here, $BE(A_i, Z_i)$ and $S(A_i, Z_i)$ are the known binding energy and ground state spin of a fragment with mass A_i and charge Z_i. The exponent η arises from an integration over the energy spectrum and equals 1.5 for volume emission via a Maxwellian distribution [20] and 1.0 for surface emission via the Weisskopf formula [21], for

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example. In either limit, this factor is very close to unity for $\Delta A = 1$ and consequently unimportant.

Experimentally one cannot measure T_o directly. Instead, one can only construct the "apparent" temperatures

$$T_{\rm app} = \frac{B}{\ln(aR_{\rm app})} \tag{5}$$

from experimentally measured fragment yield ratios $R_{\rm app}$ that are altered from the equilibrium values by the effects of sequential decays and other processes neglected by the simple theoretical model of Eq. (2).

The effects of sequential decay and the accuracy of Eq. (2) can be addressed by examining the consistency of a large number of such double ratio thermometers. Until now, tests have been performed using a few ($\leq$10) thermometers constructed from the yields of H, He, Li, Be, B, and C [13–19] nuclei. To construct a more significant test, we utilized the detailed inclusive isotope data obtained at Fermilab from proton collisions on Xe at $E_p = 80$ to 350 GeV/c by the Purdue group [22]. The set of data consists of cross section measurements for 80 species ranging from lithium to silicon isotopes. All were obtained with very low energy thresholds with time-of-flight telescopes and the angular distributions are nearly isotopic, making the extraction of total yields straight forward. Excitation energy selected data would have been preferable, but do not exist with the necessary wide range of isotopes. Furthermore, recent temperature measurements of central collisions of Au + Au reaction at 35A MeV using ten isotope ratios confirm the findings first shown in this study [16,17]. Thus we believe exclusive data would not lead to different conclusions.

Using Eq. (5), we have constructed 1326 thermometers assuming $\Delta Z = 0$ and $\Delta A = 1$. The left panel of Fig. 1 shows the distributions of apparent temperatures from these thermometers as a function of the binding energy difference B. Both positive and negative temperatures spanning magnitudes from zero to hundreds of MeV are observed. For $B > 4$ MeV, positive mean temperatures are observed. At low values of B, the distribution of extracted values of $T_{\rm app}$ breaks into distinct positive and negative values reflecting the discontinuity in Eq. (5) at $R_{\rm app} \approx 1/a$. This discontinuity can be removed if the inverse of the apparent temperature is plotted as shown in the lower right panel of Fig. 1. Reduction of the width of the apparent temperature distributions with increasing B is clearly demonstrated. From a practical point of view, the thermometers with the highest values of B are those with the least fluctuations and thus appear the most suited to measure the nuclear temperatures.

To allow the extraction of reliable temperatures from such data, one must address the fluctuations in the apparent temperatures. The most likely origin lies in the role of nuclear structure effects which raise or lower the branching ratios that govern the decay to the isotopes of interest

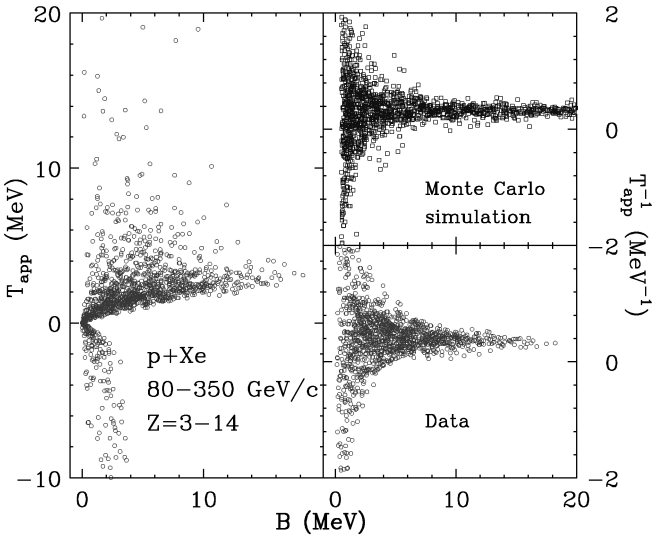

FIG. 1. Apparent temperature as a function of binding energy parameter for p + Xe system (left panel). The lower right panel shows the same data plotted as inverse apparent temperatures. The upper right panel shows the results of the Monte Carlo simulations discussed in the text.

[8–12,15]. To quantify the feeding effects, we define a correction factor, κ, for each ratio by the relationship $R_{\rm app} = \kappa R_o$ where $R_{\rm app}$ is the measured and R_o, the equilibrium values of the double isotope ratios. It follows from the definition of Eq. (5) that

$$\frac{1}{T_{\rm app}} = \frac{1}{T_o} + \frac{\ln k}{B}\,. \tag{6}$$

The general features of the fluctuations in $1/T_{\rm app}$ can be reproduced as shown in the top right panel of Fig. 1 by performing Monte Carlo simulations assuming all fragment yields in Eq. (1) fluctuate about their equilibrium values according to a single Gaussian distribution whose variance is 40% of the mean [23]. While this parameter was chosen to best describe the data, it is comparable to the variations predicted by the secondary decay calculations discussed below. To the extent the observed distributions of the measured apparent temperatures are consistent with nuclear structure effects in secondary decay, widely different apparent temperatures within a large ensemble of thermometers may not be inconsistent with a single value of primary temperature, and the existence of equilibrium. However, it should be noted that fluctuations can also arise from other effects such as nonequilibrium emission.

If such fluctuations reflect structure effects in the sequential decay, they should also appear for other reactions. We have analyzed additional isotope yield data measured for ^{14}N + Ag collisions at $E/A = 35$ MeV [8] and ^{32}S + Ag collisions at $E/A = 22$ MeV [12], where emission temperatures of 4.0 $\pm$ 0.5 and 3.5 $\pm$ 0.5 MeV were extracted, respectively, from the relative populations of excited states. For the latter two systems, however, the range of isotopes measured ($3 \leq Z \leq 8$) is more limited

TABLE I. List of thermometers with $B > 10$ MeV and the extracted correction factors $\langle \ln(\kappa)/B \rangle$.

Isotope ratio	B (MeV)	a	$\langle \ln \kappa/B \rangle$ (MeV^{-1})	Isotope ratio	B (MeV)	a	$\langle \ln \kappa/B \rangle$ (MeV^{-1})
^{13,14}C/^{11,12}C	10.54	1.96	0.021	^{15,16}N/^{15,16}O	13.17	5.00	0.131
^{17,18}O/^{11,12}C	10.68	0.64	−0.132	^{7,8}Li/^{15,16}O	13.63	2.77	0.074
^{12,13}C/^{15,16}O	10.72	4.09	0.044	^{12,13}C/^{11,12}C	13.77	7.92	0.0015
^{12,13}B/^{15,16}O	10.78	2.73	0.125	^{12,13}B/^{11,12}C	13.84	5.28	0.065
^{6,7}Li/^{11,12}C	11.47	5.90	−0.039	^{16,17}O/^{11,12}C	14.58	23.07	0.083
^{16,17}O/^{15,16}O	11.52	11.93	0.145	^{11,12}B/^{11,12}C	15.35	3.00	0.010
^{9,10}Be/^{11,12}C	11.91	1.03	−0.098	^{8,9}Li/^{11,12}C	15.78	3.35	−0.006
^{11,12}B/^{15,16}O	12.29	1.55	0.049	^{15,16}N/^{11,12}C	16.23	9.67	0.078
^{8,9}Li/^{15,16}O	12.73	1.73	0.028	^{7,8}Li/^{11,12}C	16.69	5.36	0.033

than for the p + Xe reaction. Therefore, only 18 isotope ratios ($\Delta Z = 0, \Delta A = 1$) with $B > 10$ MeV common to all three systems are studied. The ratios used in constructing the thermometers and their associated B values are listed in Table I. The apparent temperatures are plotted as open circles in Fig. 2. Thermometers that yield high (low) values for the apparent temperatures for the p + Xe reaction generally yield the same high (low) values for the other two reactions.

To verify that such fluctuations can arise from sequential feedings, the predictions of the published sequential decay calculations which include γ decays, particle unstable states, and unbound states in the continuum for the N + Ag system with an initial temperature of 4 MeV [8] are shown as open squares in the upper right panel of Fig. 2. These predicted values for $T_{\rm app}$ qualitatively follow the experimental trends seen for the three systems in Fig. 2, indicating that much of the structure information that leads to enhanced emission for one isotope relative to another is already contained in the calculations. Because of, in part, the inability for the present sequential decay calculations to reproduce isotope distributions for proton rich isotopes such as ^{11}C and ^{15}O accurately, the calculations are not in perfect agreement with the measured apparent temperatures for the N + Ag system [8] even for those thermometers with large B. These discrepancies are under current investigation [24].

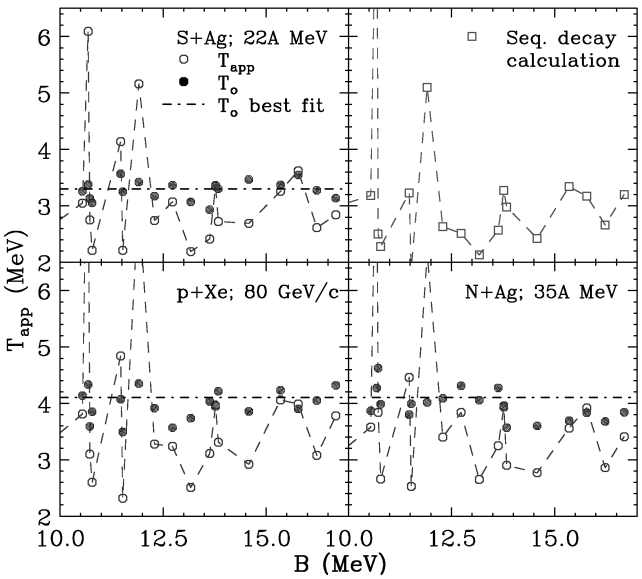

FIG. 2. Apparent temperatures as a function of $B > 10$ MeV for p + Xe, S + Ag, and N + Ag systems. The open circles are raw data and closed circles correspond to corrected temperature calculated according to Eq. (6) (see text for details). Dashed lines are drawn to guide the eye. The upper right panel shows the apparent temperatures determined from the sequential decay calculations of Ref. [8].

In the absence of a secondary decay calculation that reproduces the distributions of proton rich nuclei such as ^{11}C and ^{15}O accurately, we adopt a best fit procedure by fitting the data points shown in Fig. 2. Of the 18 isotope ratios shown in Table I, four of them, ^{12,13}C/^{15,16}O, ^{12,13}B/^{15,16}O, ^{16,17}O/^{11,12}C, and ^{11,12}B/^{11,12}C are interrelated with the remaining ratios. We used 17 parameters to fit $14 \times 3 = 42$ data points assuming (1) the $\ln \kappa/B$ factor for each thermometer is independent of reactions studied, (2) only one breakup temperature (T_o) is obtained for each reaction, and (3) the values of T_o should be consistent with the temperatures measured by the relative populations of the excited states. The resulting best fit values are $T_o = 4.1 \pm 0.4$ and 3.3 ± 0.3 MeV for the N + Ag system and the S + Ag system, respectively. The extracted T_o value for the p + Xe reaction at 80–350 GeV/c is nearly identical to the N + Ag reaction at 35A MeV incident energy. The mean values of $\ln \kappa/B$ obtained from the three reactions are listed in Table I.

As an internal consistency test, we can use the values for $\langle \ln \kappa/B \rangle$ listed in Table I and Eq. (6) to extract temperatures for each thermometer in each reaction. The resulting temperatures, shown as solid points in Fig. 2, follow the T_o values (dot-dashed line) very closely, indicating that the remaining fluctuations are much less than observed for the raw data. These remaining fluctuations may reflect nonequilibrium effects in the breakup process [8] or the possibility that these inclusive data may have contributions from several sources with different Z/A [24].

To test the Coulomb and isotope effects in determining the double ratio thermometers, the constraints $\Delta Z = 0$,

$\Delta A = 1$ have been removed from Eq. (1). The resulting temperatures exhibit similar trends as those shown in Fig. 1. However, the individual temperatures for different reactions are much more scattered than those shown in Fig. 2. This implies that effects of Coulomb interaction may not be completely removed using the double ratios. Thus, the isotope yields with $\Delta Z = 0$, $\Delta A = 1$ appear to be the most robust thermometers.

The similarities of the fluctuations independent of reactions studied suggest that the correction factors $\ln \kappa/B$ obtained in the current study can be applied to other reactions. Indeed, applying these factors to the data of Ref. [15] for the ratios $^{6,7}\mathrm{Li}/^{11,12}\mathrm{C}$ and $^{12,13}\mathrm{C}/^{11,12}\mathrm{C}$ measured for Ca + Ni, Ar + Ni, Ca + Fe, and Ar + Fe collisions at 33A MeV resolved the discrepancies between the measured temperatures ($T_{\mathrm{app}} = 5.38 \pm 0.50$ and 4.17 ± 0.26 MeV) yielding $T_o = 4.2 \pm 0.3$ and 4.1 ± 0.2 MeV, respectively. Extension to significantly high temperature will require theoretical estimations of the excitation energy dependence of κ [24].

In summary, a general systematic trend is observed for the apparent temperatures measured with double ratios of isotope yields. The overall trends suggest that the individual isotopic yield fluctuates about the thermal prediction with a standard deviation of 40% of the thermal yield. These fluctuations appear to originate from structure effects in the secondary decay process and therefore each isotope ratio shows a characteristic behavior that is independent of the reaction. A procedure has been described and tested in extracting isotope temperatures simultaneously from many thermometers with high B. We found that the correction factors for individual isotope thermometers obtained in the present study can be applied to other reactions. However, more studies are needed to calibrate $\ln \kappa/B$ and to understand the dependence of $\ln \kappa/B$ upon excitation energy and system size.

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