Resistivity due to a Domain Wall in Ferromagnetic Metal

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The resistivity due to a domain wall in ferromagnetic metallic wire is calculated based on the linear response theory. The interaction between conduction electrons and the wall is expressed in terms of a classical gauge field which is introduced by the local gauge transformation in the electron spin space. It is shown that the wall contributes to the decoherence of electrons and that this quantum correction can dominate over the Boltzmann resistivity, leading to a *decrease* of resistivity by nucleation of a wall. Conductance fluctuations due to the motion of the wall are also investigated. The results are compared with recent experiments. [S0031-9007(97)03126-8]

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The interplay between electron transport properties and a magnetic object such as the magnetization or the domain wall (DW) has recently been attracting much attention. Of particular interest is the case of a mesoscopic system where the magnetization or the motion of a DW can be driven by quantum fluctuation and described as a macroscopic quantum phenomena [1]. In the case of the quantum depinning of a DW [2], for example, a theoretical study [3] indicates that the depinning can be affected by the dissipation caused by the conduction electron if the thickness of the wall, λ, is small, e.g., a few lattice constants. The change of the magnetization associated with such a depinning of a mesoscopic DW is very small and so it is very difficult to observe such small magnetic objects directly, e.g., by SQUID. The transport properties, on the other hand, can detect a very small change of the magnetization as a change of resistance. Indeed, recently in a mesoscopic wire of Ni with a diameter of 300 Å several small discontinuous changes of the resistivity have been observed as the magnetic field is swept [4]. It is argued there that these jumps are due to the change of the total magnetization by depinning of a DW, and the displacement of the wall has been estimated from the value of magnetoresistance to be $\sim 1.2\ \mu$m [4]. These considerations on the transport properties are based on the classical approximation. Other possible origins of this jump are proposed in this paper. Our study indicates the important role played by the quantum interference among the conduction electron, as in the case of the conductance fluctuation (CF) [5,6] in the weakly localized regime, which is shown to be sensitive to even a motion of a single impurity atom [7,8] or a small magnetization of $\sim 50\mu_B$ [9]. Here we discuss a new effect on the quantum transport properties due to a magnetic domain wall.

Not only in these mesoscopic systems the interplay between the magnetic structure and the electronic transport properties may play important roles in the bulk system; e.g., in films [10] and in double exchange systems like $La_{1-x}Sr_xMnO_3$, where scattering by DWs is considered as a possible origin of low temperature magnetoresistance [11,12].

In this paper we study the resistivity in ferromagnetic metals arising from the scattering by a DW due to the exchange coupling on the basis of linear response theory by taking account of the impurity scattering at the same time. The case of $\lambda \ll l$ (l being the elastic mean free path) has been studied by Cabrera and Falicov [13] in the classical Boltzmann approximation. Their result indicates that the resistivity becomes large only when the spin splitting is comparable to the Fermi energy and $k_F\lambda \lesssim 1$ (k_F being the Fermi momentum). In their study, however, electronic motions have been assumed to be one dimensional, which is not realistic, at least at present, in actual metallic wires. The force acting on the wall as a consequence of the electronic current has been studied, based on the classical transport equation by Berger [14]. It has also been argued that the eddy current due to the DW can lead to excess resistivity if the sample is not too small [15]. Here we study the effect of a DW on resistivity in a mesoscopic wire with width $L_\perp$ satisfying $\lambda \gtrsim L_\perp \gg k_F^{-1}$, thus treating the electron as three dimensional. The length is L and the wire direction has been chosen as the z axis. We investigate the quantum corrections to the resistivity by a wall as well as the Boltzmann resistivity. The CF arising from the motion of the wall has also been calculated.

We consider explicitly the case described by a single-band Hubbard model in the Hartree-Fock approximation [16]. The calculation is carried out at zero temperature. The Lagrangian of the electron (denoted by $c^{(0)}$) in the imaginary time (s) is given as

$$L = \sum_{\mathbf{k}\sigma} c^{(0)\dagger}_{\mathbf{k}\sigma}(\partial_s + \epsilon_{\mathbf{k}})c^{(0)}_{\mathbf{k}\sigma} - U\int d^3x\, \mathbf{M}(\mathbf{x})\,(c^{(0)\dagger}\boldsymbol{\sigma}c^{(0)})_{\mathbf{x}}\,, \qquad (1)$$

where $\epsilon_{\mathbf{k}} \equiv \hbar^2\mathbf{k}^2/2m - \epsilon_F$ (ϵ_F being the Fermi energy) and U is the Coulomb interaction. The spin index is denoted by $\sigma = \pm$ and $\boldsymbol{\sigma}$ is the Pauli matrix. The magnetization is written as $\mathbf{M}$, whose configuration is determined by the ferromagnetic Heisenberg model [3],

$$H_M = \int d^3x \left[\frac{J}{2} |\nabla \mathbf{M}|^2 - \frac{K}{2} M_z^2 \right], \tag{2}$$

where J is the effective exchange energy determined by U and K is the magnetic anisotropy energy introduced phenomenologically [17]. Here we are interested in the solution of a single DW. In terms of the polar coordinates, (θ, ϕ), that represent the direction of $\mathbf{M}$, the solution of a DW is given by $\cos\theta = \tanh\frac{z}{\lambda}$ and constant ϕ, where $\lambda = \sqrt{K/J}$.

In Eq. (1) the last term represents the interaction between the magnetization and the electron. For the perturbative calculation of resistivity, we need to rewrite this term by the use of the local gauge transformation in the spin space,

$$c_\sigma \equiv \sigma\left(\cos\frac{\theta}{2}\, c_\sigma^{(0)} - i\sin\frac{\theta}{2}\, c_{-\sigma}^{(0)}\right). \tag{3}$$

In terms of the new electron operator, c, the Lagrangian is written as [3] $L = \sum_{\mathbf{k}\sigma} c^\dagger_{\mathbf{k}\sigma}(\partial_s + \epsilon_{\mathbf{k}\sigma})c_{\mathbf{k}\sigma} + H_{\rm int}$, where $\epsilon_{\mathbf{k}\sigma} \equiv \epsilon_{\mathbf{k}} - \sigma\Delta$ with $\Delta \equiv U|\mathbf{M}|$ being half the splitting between the up and down spin electrons. The interaction is obtained as

$$H_{\rm int} = \frac{\hbar^2}{2m}\sum_{\mathbf{k}}\sum_{q\|z}\Bigg[-\left(k_z + \frac{q}{2}\right) a_q c^\dagger_{\mathbf{k}+q}\sigma_x c_{\mathbf{k}} + \frac{1}{4}\sum_{p\|z} a_p a_{-p+q} c^\dagger_{\mathbf{k}+q} c_{\mathbf{k}}\Bigg]. \tag{4}$$

Here $a_q \equiv (1/V)\sum_{\mathbf{x}} e^{-iqz}\nabla_z\theta = (\pi/L)e^{-iqz_i}[1/\cosh(\pi q\lambda/2)]$ ($V \equiv L_\perp^2 L$ and z_i being the center coordinate of the DW). Because of this gauge transformation, the electronic current in the z direction is changed to be $J_z = J_z^0 + \delta J$, where $J_z^0 \equiv (e\hbar/m)\sum_{\mathbf{k}} k_z c^\dagger_{\mathbf{k}} c_{\mathbf{k}}$ and

$$\delta J \equiv -\frac{e\hbar}{2m}\sum_{\mathbf{k},q\|z} a_q c^\dagger_{\mathbf{k}+q}\sigma_x c_{\mathbf{k}}\,. \tag{5}$$

By the use of the Kubo formula, the conductivity for the current along the wire is calculated from the current-current correlation function. By assuming that the scattering due to normal impurities is dominant, we estimate the effect of DW on the correction to the conductivity perturbatively to the second order of a_q. (The first order contribution vanishes.) The second order contributions to the Boltzmann conductivity are shown in Fig. 1. The process Q_1 arises from the correlation of δJ and Q_3 is due to δJ and an interaction with the wall. Q_2 and Q_4 are the self-energy corrections due to the wall and Q_5 is the vertex correction to the correlation of J_z^0. After straightforward calculation and by the use of the particle-hole symmetry, which we assume, $\Delta Q \equiv \sum_{i=1}^5 Q_i$ is shown to be

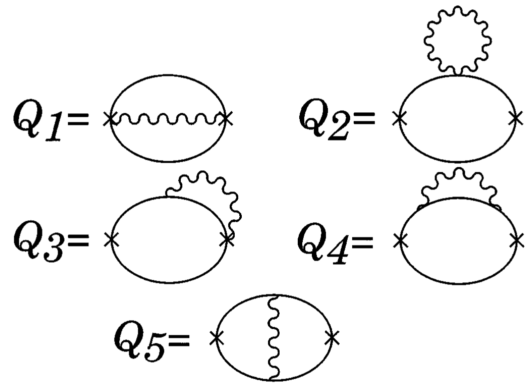

FIG. 1. The contributions to the Boltzmann conductivity which are the second order with respect to the interaction with the domain wall, denoted by wavy lines. Solid lines indicate the electron Green functions and the current vertex (expressed by crosses) with wavy line represents δJ.

$$\Delta Q(i\omega) = \frac{1}{2}\left(\frac{e\hbar\Delta}{m}\right)^2 \int \frac{d\omega'}{2\pi}\frac{1}{V} \times \sum_{\mathbf{k}q\sigma} |a_q|^2 G_{\mathbf{k}-\frac{q}{2},\omega',\sigma} G_{\mathbf{k}-\frac{q}{2},\omega'+\omega,\sigma} \times G_{\mathbf{k}+\frac{q}{2},\omega',-\sigma} G_{\mathbf{k}+\frac{q}{2},\omega'+\omega,-\sigma}\,. \tag{6}$$

Here the Green function is given by $G_{\mathbf{k},\omega',\sigma} \equiv 1/\{i[\omega' + (\hbar/2\tau)\mathrm{sgn}(\omega')] - \epsilon_{\mathbf{k}\sigma}\}$, where τ is the lifetime due to the normal impurity scattering and $\mathrm{sgn}(\omega') = 1$ and -1 for $\omega' > 0$ and $\omega' < 0$, respectively.

Hence the correction to the Boltzmann conductivity by a DW, $\Delta\sigma$, is obtained as $\Delta\sigma = -\sigma_0 A$ where $\sigma_0 \equiv e^2 n\tau/m$ (n being the electron density) is the Boltzmann conductivity without the wall and A is given by

$$A = \frac{\pi}{\hbar}\frac{\Delta^2\tau}{2nV} n_{\rm w}\sum_{\sigma,\pm}\frac{N_\sigma}{k_{{\rm F}\sigma}} \times \int_0^\infty \frac{dx}{x}\frac{1}{\cosh^2 x}\tan^{-1}\left(\frac{2l_\sigma}{\pi\lambda}x \pm 2\Delta\frac{\tau}{\hbar}\right). \tag{7}$$

Here $l_\sigma \equiv \hbar k_{F\sigma}\tau/m$, $n_{\rm w} \equiv 1/L$ being the density of the wall, and $N_\sigma \equiv (mk_{F\sigma}V/2\pi^2\hbar^2)$ is the density of states at the Fermi energy of the electron with spin σ. The wall contribution to the resistivity is given as $\rho_{\rm w} \equiv \sigma_0^{-1}[(1-A)^{-1} - 1] \simeq \sigma_0^{-1}A$.

We consider a ferromagnet where $\Delta\tau/\hbar \gg 1$ is satisfied. Then Eq. (7) reduces to

$$A \simeq \frac{3n_{\rm w}}{2mn\lambda}\sum_\sigma \frac{N_\sigma}{V}\,. \tag{8}$$

Let us look into the effect of the wall on quantum transport properties in the disordered system, where the interference effect, which is represented by the maximally crossed diagram (Cooperon), becomes important. The processes which describe the effect of the wall on the quantum correction at low energy are shown in Fig. 2. They both contribute to the dephasing of the electron, but the vertex type process (b) includes Cooperons which connect the electrons with different spin, and thus is suppressed in ferromagnets we are considering due to the condition $\Delta\tau/\hbar \gg 1$. Hence only the self-energy type (a) is important here. The higher order contributions similar to this process can be summed up giving rise to the mass of the Cooperon. The quantum correction by the wall is then obtained as

$$\sigma_Q = \frac{2\hbar e^2 k_F^2 \tau}{3\pi m^2} \frac{1}{V} \sum_q \left(\frac{1}{Dq^2} - \frac{1}{Dq^2 + (1/\tau_{\mathrm{w}})} \right), \quad (9)$$

where $D \equiv (k_F^2 \hbar^2 \tau / 3m^2)$ and τ_{w} is the lifetime due to the wall given by $1/\tau_{\mathrm{w}} = (n_{\mathrm{w}}/6\lambda k_F^2)(\epsilon_F/\Delta)^2/\tau$. In the case where $DL_\perp^{-2} > \tau_{\mathrm{w}}^{-1}$, which we assume, the q summation should be carried out along the one dimension with a cutoff of L^{-1} for small q. The result for $L/l \gg 1$ and $\kappa \equiv \tau/\tau_{\mathrm{w}} \ll 1$ is

$$\frac{\sigma_Q}{\sigma_0} \simeq \frac{6}{k_F^2 L_\perp^2} \left(\frac{L}{l} - \frac{\tan^{-1}(\sqrt{3\kappa}\, L/l)}{\sqrt{3\kappa}} \right). \quad (10)$$

Note that σ_Q is positive, since the DW suppresses the interference due to random impurity scattering.

So far we have studied a static wall. Let us now discuss the CF [5,6] due to the motion of the wall. In this case a small jump of a wall can result in substantial change in resistivity, in contrast to the change due to the effect of classical magnetoresistance [4], which becomes important only when the wall moves over a distance comparable to L. The calculation goes in the similar way as the CF due to the motion of a single atom in a disordered metal [7]. The square of the conductance change δG due to the motion of a wall over a distance of r is evaluated by calculating the diagram with two bubbles with the wall position at $z = r$ and $z = 0$ connected by impurities and the wall. A typical diagram is shown in Fig. 3. The DW line here represents the motion of the wall and Cooperons include the mass arising from the wall, $1/\tau_{\mathrm{w}}$. There are other diagrams with the contribution of the same order

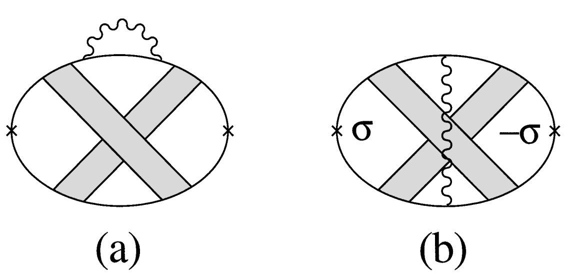

FIG. 2. (a) The dominant process to the quantum correction of the conductivity. Hatched square denotes the particle-particle ladder (Cooperon) due to the impurity scattering. Process (b), which contains Cooperons connecting the electrons with different spin (denoted by σ and $-\sigma$), is unimportant in ferromagnets due to the present condition $\Delta\tau/\hbar \gg 1$.

which contains one or two more impurity ladders [5] and the result of δG is obtained as

$$\frac{\delta G(r)}{e^2/h} = \sqrt{2}\,\frac{4\pi}{3}\,\epsilon^2\kappa\alpha \times \begin{cases} \frac{1}{6}(\frac{r}{\lambda})^2 & (r \ll l, \lambda), \\ 1 & (r \gg l, \lambda), \end{cases} \quad (11)$$

where $\epsilon \equiv l/L$ and $\alpha \equiv [\sum_q (Dq^2\tau + \kappa)^{-4}]^{(1/2)}$ is calculated for $\epsilon, \kappa \ll 1$ as

$$\alpha \simeq \frac{9}{2\pi}\frac{1}{\epsilon\kappa}\left[\frac{5}{24\kappa^2}\left(\frac{\tan^{-1}(\epsilon/\sqrt{3\kappa})}{\sqrt{3\kappa}} - \frac{\epsilon(\epsilon^2 + 5\kappa)}{(\epsilon^2 + 3\kappa)^2} \right) - \frac{\epsilon}{(\epsilon^2 + 3\kappa)^3} \right]. \quad (12)$$

Let us give a numerical estimate of our theoretical conclusions. Consider a wire of Ni or Fe with $L = 10\ \mu$m and $L_\perp = 300$ Å, where $\lambda \sim 500$ Å [4]. If we consider d electron ($k_F^{-1} \sim 1.5$ Å, $\Delta/\epsilon_F \sim 0.2$) and choose $l \sim 1000$ Å, then $\Delta\tau = 150$ and Eq. (8) leads to a very small Boltzmann contribution of $A \simeq 1.4 \times 10^{-8}$. For s electron, Δ/ϵ_F will be smaller by a factor of about 10^{-2}, and then from Eq. (7) we obtain $A \simeq 2.7 \times 10^{-9}$. On the other hand, the quantum correction becomes larger for the above values of parameters (κ is $\sim 3.7 \times 10^{-4}$); $\sigma_Q/\sigma_0 = 1.6 \times 10^{-3}$. Thus DWs will contribute to a decrease of resistivity in a ferromagnetic wire of transition metals. If the wall moves over a distance of $r \sim 100$ Å in this situation, the expected conductance change is $\delta G \simeq 5.0 \times 10^{-3}(e^2/h)$.

In the experiment on Ni [4], a discrete increase of resistivity of about 0.2% [$\delta\rho \simeq 2 \times 10^{-9}\ \Omega$ cm or $\delta G \simeq 5 \times 10^{-3}(e^2/h)$] has been observed as the magnetic field is swept above the coercive field, at which the minimum of resistivity appears. Comparison with our study may suggest two possibilities for the cause. One is that $\delta\rho$ might be due to the annihilation of a wall. The other is that $\delta\rho$ can be the fluctuation due to a motion of wall over a distance of $r \sim 100$ Å. Further studies are needed to determine which is the true origin. In this context it is interesting to note that a recent experiment on Fe wire with width of 3000 Å has disclosed the existence of a negative jump of ρ followed by a positive one close to the field where ρ becomes minimum [18]. This result may suggest that the jumps are due to the nucleation and subsequent annihilation of a wall.

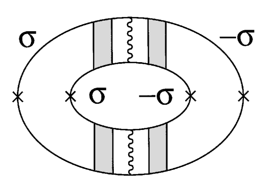

FIG. 3. An example of diagrams which contributes to the conductance fluctuation due to the motion of the wall. Wavy lines represent the motion of the domain wall, and the Cooperons here include the mass due to the domain wall.

To summarize, the resistivity arising from the scattering of the conduction electron by a domain wall in a wire of ferromagnetic metal is calculated based on the linear response theory. The interaction with the wall is expressed as a classical gauge field acting on the electron, which we examined in the second order perturbation theory. In addition to the Boltzmann resistivity, we have investigated the effect of the wall on quantum transport properties in disordered metals. The wall suppresses the interference between the electron, and hence decreases the resistivity in the weakly localized regime. It will be interesting to observe in magnetic wires this reduction of resistivity by the nucleation of domains in more definitive ways. It has been shown that a small motion of a wall can lead to substantial conductance fluctuation. The present calculation provides a first quantitative estimate of the effect of a domain wall on the mesoscopic transport properties, which, we hope, will be useful in the interpretation of the experimental results in the near future.

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