# Constraints on a Primordial Magnetic Field

John D. Barrow,[1,2] Pedro G. Ferreira,[1] and Joseph Silk[1]
[1]*Center for Particle Astrophysics, and Departments of Astronomy and Physics, University of California, Berkely, California 94720-7304*
[2]*Astronomy Centre, University of Sussex, Brighton BN1 9QH, United Kingdom*
(Received 11 December 1996)

We derive an upper limit of $B_0 < 3.4 \times 10^{-9}(\Omega_0 h_{50}^2)^{1/2}$ G on the present strength of any primordial homogeneous magnetic field. The microwave background anisotropy created by cosmological magnetic fields is calculated in the most general flat and open anisotropic cosmologies containing expansion-rate and 3-curvature anisotropies. Our limit is derived from a statistical analysis of the 4-year Cosmic Background Explorer (COBE) data for anisotropy patterns characteristic of homogeneous anisotropy averaged over all possible sky orientations with respect to the COBE receiver. The limits we obtain on homogeneous magnetic fields are stronger than those imposed by nucleosynthesis. [S0031-9007(97)03145-1]

PACS numbers: 98.70.Vc, 98.58.Ay, 98.80.Cq, 98.80.Hw

The origin of large-scale magnetic fields whether observed in galaxies or galaxy clusters is still a mystery. Intracluster fields are largely dominated by ejecta from galaxies. The invocation of protogalictic dynamos to explain the magnitude of the galactic field involves many uncertain assumptions but still requires a small primordial (pregalactic) seed field [1]. Hence the possibility of a primordial field merits serious consideration. Other attempts to find an origin for the field in the early Universe have appealed to battery effects, the electroweak phase transitions, or to fundamental changes in the nature of the electromagnetic interaction. All introduce further hypotheses about the early Universe or the structure of the electroweak interaction [2]. All aim to generate fields by causal processes when the Universe is of finite age. Therefore, any magnetic field created by these means will exist only on very small scales with an energy density that is a negligible fraction of the background equilibrium radiation energy density.

Nevertheless, while such fields might still provide the seeds for nonlinear dynamos in the postrecombination era, any *large-scale* magnetic field with a strength of order $B \simeq 10^{-8}$ G, comparable to that inferred from the lowest measured intergalactic fields and close to the observation all upper limits via Faraday rotation measurements [3], may well be of cosmological origin. A similar pregalactic (or protogalactic) field strength is inferred from the detection of fields of order $10^{-6}$ G in high redshift galaxies [4] and in damped Lyman-alpha clouds [5], where the observed fields are likely to have been adiabatically amplified during protogalactic collapse. In the absence of a plausible dynamo for generating large-scale pregalactic fields, it is of interest to reconsider the limits on a large-scale primordial field in view of new observational constraints that we outline below.

Primordial magnetic field can leave observable traces of their influence on the expansion dynamics of the Universe because they create anisotropic pressures, and these pressures require an anisotropic gravitational field to support them. The influence of a magnetic field on reaction rates at nucleosynthesis only limits the equivalent current epoch field to be less than about $3 \times 10^{-7}$ G [6]. This value is only slightly stronger than the aforementioned dynamical constraint at nucleosynthesis [7]. We show in this Letter that the cosmic microwave background isotropy provides a stronger limit on the strength of a homogeneous component of a primordial magnetic field.

We consider the cosmological evolution of the most general homogeneous magnetic fields, calculate their gravitational effects on the temperature anisotropy of the microwave background radiation, and hence derive a strong limit on the strength of any homogeneous cosmological magnetic field by using the 4-year Cosmic Background Explorer (COBE) microwave background isotropy measurements [8]. We employ statistical sampling techniques appropriate for the non-Gaussian statistics of the large-scale temperature anisotropy pattern created by a general homogeneous cosmological magnetic field and allow for the randomness of the angle at which COBE views the characteristic anisotropy pattern on the sky [9]. The addition of a homogeneous cosmological electric field will not be considered: a homogeneous intergalactic electric field would create a current of charged particles, and rapidly decay.

Our limit derives from the nonlinear coupled evolution of the shear anisotropy and magnetic field density during the radiation era which we shall discuss below. In the presence of an equilibrium background of blackbody radiation with isotropic momentum distribution, pressure $p_r$, density $\rho_r$, and equation of state $p_r = \frac{1}{3}\rho_r$, the anisotropic magnetic pressure prevents the rapid decay of the expansion shear anisotropy [10]. The isotropic expansion is stable at second order, and the anisotropies decay only logarithmically in time relative to the mean expansion rate. Physically, the evolution of the anisotropy is governed by the magnetic pressure anisotropy.

We need to describe the evolution of the magnetic field strength and the accompanying shear anisotropy, which will distort the microwave background in the most general homogeneous and anisotropic universes. The pattern of evolution of magnetic universes during the radiation era can be deduced from earlier studies of the evolution of the most general homogeneous universes in the absence of magnetic fields [11,12]. In general, if the Universe contains a noninteracting mixture of isotropic blackbody radiation and any matter source possessing an energy-momentum tensor with zero trace (e.g., magnetic or electric fields, long-wavelength gravitational waves, or an anisotropic distribution of collisionless massless or relativistic particles [13]), then the evolution follows the same characteristic pattern. The most general anisotropic flat and open universes (Bianchi type VII) which asymptotically include the isotropic model are equivalent to simpler flat or open anisotropic spacetimes (Bianchi types I or V) to which gravitational waves have been added [11,12]. The Einstein tensor for the more general homogeneous models can be split into two pieces: one corresponding to a simpler anisotropic universe, the other to an effective energy-momentum tensor for the gravitational waves (with zero trace). This means that simple homogeneous anisotropic universes containing blackbody radiation plus trace-free matter sources with anisotropic pressures have similar time evolution to the most general homogeneous anisotropic universes containing blackbody radiation and a magnetic field. The 3-curvature anisotropies and the magnetic stresses display the same asymptotic time evolution.

When the deviations from isotropy are small, the evolution of the shear and the energy density of any trace-free matter field with anisotropic pressures is well approximated by setting the blackbody density and the volume-averaged Hubble expansion rate equal to their values in the isotropic flat Friedmann universe ($\rho_r = 3/4t^2$ and $H = 1/2t$, since we set $8\pi G = 1$). One then solves for the nonlinear evolution of the shear anisotropy and the magnetic field. The anisotropy is conventionally parametrized by the dimensionless ratio of the shear anisotropy, $\sigma$, to the mean Hubble rate, $H$, and determines the overall amplitude of the microwave anisotropies directly. This ratio has a generic behavior for $p \leq \frac{1}{3}\rho$; the ratio $\sigma/H$ relaxes to a constant value determined by the ratio of the total energy densities in anisotropic traceless fluids and 3-curvature anisotropies, $\rho_{ga}$, and magnetic fields, $\rho_B$, to that of the isotropic perfect fluid density, $\rho$ (radiation or dust). When $p > \frac{1}{3}\rho$ the anisotropic stresses dominate at large $t$ and the solution ceases to be a small perturbation of an isotropic Friedmann universe as $t \to \infty$.

For an isotropic fluid with an equation of state $p = (\gamma - 1)\rho$ and $0 < \gamma \leq 4/3$, the time evolution is determined by the Einstein equations

$$\frac{d}{dt}\left(\frac{\sigma}{H}\right) = \left(\frac{\sigma}{H}\right)\left(\frac{\gamma - 2}{\gamma t}\right) + \frac{4}{\gamma t}\left(\frac{\rho_{ga} + \rho_B}{\rho}\right),$$
$$\frac{d}{dt}\left(\frac{\rho_{ga} + \rho_B}{\rho}\right) = -\frac{2}{9\gamma y}\left(\frac{\rho_{ga} + \rho_B}{\rho}\right) \times \left(4\frac{\sigma}{H} + 9\gamma - 12\right). \tag{1}$$

Here, $\rho_B = B^2/8\pi$ is the magnetic field density. Hence, as $t \to \infty$, we have $\sigma/H \to$ const. In the radiation era ($\rho = \rho_r, \gamma = 4/3$), at redshifts $1 + z > 1 + z_{\text{eq}} = 8 \times 10^3 \Omega_0 h_{50}^2$, where $\Omega_0 \leq 1$ is the cosmological density parameter and $h_{50}$ is the present value of the Hubble constant in units of $50\ \text{K ms}^{-1}\,\text{Mpc}^{-1}$. We have

$$\rho_B/\rho_r \to Q/[1 + 4Q\ln(t/t_0)];\ \text{with } Q \text{ constant}. \tag{2}$$

During the dust era ($\rho = \rho_d, \gamma = 1$), when $z < z_{\text{eq}}$ the evolution is determined at linear order and $(\rho_{ga} + \rho_B)/\rho_d \propto 1 + z$ falls linearly with redshift.

In general, the shear distortion created by magnetic fields and any other trace-free anisotropic stresses is given by

$$\frac{\sigma}{H} = \frac{4}{2 - \gamma}\left(\frac{\rho_B}{\rho} + \frac{\rho_{ga}}{\rho}\right) + \delta t^{(\gamma - 2)/\gamma}, \tag{3}$$

where $\delta$ is a constant. The $\delta$ term gives the simple shear decay for universes with isotropic 3-curvature containing only matter with isotropic pressure; this term becomes negligible at late times. In both dust and radiation eras the anisotropic and magnetic stress terms dominate the $\delta$ term at late times and produce a slower decay of the shear distortion [7,14,15]. Note that the presence of curvature anisotropy or any ansiotropic trace-free matter stress changes the shear evolution from the simple delta term that is usually studied in the literature (e.g., in [16]). Also, note that the ratio of the magnetic and blackbody radiation densities is not constant (as assumed, for example, in [6] and [17]), but falls logarithmically during the radiation era.

The magnetic field and the accompanying shear distort the microwave background temperature isotropy in accord with the most general anisotropic universes of interest. By combining Eq. (3) and the expression for the angular anisotropy pattern in type VII [18],

$$\Delta T_A(\hat{\mathbf{r}}) = \left(\frac{\sigma}{H}\right)_{\text{ls}} Y(\theta, \phi; \Omega_0, h_{50}x, z_{ls}), \tag{4}$$

where ls denotes the epoch of last scattering. The exact form of the pattern function $Y$ can be read off from Eqs. (4.11)–(4.16) of Barrow, Jaszkiewicz, and Sonoda [18]. The constant parameter $x$, introduced by Collins and Hawking [11] is a measure of the 3-curvature anisotropy configuration in these anisotropic universes, and corresponds physically to the characteristic wavelength over which the principal axes of shear charge orientation. The angular pattern is a spiral with $2/\pi x$ twists to the angular pattern [18]. If $\Omega_0 < 1$ there will be a focusing of the quadrupole towards the axis of anisotropy, generating a hot spot.

The problem of constraining global anisotropy is substantially different from the traditional statistical task of estimating parameters in Gaussian models. In the latter case, the ensemble is entirely characterized by the power spectrum while in the former, a given set of parameters corresponds to a completely specified pattern in the sky, up to an arbitrary rotation. This problem was dealt with in some detail in [9]. A brief outline of our procedure is as follows. One can model the microwave background signal as the sum of two components: a *statistically isotropic* Gaussian random field $\Delta T_I$, which we assumed to have a scale invariant power spectrum on the scales we are interested in, and a *global, anisotropic* pattern, $\Delta T_A$, as in Eq. (4), which is uniquely defined by the set of parameters $x$, $\Omega_0$, $h_{50}$, $(\sigma/H)_0$, and $\theta$, $\phi$ (its orientation on the sky). Each pixel of a data set of measured microwave background anisotropies is given by $d_i = (\Delta T \star \beta)(\mathbf{r}_i) + N_i$ where $\beta$ represents the differential microwave radiometer (DMR) beam pattern, $\mathbf{r}_i$ is a unit vector pointing in the direction of pixel $i$, $N_i$ is the noise in pixel $i$, and $\star$ is the convolution operator. To an excellent approximation, one can assume that $N$ is Gaussian "white" noise, i.e., $\langle N_i N_j \rangle = s_i^2 \delta_{ij}$.

Our task is, given a pair $(x, \Omega_0)$, to find the orientation $(\theta, \phi)$ which allows the maximum observed value of $(\sigma/H)_0$. One can do this using standard frequentist statistical methods: we define a goodness-of-fit statistic that depends on the data, compute its value for the actual data, and then compute the probability that a random data set would have given a value as good as the actual data. In [6] $\eta$ was defined to be

$$\eta = \min_{\sigma,\theta,\phi} \eta_1, \text{ where } \eta_1 = \frac{\Delta_0^2 - \Delta_1^2}{\Delta_0^2}; \tag{5}$$

$\Delta_0^2$ is the noise-weighted mean-square value of the data and $\Delta_1^2$ is the noise-weighted mean-square value of the residuals after we have subtracted off the anisotropic part. Note that removing the incorrect anisotropic portion will only increase the residuals to the difference between the two terms is an obvious choice for a goodness-of-fit. Dividing by $\Delta_0^2$ ensures a weak dependence on the amplitude of the isotropic component, while defining the statistic as the minimum of $\eta_1$ allows us to deal with the uncertainty in $(\theta, \phi)$.

This statistical method was applied to the 4-yr COBE DMR data set. The two 53 GHz and the two 90 GHz maps were averaged together, each pixel weighted by the inverse square of the noise level, to reduce the noise level in the average map. All pixels within the galactic cut were removed so as to reduce galactic contamination, and a best-fit monopole and dipole were subtracted out. The map was degraded from pixelization 6 to pixelization 5 (i.e., binning pixels in groups of four). Simulations were performed for a set of models from the $(\Omega_0, x)$ plane; for each choice of the three parameters $[\Omega_0, x$, and $(\sigma/H)_0]$ approximately 200 to 500 random DMR sets were generated, so allowing us to determine an approximate fit to the probability distribution function of $\eta$.

The most conservative limit on the cosmological magnetic field arises when we assume $\rho_{ga} < \rho_B$ so the whole anisotropy is contributed by the magnetic field stresses. A reasonable fit of the upper bound at a 95% confidence level is

$$\left(\frac{\rho B}{\rho}\right)_0 < \frac{(2-\gamma)}{4} f(x, \Omega_0) \times 10^{-9},$$
$$f \simeq 2.1 \Omega_0^{0.33} x^{-0.01/\Omega_0}, \tag{6}$$
$$\text{with } x \in [0.01, 3] \text{ and } \Omega_0 \in [0.1, 1],$$

where we have considered the largest possible contribution from the magnetic component. Note that the "shape" factor is roughly bounded by $0.6 < f < 2.2$. This gives us a final bound on the magnitude of the magnetic field today of

$$B_0 < 3.5 \times 10^{-9} f^{1/2} (\Omega_0 h_{50}^2)^{1/2} \text{ G}. \tag{7}$$

This bound can be improved by a factor of $\sqrt{3}$ if one considers the results from [19]. In this case, a slightly different goodness-of-fit statistic is used: instead of working with the noise-weighted quantities, $\Delta_0^2$ and $\Delta_1^2$, the authors chose to weight the pixels with the covariance matrix of the total Gaussian components (i.e., the noise and isotropic cosmological components). This gives a limit of

$$B_0 < 2.3 \times 10^{-9} f^{1/2} (\Omega_0 h_{50}^2)^{1/2} \text{ G}. \tag{8}$$

Note that the microwave background limits on the amplitude of anisotropies are much stronger than those imposed by nucleosynthesis [15]. In unrealistic models with no anisotropic matter stresses (and therefore no magnetic field) and isotropic curvature, the shear falls off rapidly in accord with the $\delta$ term in (3) and the limits from nucleosynthesis would be stronger. But with anisotropic matter stresses, magnetic fields, or anisotropic curvature, the anisotropy falls only logarithmically during the radiation era. The limits on $\rho_B/\rho_r$ at nucleosynthesis are only $O(1)$ and the logarithmic decay means they are weaker than limits $O(10^{-5})$ imposed at $z = 1.1 \times 10^3$ by the microwave background. If there is reheating and last scattering occurs at $z \ll 1100$ then the analysis is slightly changed, but last scattering would need to occur at a redshift lower than $6\Omega_0 h_{50}^2$ for the nucleosynthesis limit to be competitive with the microwave limit. This never happens.

Adams *et al.* [17] have argued that a cosmological magnetic field could lead to observable distortions of the acoustic peaks in the microwave background. Our limit on $B_0$ rules out any observable effect of a *homogeneous* magnetic field on the acoustic peaks. In fact, when the $(\ln t)^{-1}$ decay of $\rho_B/\rho_r$ of Eq. (2) is taken into account, the nucleosynthesis limit on a homogeneous field is strong enough to render the acoustic peak distortions unobservable. A large scale, inhomogeneous, magnetic field may, however, introduce observable distortions in the acoustic peaks. Our limit permits a field strength of $10^{-9}$ G required to induce a measurable Faraday rotation in the polarization of the microwave background [20].

Any period of inflation long enough to explain the horizon and flatness problems would necessarily reduce homogeneous magnetic field effects and their associated anisotropies to unobservably low levels. If $N$ $e$-folds of de Sitter inflation occur ($p = -\rho = \text{const}$) then $\sigma/H$ will be reduced by $\exp(-3N)$ and $\rho_B/\rho$ will be reduced by a factor $\exp(-4N)$ and $N \sim 70$ is sufficient to solve the horizon problem [21]. The formula (3) for $\sigma/H$ applies to the case of inflation if the $\delta$ term is changed to $\delta \exp(-3N)$. Note that if generalized inflation occurs ($0 \leq \gamma < 2/3$), the $\delta$ term of Eq. (3) always dominates the $(\rho_{ga} + \rho_B)/\rho$ term as $t \rightarrow \infty$ unlike in the noninflationary case when $2/3 < \gamma < 4/3$. All anisotropies decay in accord with the no-hair theorem when the curvature is nonpositive because the anisotropic trace-free stresses obey the strong energy condition [22]. Any variants on inflation designed to generate large-scale fields would presumably result in a constant curvature spectrum of magnetic inhomogeneities rather than in a homogeneous field. The discovery of microwave background patterns characteristic of large-scale homogeneous anisotropy or of homogeneous primordial magnetic fields in future observational programs would certainly challenge the standard picture of inflation.

In summary, we have used the equivalence between the dynamics of homogeneous, anisotropic universes with some matter content, and universes with a homogeneous magnetic field to relate the shear induced by anisotropic curvature with that induced by a magnetic field. In doing so we have been able to relate the amplitude of the magnetic field to the amplitude of the microwave background anisotropies on large scales. We have then used the COBE 4-year data set to constrain this amplitude. In restricting ourselves to the case of a homogeneous field, our results are insensitive to higher resolution. In fact, the only limitation on these larger angular scales is given by cosmic and sample variance. This will not improve greatly on the scales of interest with the next generation of satellite experiments.

We acknowledge partial support by NASA and DOE. P.F. was supported by the Center for Particle Astrophysics, a NSF Science and Technology Center at U.C. Berkely, under Cooperative Agreement No. AST 9120005. J.D.B. was also supported by the PPARC. We would like to thank J. Adams, K. Subramanian, and T. Vachaspati for helpful comments.

[1] R. Pudritz and J. Silk, Astrophys. J. **342**, 650 (1989); R. Kulsrud, R. Cen, J.P. Ostriker, and D. Ryu, 1996, Report No. astro-ph/960714.

[2] E.R. Harrison, Mon. Not. R. Astron. Soc. **147**, 279 (1970); **165**, 185 (1973); A.D. Dolgov, Phys. Rev. D **48**, 2499 (1993); M.S. Turner, and L.M. Widrow, Phys. Rev. D **37**, 2743 (1988); K. Enqvist and P. Olensen, Phys. Lett. B **329**, 195 (1994); T. Vachaspati, Phys. Lett. B **265**, 258 (1991); A.D. Dolgov and J. Silk, Phys. Rev. D **47**, 3144 (1993); C. Hogan, Phys. Rev. Lett. **51**, 1488 (1983); J. Quashnock, A. Loeb, and D.N. Spergel, Astrophys. J. **344**, L49 (1989); B. Ratra, Astrophys. J. **391**, L1 (1992); Phys. Rev. D **45**, 1913 (1992).

[3] K.T. Kim, P.C. Tribble, and P.P. Kronberg, Astrophys. J. **379**, 80 (1991); R. Perley and G. Taylor, Astrophys. J. **101**, 1623 (1991); P.P. Kronberg, Rep. Prog. Phys. **57**, 325 (1994).

[4] P.P. Kronberg, J.J. Perry, and E.L. Zukowski, Astrophys. J. **387**, 528 (1992).

[5] A.M. Wolfe, K. Lanzatta, and A.L. Oren, Astrophys. J. **388**, 17 (1992).

[6] D. Grasso and H.R. Rubinstein, Phys. Lett. B **379**, 73 (1996); B. Cheng, A.V. Olinto, D.N. Schramm, and J.W. Truran, Phys. Rev. D **54**, 4714 (1996).

[7] K.S. Thorne, Astrophys. J. **148**, 51 (1967); A.G. Doroshkevich, Astrophysics **1**, 138 (1967); K. Jacobs, Astrophys. J. **155**, 379 (1969).

[8] K.M. Gorski *et al.*, Report No. astro-ph/9601063.

[9] E.F. Bunn, P.G. Ferreira, and J. Silk, Phys. Rev. Lett. **77**, 2883 (1996).

[10] Y.B. Zeldovich, Sov. Astron. **13**, 608 (1970); C.B. Collins, Commun. Math. Phys. **27**, 37 (1972); I.S. Shikin, Sov. Phys. JETP **36**, 811 (1972). If one linearizes the magnetic radiation universe Einstein equations about the isotropic Friedman universe (magnetic field = shear anisotropy = 0) then there is a zero eigenvalue associated with the evolution of the ratio of magnetic to blackbody radiation density [Eq. (1) with $\gamma = 4/3$] and the stability is decided at second order. This mathematical structure is characteristic of the evolution of any trace-free energy momentum tensor in a cosmology dominated by isotropic blackbody radiation; see also J.D. Barrow and D. Sonoda, Phys. Rep. **139**, 1 (1986).

[11] V.N. Lukash, Nuovo Cimento B **35**, 269 (1976); A.G. Doroshkevich, V.N. Lukash, and I.D. Novikov, Sov. Phys. JETP **37**, 739 (1973); C.B. Collins and S.W. Hawking, Astrophys. J. **181**, 317 (1972).

[12] J.D. Barrow, Phys. Rev. D **51**, 3113 (1995).

[13] V.N. Lukash and A.A. Starobinskii, Sov. Phys. JETP **39**, 742 (1974).

[14] R. Maartens, G.F.R. Ellis, and W.R. Stoeger, Phys. Rev. D **51**, 1525 (1995); J.D. Barrow, Report No. gr-qc/9701038.

[15] J.D. Barrow, Mon. Not. R. Astron. Soc. **175**, 359 (1976).

[16] E. Martínez-Gonzalez and J.L. Sanz, Astron. Astrophys. **300**, 346 (1995).

[17] J. Adams, U.H. Danielsson, D. Grasso, and H. Rubinstein, Phys. Lett. B **388**, 253 (1996).

[18] J.D. Barrow, R. Juszkiewicz, and D.H. Sonoda, Mon. Not. R. Astron. Soc. **213**, 917 (1985); J.D. Barrow, Can. J. Phys. **164**, 152 (1986); I.D. Novikov, Sov. Astron. **12**, 427 (1968).

[19] A. Kogut, G. Hinshaw, and A. Banday, Phys. Rev. D (to be published).

[20] A. Kosowsky and A. Loeb, Report No. astro-ph/9601055 [Astrophys. J. (to be published)].

[21] A. Guth, Phys. Rev. D **23**, 347 (1981).

[22] R. Wald, Phys. Rev. D **28**, 2118 (1983); J.D. Barrow, Phys. Lett. B **180**, 335 (1987).