

## Spinning Down a Black Hole with Scalar Fields

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We study the evolution of a Kerr black hole emitting purely scalar radiation via the Hawking process. We show that the rate at which mass and angular momentum are lost by the black hole leads to a final evolutionary state with nonzero angular momentum, namely,  $a/M \approx 0.555$ . [S0031-9007(97)03072-X]

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The conventional view of black hole evaporation is that, regardless of its initial state, Hawking radiation will cause a black hole to approach an uncharged, zero angular momentum state long before all its mass has been lost. Thus, as the evolution nears the Planck scale, where quantum gravity will be required to determine its future development, the final asymptotic state is assumed to be described by the Schwarzschild solution.

Although calculations have shown that any charge on the black hole will be rapidly neutralized [1], qualitative arguments [2] suggest that the mass and angular momentum loss proceeds on cosmologically long time scales. Such arguments, however, do not determine if the angular momentum will tend to zero before the mass does. Using Hawking's results on quantum emission from black holes [3], Page has been able to address this and other related questions. By performing an exhaustive, quantitative, study of the emission of particles associated with fields of spin 1/2, 1, and 2 for both rotating [4] and nonrotating holes [5], he concludes that for these fields the specific angular momentum does tend to zero more rapidly than the mass. This implies that the final asymptotic state of black hole evaporation is indeed the Schwarzschild solution.

However, Page did provide some indirect evidence to suggest that if there were a sufficiently large number of, as yet unknown, massless scalar fields present in nature, then the dimensionless ratio of the black hole's specific angular momentum to its mass,  $a_* = a/M$ , might evolve to a stable nonzero value. In this case, a microscopic evaporating black hole approaching the Planck scale would be described by a Kerr solution, characterized by a mass  $M$  and specific angular momentum  $a$ , rather than Schwarzschild. First, at low  $a_* = a/M$ , Page found that the dominant angular modes for the spin- $s$  fields,  $s = 1/2, 1$ , and  $2$ , were those with  $l = s$ . If the same is true for the scalar field, with  $s = 0$ , then the dominant mode would be  $l = 0$ , which carries off energy, but no angular momentum. In that case, one could imagine a scenario in which the mass loss occurs much more rapidly than the angular momentum loss, causing the black hole to evolve toward a state with  $a_*$  nonzero. Second, Page noted in his results a simple, perhaps accidental linear relationship

between the spin of the field and the (suitably normalized) ratio of the angular momentum loss rate to the mass loss rate,  $h(a_*)$ , as  $a_*$  tended to zero. Page pointed out that if this relation were extrapolated to the case of a scalar field,  $s = 0$ , then the loss rates for emission of scalar particles would result in a final state of nonzero angular momentum.

In this Letter we investigate, in some detail, the evolution of a Kerr black hole emitting purely scalar radiation via the Hawking process. For clarity we have restricted our attention to the mass and angular momentum loss rates, and their effect on the final evolutionary state of the hole. We find that the hole does indeed evolve to a final asymptotic state with nonzero angular momentum, confirming the conjecture of Page. Our numerical results allow us to conclude that the final state will be described by a specific angular momentum  $a = 0.555M$ . In addition, we find that the linear relationship between the spin of the field and  $h(a_*)$ , found by Page, breaks down for  $s = 0$ . We find that for a scalar field,  $h(a_* = 0) = -0.806$ , rather than the value  $-1.195$  which Page obtained by extrapolation. The details of this work will appear elsewhere.

The notation here and throughout follows that of Press and Teukolsky [6] and Page [4]. We make the assumption that the black hole has existed for a sufficient period of time so that any charge it possessed has been lost, and hence is adequately described by a Kerr solution. In terms of Kerr-ingoing coordinates  $(v, r, \theta, \tilde{\phi})$ , the scalar wave equation,  $\square\phi = 0$ , separates by writing  $\phi = R(r)S(\theta)e^{-i\omega v}e^{im\tilde{\phi}}$ , where the angular function  $S(\theta)$  is a spheroidal harmonic [7]. The radial function,  $R(r)$ , satisfies

$$(\partial_r \Delta \partial_r - 2iK \partial_r - 2i\omega r - \lambda)R(r) = 0, \quad (1)$$

where  $\Delta = r^2 - 2Mr + a^2$ ,  $K = (r^2 + a^2)\omega - am$ ,  $\lambda = E_{lm\omega} - 2am\omega + a^2\omega^2$ , and  $E_{lm\omega}$  is the separation constant. While the solutions to Eq. (1) are not, in general, expressible in terms of known functions, their asymptotic behavior is easily obtained [6],

$$R \rightarrow \begin{cases} Z_{\text{hole}}, & r \rightarrow r_+, \\ Z_{\text{in}} r^{-1} + Z_{\text{out}} r^{-1} e^{2i\omega r}, & r \rightarrow \infty. \end{cases} \quad (2)$$

The subscript "in" refers to an ingoing wave originating from past null infinity, "out" refers to the reflected

component of the wave that propagates outward, toward future null infinity, and “hole” refers to the transmitted component that crosses the black hole event horizon at  $r = r_+$ . The amplification,  $Z$  (the fractional gain of energy in a scattered wave), is

$$Z = \left| \frac{Z_{\text{out}}}{Z_{\text{in}}} \right|^2 - 1. \quad (3)$$

Page [4] has shown that, in terms of the scale invariant quantities  $f \equiv -M^2 dM/dt$  and  $g \equiv -Ma_*^{-1} dJ/dt$ , the rate at which the mass and angular momentum of an evaporating black hole decrease is given by

$$\begin{pmatrix} f \\ g \end{pmatrix} = - \sum_{l,m} \frac{1}{2\pi} \int_0^\infty dx \frac{Z}{e^{2\pi k/\kappa} - 1} \begin{pmatrix} x \\ ma_*^{-1} \end{pmatrix}, \quad (4)$$

where  $k = \omega - m\Omega$ ,  $\Omega = a_*/2r_+$  is the surface angular frequency,  $\kappa = \sqrt{(1 - a_*^2)}/2r_+$  is the surface gravity of the hole, and following Page [4] we have defined  $x = M\omega$  and  $a_* = a/M$ . In order to investigate how  $a_*$  varies with  $M$  as the black hole loses mass and angular momentum, we define the function  $h(a_*)$

$$h(a_*) \equiv \frac{d \ln a_*}{d \ln M} = \frac{g(a_*)}{f(a_*)} - 2. \quad (5)$$

The rate of change of  $a_*$  is then given by

$$\frac{da_*}{dt} = - \frac{a_* f h}{M^3}. \quad (6)$$

If there is a nonzero value of  $a_*$  for which  $h$  is zero, then  $da_*/dt$  will be zero there. Since  $f$  is nonnegative, if  $h$  is positive above this value and negative below it, then an evaporating black hole will evolve towards a stable state at  $h(a_*) = 0$ . We now have all the necessary machinery needed to study the mass and angular momentum loss rates of a purely rotating black hole.

For clarity we have limited our attention to the behavior of the mass and angular momentum loss rates, due solely to the emission of massless scalar particles, and their effect on the final asymptotic state of the evaporating hole. We calculated the functions  $f(a_*)$  and  $g(a_*)$  at 18 values of  $a_*$  ranging from  $a_* = 1 \times 10^{-4}$  to  $a_* = 0.99$  and used a clamped cubic spline to extrapolate these values to  $a_* = 0$  and  $a_* = 1$ . The same spline was used to interpolate for points of interest.

Figure 1 shows the behavior of the mass loss rate as a function of the specific angular momentum, described in a scale invariant way by the function  $f(a_*)$ . The loss of mass energy from the hole by emission of scalar particles is more effective at high values of  $a_*$ . The fact that emission still occurs at  $a_* = 1$ , even though the hole has zero temperature, is due to the nonzero chemical potential associated with the angular momentum of the hole; this results in spontaneous emission into the superradiant modes, first discovered by Zel'dovich [8]. An interesting feature of Fig. 1 is the existence of a minimum at  $a_* = 0.574$ , which does not occur for fields of nonzero spin. The unusual behavior of the mass loss,

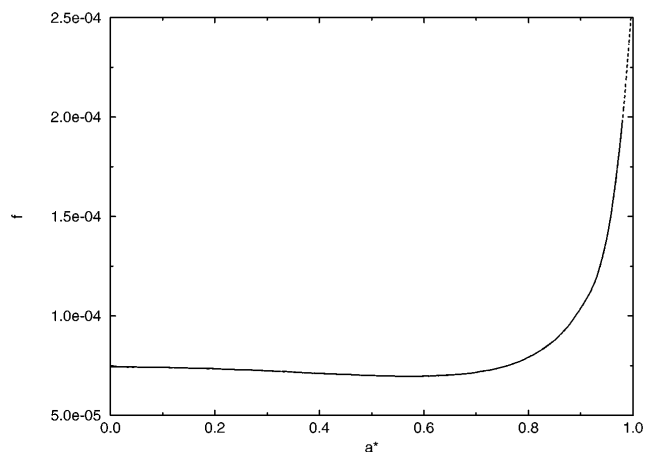


FIG. 1. The mass loss rate, due to scalar particle emission, for a Kerr black hole. At low rotation the mass loss rate approaches the value  $7.439 \times 10^{-5}$ , while at the extreme limit  $a_* = 1$ , it reaches  $2.601 \times 10^{-4}$ . There is a minimum in the emission at  $a_* = 0.574$ .

in this case, is mainly attributable to the fact that a scalar field, unlike higher spin fields, is able to radiate in an  $l = 0$  mode, which is not a superradiant mode. A plot of the mass loss rate, due solely to the  $l = 0$  mode of the field, reveals  $f_{l=0}$  to be a monotonically decreasing function of  $a_*$ . This suggests that the hole couples most strongly to the  $l = 0$  mode at low rotation. At larger values of  $a_*$  the contribution to  $f$  from the higher  $l$  modes becomes more significant, as the effects of superradiant scattering increase. This has the effect of increasing the mass loss rate, hence  $f$ , at high  $a_*$ . The combination of these two effects, at intermediate  $a_*$ , results in the appearance of a minimum in  $f$  as shown.

Figure 2 displays the angular momentum loss rate,  $g$ , versus  $a_*$ . Since the  $l = 0$  mode cannot carry off any

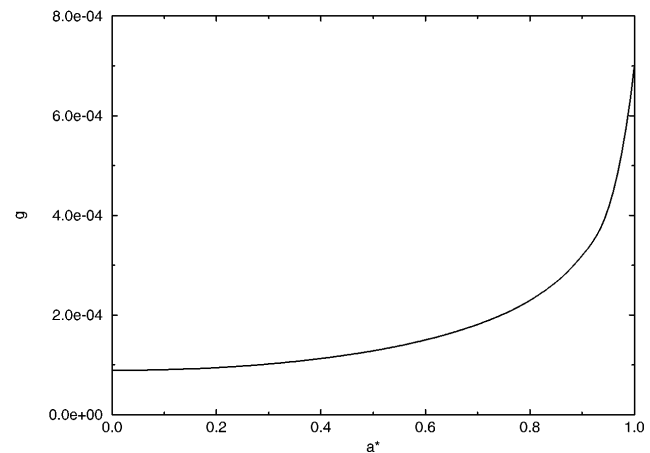


FIG. 2. The angular momentum loss rate, due to scalar particle emission, for a Kerr black hole. At low rotation, the angular momentum loss rate approaches  $8.886 \times 10^{-5}$ . At the extreme limit,  $a_* = 1$ , the emission rate is  $6.853 \times 10^{-4}$ .

angular momentum, the behavior of  $g$  can be understood purely in terms of superradiance. As  $a_*$  increases emission into the superradiant modes becomes more effective, causing the angular momentum loss rate to increase monotonically. Again, at the extreme limit, scalar particle emission continues to carry off angular momentum despite the temperature of the hole being zero, by spontaneous emission into the superradiant modes.

Finally, Figure 3 shows the behavior of  $h(a_*)$ . The most important feature is the existence of a zero in  $h$ , at  $a_* = 0.555$ . A black hole formed with  $a_* < 0.555$  or  $a_* > 0.555$  will evolve, by the loss of mass and angular momentum, until it reaches an asymptotic state characterized by  $a_* = 0.555$ . This confirms the conjecture made by Page [4] that a black hole emitting scalar radiation, via the Hawking process, could evolve to a final state with nonzero angular momentum.

From his examination of spin 1/2, 1, and 2 fields, Page found a remarkable, linear relationship between  $h(a_*)$  and the spin  $s$  of the fields at  $a_* = 0$ . Extrapolating this to the case  $s = 0$  he noted that the relation predicted  $h(a_* = 0) = -1.195$ . Our numerical results have shown that, in fact,  $h(a_* = 0) = -0.806$ , accurate to one part in  $10^4$ .

We have studied the evolution of a rotating black hole emitting purely massless scalar particles via the Hawking process. We have concluded that rather than evolve to a nonrotating state with  $a_* = 0$ , as is often assumed, the hole approaches a nonzero value of  $a_*$ . Our numerical results have allowed us to ascertain that this value is  $a_* = 0.555$ . This result confirms the conjecture of Page, and, for the first time, reveals the exact value of  $a_*$  to which the black hole will relax. Additional details of this study will be given in a separate publication, where we shall also report on additional aspects of black-hole scalar-field interactions.

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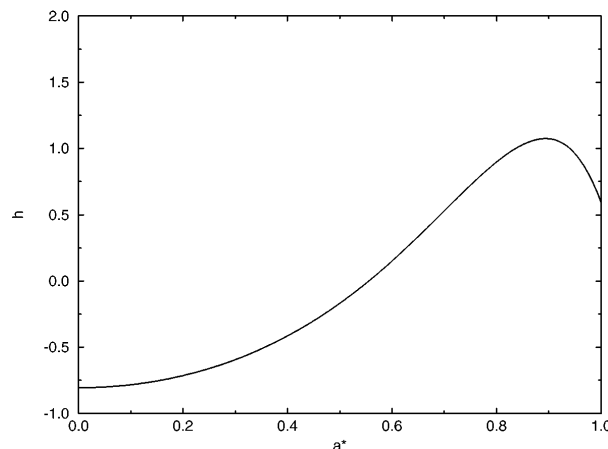


FIG. 3. The angular momentum loss rate compared to the mass loss rate described in a scale invariant way by  $h(a_*)$ , for a Kerr black hole emitting scalar particles. The point at which  $h(a_*) = 0$  occurs at  $a_* = 0.555$ . A hole formed with  $a_*$  on either side of this value will evolve to a state characterized by this value. As  $a_* \rightarrow 0$ ,  $h(a_*)$  approaches the value  $-0.806$ .

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