Inelasticity Distributions in High-Energy p-Nucleus Collisions

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Inelasticity distributions in high-energy p-nucleus collisions are computed in the framework of the interacting gluon model, with the impact-parameter fluctuation included. A proper account of the peripheral events by this fluctuation has shown to be vital for the overall agreement with several reported data. The energy dependence is found to be weak. [S0031-9007(97)02894-9]

Inelasticity, i.e., the fraction of the incident energy E_0 which is transformed into produced particles, is one of the basic quantities in high-energy hadronic and nuclear collisions. It is crucial in cosmic-ray data analysis where the primary mass composition, which is an important piece of information about the Universe, is deduced by using cascade models of the development of the mass composition with appropriate inelasticity distributions [1] and cross sections as the inputs. So, it is natural that since early times the study of the inelasticity has received special attention by both experimentalists and theoreticians. More recently, it has also aroused interest in connection with the production of a quark-gluon plasma in heavy-ion collisions [2–7]. Yet, experimental data are rather scarce and the theoretical understanding of several aspects of the inelasticity, such as its distribution and E_0 dependence, is far from being satisfactory. The main obstacle in accelerator studies, especially with colliding beams, is the increasing difficulty in detecting those particles which carry the main fraction of E_0 as E_0 increases. As for the models, they are largely in conflict with each other even in explaining such a simple aspect as the E_0 dependence of the inelasticity.

A classical model of multiparticle production, which is still popular, is the hydrodynamic model [8]. In any variant of it, the concept of inelasticity is essential, because it defines one of the ingredients, namely, the available energy for particle production. To other types of models, it is just one more variable, which can be computed, without playing a vital role.

One of the main features of high-energy hadronic or nuclear collisions is the large event-by-event fluctuation, exhibited in several observed quantities. Thus, in a given experimental setup and even under the same initial conditions for the colliding objects, events with different final-state configurations occur. For example, in $\bar{p}p$ collisions at $\sqrt{s} = 540$ GeV [9], the number of charged particles produced in an event varies from 2 to more than 100. It was also reported [10] that, in the CERN intersecting storage ring (ISR), the so-called leading particles, those which carry the largest momentum in the center of mass frame in either direction, are uncorrelated and their mo-

mentum distribution is more or less uniform. This implies, in turn, that also the inelasticity varies from event to event. Such fluctuations have either a quantum mechanical or a statistical origin, or even simply associated with the impact parameter. The interacting gluon model (IGM) [7] is a simple QCD-based model, which is especially designed to create the initial conditions for hydrodynamic descriptions by incorporating in an intuitive way the microscopic fluctuations in the initial stage of the collision. It is based on the idea [11] that in high-energy collisions valence quarks weakly interact so that they almost pass through the interaction zone, whereas gluons interact strongly, producing an indefinite number of mini-fireballs, which eventually form a unique large central fireball (all possible $q\bar{q}$ sea quarks are, in this model, "converted" to equivalent gluons). This feature of the IGM makes it quite attractive because it allows us to study not only the leading-particle spectrum [2–6] but also other relevant quantities as momentum distributions [12], correlations, etc.

However, the main drawback of the original version of the IGM was the neglect of the impact parameter. As is clearly seen in Ref. [1], it is also the model which predicts the most pronounced decrease of the average inelasticity with E_0, in clear conflict with the estimates of this quantity based on data. In a previous work [12] (hereafter called I), we have improved the IGM by including the impact-parameter fluctuation. Conceptually, this is indeed necessary in any realistic description of hadronic or nuclear collisions, but it also affects the observables in a significant way. In I, we studied the effects of the initial-condition fluctuations in hydrodynamic models and thereby focused our attention mainly on the rapidity distributions in p-p collisions. In the present Letter, we shall focus upon the inelasticity both in p-p and p-nucleus collisions and show that a proper inclusion of the impact-parameter fluctuation improves considerably the agreement with data.

The impact parameter $\vec{b}$ defines, in the first place, the *probability density of occurrence of a reaction* (apart from the normalization) $F(\vec{b}) = 1 - |S(\vec{b})|^2$, where the eikonal function is written as

$$|S(\vec{b})|^2 = \exp\left[-C \int d\vec{b}' \int d\vec{b}'' D_p(\vec{b}')\right.$$
$$\left. \times D_A(\vec{b}'') f(\vec{b} + \vec{b}' - \vec{b}'')\right]$$
$$= \exp[-Ch(\vec{b})], \tag{1}$$

with $D_p(\vec{b})$ the gluon thickness function of proton, $D_A(\vec{b})$ the one for the nucleus A, and C is an energy-dependent parameter to be determined by the condition $\int F_{pp}(\vec{b}) \, d\vec{b} = \sigma_{pp}^{\text{inel}}(\sqrt{s})$ for pp collisions. Notice that, because of this condition, once the pp cross section is fixed, the pA cross section $\sigma_{pA}^{\text{inel}}(\sqrt{s}) = \int F_{pA}(\vec{b}) \, d\vec{b}$ may be calculated by using Eq. (1). We have taken

$$\sigma_{pp}^{\text{inel}} = 56(\sqrt{s})^{-1.12} + 18.16(\sqrt{s})^{0.16} \tag{2}$$

as an input [13]. The function $f(\vec{b})$ in (1), subject to the constraint $\int f(\vec{b}) \, d\vec{b} = 1$, accounts for the finite *effective* gluon interaction range (with the screening effect taken into account). The simplest choice of $f(\vec{b})$ would be a point interaction $\delta(\vec{b})$, but we preferred to parametrize it as a Gaussian with a range ≈ 0.8 fm, which is more consistent with the character of the strong interaction and also describes better the data. For $D_p(\vec{b})$ we take a Gaussian distribution. So, we have eventually $D_p(\vec{b}) = f(\vec{b}) = (a/\pi)\exp(-ab^2)$, with $a = 3/(2R_p^2)$, where $R_P \approx 0.8$ fm is the proton radius. For $D_A(\vec{b})$, we take

$$D_A(\vec{b}) = \int_{-\infty}^{+\infty} \rho_A(\vec{b}, z) \, dz$$
$$= \int_{-\infty}^{+\infty} \frac{\rho_0}{1 + \exp[(r - R_0)/d]} \, dz, \tag{3}$$

where $R_0 = r_0 A^{1/3}$, $r_0 = 1.2$ fm, $d = 0.54$ fm, and $\rho_A(\vec{r})$ is normalized to A. Thus we get

$$h(\vec{b}) = a \int_0^\infty db' \, b' D_A(\vec{b}') I_0(abb') e^{-a(b^2 + b'^2)/2}, \tag{4}$$

where I_0 is a modified Bessel function.

Secondly, the impact parameter determines *the size of the fireball*, because as b increases the average fireball mass becomes smaller. We incorporate this effect by writing the gluon momentum distribution functions as

$$G_p(x, \vec{b}) = D_p(\vec{b})/x, \qquad G_A(y, \vec{b}) = D_A(\vec{b})/y, \tag{5}$$

where x and y are the Feynman variables of gluons in p and A, respectively, in the equal-velocity (e.v.) frame. With this notation, the density of gluon pairs that fuse contributing to the final fireball may be expressed as

$$w(x, y; \vec{b}) = \int d\vec{b}' \int d\vec{b}'' G_p(x, \vec{b}'') G_A(y, \vec{b}'') \sigma_{gg}(x, y)$$
$$\times f(\vec{b} + \vec{b}' - \vec{b}'')\theta(xy - M_{\min}^2/s)$$
$$= h(\vec{b})w(x, y), \tag{6}$$

with

$$w(x, y) = [\sigma_{gg}(x, y)/xy]\theta(xy - M_{\min}^2/s), \tag{7}$$

where $M_{\min} = 2m_\pi$ and the gluon-gluon cross section is parametrized as [14] $\sigma_{gg}(x, y) = \alpha/xys$, with $\alpha = 21.35$, determined by using the pp inelasticity data [15]. Observe that in (6), b dependence is factorized out. It is also presumed that *the same physics describes both pp and pA collisions.*

Now, we shall give a brief account of how to obtain the probability density $\chi(E, P; \vec{b})$ of forming a fireball with energy E and momentum P at a fixed $\vec{b}$. We assume that the colliding objects form a fireball, via gluon exchanges, depositing in it momenta $x(\vec{b})\sqrt{s}/2$ and $-y(\vec{b})\sqrt{s}/2$, respectively. Let n_i be the number of gluon pairs that carry momenta $x_i\sqrt{s}/2$ and $-y_i\sqrt{s}/2$. Thus,

$$\sum_i n_i x_i = x(\vec{b}) \quad \text{and} \quad \sum_i n_i y_i = y(\vec{b}). \tag{8}$$

In what follows, we will omit the explicit $\vec{b}$ dependence of x and y in order not to overload the notation. The energy and momentum of the central fireball in the e.v. frame of the incident particles are given by

$$E = (x + y)\sqrt{s}/2, \qquad P = (x - y)\sqrt{s}/2, \tag{9}$$

and its invariant mass M and rapidity Y are, respectively,

$$M = \sqrt{sxy} \equiv \kappa\sqrt{s} \quad \text{and} \quad Y = (1/2)\ln(x/y). \tag{10}$$

With these notations, we can follow the prescription given in [7] and write the relative probability of forming a fireball with a specific energy and momentum as

$$\Gamma(x, y; \vec{b}) \simeq \exp[-\mathbf{X}^T \mathbf{G}^{-1} \mathbf{X}]/[\pi\sqrt{\det(\mathbf{G})}], \tag{11}$$

where

$$\mathbf{X} = \begin{pmatrix} x - \langle x \rangle \\ y - \langle y \rangle \end{pmatrix}, \qquad \mathbf{G} = 2\begin{pmatrix} \langle x^2 \rangle & \langle xy \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix},$$

with the notation

$$\langle x^m y^n \rangle = \int dx' \int dy' \, x'^m y'^n w(x', y'; \vec{b}). \tag{12}$$

In terms of E and P, $\Gamma(E, P; \vec{b})$ reads

$$\Gamma(E, P; \vec{b}) \simeq [2\sqrt{a_1 a_2}/\pi]$$
$$\times \exp\{-a_1[E - \langle E \rangle]^2 - a_2 P^2\}, \tag{13}$$

where $a_1 = [s(\langle x^2 \rangle + \langle xy \rangle)]^{-1}$, $a_2 = [s(\langle x^2 \rangle - \langle xy \rangle)]^{-1}$, and $\langle E \rangle = (\langle x \rangle + \langle y \rangle)\sqrt{s}/2$ [*do not confuse this notation with the average value; it is not because $w(x, y; \vec{b})$ is not normalized*]. Apparently, $\Gamma(E, P; \vec{b})$ in (13) is normalized. However, both E and P are bounded because of the energy-momentum conservation constraint. It is also constrained by $M > M_{\min} = 2m_\pi$. So, we put some additional factor $\chi_0(\vec{b})$,

$$\chi(E, P; \vec{b}) = \chi_0(\vec{b})\Gamma(E, P; \vec{b}), \tag{14}$$

such that

$$\int dP \int dE \, \chi(E, P; \vec{b}) \times$$
$$\theta(\sqrt{E^2 - P^2} - M_{\min}) = \frac{F_{pA}(\vec{b})}{\sigma_{pA}^{\text{inel}}}. \tag{15}$$

As implied by (5), the gluon momentum distribution is independent of the particular type of nucleus, the only difference being their density. So, in the integral (12), x' and y' vary from some lower limit, defined by $\sqrt{sx'y'} = M_{min}$, up to 1, corresponding to the complete neglect of any collective effect of the nucleons in a nucleus. On the other hand, the integration limits of (15) are chosen differently. y in (8) may be larger than 1, because gluons from different nucleons may contribute to give the fireball a momentum transfer that is larger than $\sqrt{s}/2$, which is just the incident momentum of a single nucleon in our e.v. frame. We take as the upper limit of y the overlap $h(\vec{b})$, whenever it is larger than 1. When $h(\vec{b}) < 1$, we take it $= 1$, because in such a case the proton interacts just with a single nucleon. It is clear that x remains ≤ 1.

Once $\chi(E, P; \vec{b})$ is determined, we are ready to compute the inelasticity distribution, which is the main object of this work. In I, following Ref. [7], we have defined the inelasticity as the variable κ appearing in (10). However, the usual definition is $k = (E_0 - E')/E_0$, where E' is the leading (or surviving) particle energy. We shall adopt this terminology here and distinguish it from κ. There is also some difference between k defined in the laboratory frame and the one given in the e.v. frame. However, since this is quite negligible (except when $k \to 1$), we will not make any distinction in this note. The κ distribution has been obtained in I and reads

$$\chi(\kappa) = \int d\vec{b} \int dE \int dP \, \chi(E, P; \vec{b})$$
$$\times \, \delta(\sqrt{(E^2 - P^2)/s} - \kappa)$$
$$\times \, \theta(\sqrt{E^2 - P^2} - M_{min}). \quad (16)$$

Then, by fitting the only existing $\chi(\kappa)$ data [15] at $\sqrt{s} = 16.5$ GeV, we fix the parameter α of the model. A comparison with the data is shown in Fig. 1, where we have also put the result of [7]. It is seen that the impact-parameter fluctuation enhances the small-κ events and makes the overall shape flatter, in better agreement with the data. The enhancement of large-κ events is simply due to the larger value of α which is necessary now.

The computation of the inelasticity distribution $\chi(k)$ is similar. We have $k = x$, so, by using (9),

$$\chi(k) = \int d\vec{b} \int dE \int dP \, \chi(E, P; \vec{b})$$
$$\times \, \delta((E + P)/\sqrt{s} - k)$$
$$\times \, \theta(\sqrt{E^2 - P^2} - M_{min}). \quad (17)$$

We show, in Fig. 2, the results for several pA collisions at $\sqrt{s} = 550$ GeV. No accelerator data at such a high energy exist, but it is seen that $\chi(k)$ is nearly k independent for pp, in agreement with ISR data [10,16]. In a recent cosmic-ray experiment [17], hadron-Pb inelasticity distribution at an average energy of $\langle\sqrt{s}\rangle = 550$ GeV has been estimated. The result is $\chi(k) = 3.3k^{2.3}$. We find that the
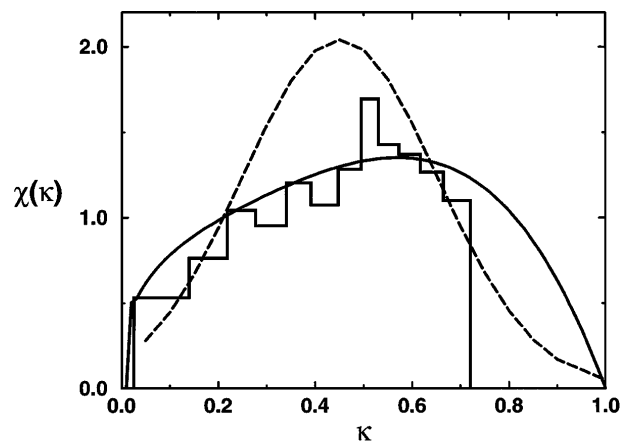

FIG. 1. κ distribution for p-p at $\sqrt{s} = 16.5$ GeV. The data are from [15]. The solid line is our result, whereas the dashed one is from [7].

qualitative features of our result agree with this estimate. Some of the origins of the quantitative discrepancy may be the difference between π-Pb and p-Pb collisions and the possible inclusion of hadron diffractive dissociation in their analysis. We show in Fig. 3 the average inelasticity $\langle k \rangle$ as a function of $\sqrt{s}$, for several target nuclei. It still decreases as $\sqrt{s}$ increases but, compared with the results of [7], the energy dependence is quite small now and compatible with the estimates obtained in [1] using cosmic-ray data. The main origin of this contrast is the factor σ_{pp}^{inel} which has been dropped out in (7), because it is not necessary in our version.

A related quantity is the leading-particle spectrum, as shown in Fig. 4 at $\sqrt{s} = 14$ GeV [18]. Since data on p_T dependence are scarce, we have assumed an approximate factorization of $x_l(= 2p_l/\sqrt{s})$ and p_T dependences,

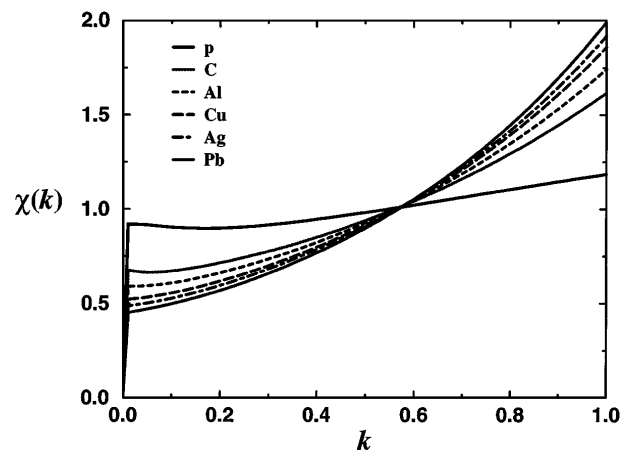$$E_l(d^3\sigma/dp^3) \approx f(x_l)h(p_T), \quad (18)$$

FIG. 2. Inelasticity distribution for p-A collisions with several targets at $\sqrt{s} = 550$ GeV.
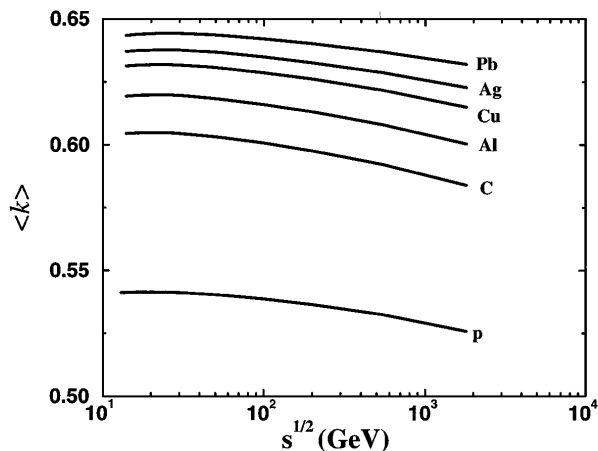
FIG. 3. Energy dependence of the average inelasticity for p-A collisions.

where

$$f(p_l) = \int d\vec{b} \int dP \int dE \, \chi(E, P; \vec{b})$$
$$\times \, \theta(\sqrt{E^2 - P^2} - M_{\min})$$
$$\times \, \delta([\sqrt{s} - (E + P)]/2 - p_l), \quad (19)$$

and parametrized $h(p_T)$ as

$$h(p_T) = (\beta^2 / 2\pi) e^{-\beta \, p_T}, \quad (20)$$

determining the average β by using Table II of [18]. The curves obtained with these β values (with an interpolation for Al and Ag) are shown in Fig. 4. One sees that the agreement is almost perfect. The result of [7] for pp is also shown for comparison. We did not put their curves for the other targets, but the behavior is similar; namely, they are more bent showing a definite deviation from the data in the largest-x_l region. This is a consequence
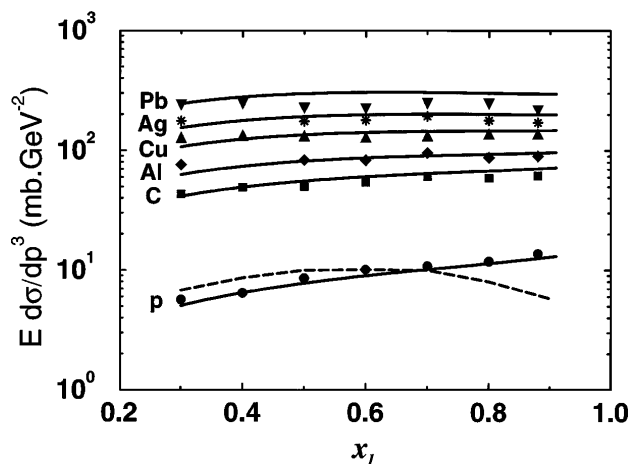

FIG. 4. Leading-particle spectra as a function of x_l at $p_T = 0.3$ GeV. The data are from [18] at $\sqrt{s} = 14$ GeV. The solid curves are our results, whereas the dashed one is from [7]. The slope parameter has been extracted from [18] as $\beta = 4.20, 3.22, 3.21, 3.26, 3.47$, and 3.78 GeV^{-1} for p, C, Al, Cu, Ag, and Pb targets, respectively.

of the neglect of the peripheral events there. Some authors [3,5,6] have obtained good fits to pA data, but in those works it is not clear which is the connection to other relevant quantities such as momentum distributions, correlations, etc., of the secondary particles. Also, pp is usually treated as a separate case.

We conclude the present Letter by summarizing that, except for the diffractive component, the IGM seems to describe well the p-A inelasticity, provided the peripheral events are correctly treated, by considering the impact-parameter fluctuation. The average inelasticity decreases very slowly with the energy, in this description.

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