Preliminary Results of a Determination of the Newtonian Constant of Gravitation: A Test of the Kuroda Hypothesis

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Two preliminary determinations of the Newtonian constant of gravitation have been performed at Los Alamos National Laboratory employing low-Q torsion pendulums and using the time-of-swing method. Recently, Kuroda has predicted that such determinations have an upward bias inversely proportional to the oscillation Q, and our results support this conjecture. If this conjecture is correct, our best value for the constant is $(6.6740 \pm 0.0007) \times 10^{-11}\ \mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$. [S0031-9007(97)03004-4]

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Interest in the value of the Newtonian gravitational constant, G, has increased recently with the publication of several disparate results [1–4]. This discrepancy, as much as 50 standard deviations, is an error unheard of in the measurement of any other of the fundamental constants. A torsion pendulum instrument has been assembled at Los Alamos National Laboratory which determines G by the method of Heyl, also called the time-of-swing method, and preliminary results of this effort are reported herein. Kuroda has shown [5] that popular models of anelasticity predict a bias in these determinations where the damping of the pendulum is caused by losses in the suspension fiber, and this upward fractional bias should be $1/\pi Q$. Two determinations were carried out employing systems which differ in Q by a factor of 2, and the disagreement of their results is consistent with that predicted by Kuroda.

In a time-of-swing, or Heyl-type, measurement, the oscillation frequency of a torsion pendulum is perturbed by the presence of source masses. In their absence, the free oscillation frequency, squared, is related to the moment of inertia, I, and fiber torsion constant K_f by

$$\omega_n^2 = K/I\,. \tag{1}$$

The interaction potential energy of the pendulum and source masses, $U_g(\varphi)$, contributes a "gravitational torsion constant,"

$$K_g(\varphi) = \frac{d^2 U_g(\varphi)}{d\varphi^2}\,, \tag{2}$$

where φ is the angle between the axis of the pendulum and that of the source masses. Therefore, the frequency of small oscillations of the pendulum is

$$\omega^2 = (K_f + K_g)/I\,. \tag{3}$$

The gravitational torsion constant is at a maximum when the pendulum is in line with the source masses ($\varphi = 0$, or "near" position), and at a minimum when it is perpendicular ($\varphi = \pi/2$, or "far" position). It is proportional to G, and is calculable from the geometry and densities of the pendulum and masses, so the gravitational constant is given by

$$G = \frac{\Delta(\omega^2) I}{\kappa_g(\varphi = 0) - \kappa_g(\varphi = \pi/2)}\,, \tag{4}$$

where $\kappa_g = K_g/G$, and $\Delta(\omega^2)$ is the difference of the square of the frequencies recorded at the two orientations.

The above derivation assumes that κ_g remains the same at each orientation, but this assumption has been called into question recently. Interest in gravitational wave detectors has spurred research into the anelastic properties of suspension materials at low frequencies, and one model of anelasticity has been shown [6–8] to predict accurately the behavior of several different materials. This model treats a physical spring as a perfectly elastic spring in parallel with a continuous number of Maxwell units, characterized by a spectrum of relaxation times. The model predicts that the torsion constant is a function of oscillation frequency. Kuroda [5] has shown that, for a Heyl-type measurement of G where the principle source of damping is the fiber, the measured value of G will be biased upward by a factor of $(1 + 1/\pi Q)$. [Newman has further demonstrated [9] that, for any function of damping strength versus damping time, the maximum possible bias predicted by the model is $(1 + 1/2Q)$.]

Therefore, we performed two determinations of G having oscillation quality factors which differed by a factor of 2. Different tungsten wires were used to suspend the same pendulum in the same apparatus, one wire having a gold coating which made it more lossy. The damping of the system was known to be produced by the losses in the wire since the system was under the same vacuum pressure in each case, and, in a previous experiment, the same pendulum at a similar vacuum achieved a Q an order of magnitude greater when suspended from fused silica fiber. The determinations therefore test the prediction of Kuroda. The damping now occurs in the malleable losses in the gold coating, and the frequency is determined by the elastic constants in the tungsten core. While these seem to be different from the model defined by Kuroda, we believe that the disparity is negligible and remains a good test of his hypothesis.

The logic of the experiment may be summarized as follows: The same apparatus, at the same vacuum, using a quartz fiber chosen to have the same oscillation frequency, has a Q greater than 10^4. (The lack of conductivity of the quartz fiber prevented an accurate determination of G.) A tungsten fiber, again with the same bob, the same frequency, the same pressure, attained a Q of approximately 10^3. Again, with a gold-coated tungsten fiber, the Q was approximately 500. Therefore, the damping in the gold-coated and bare tungsten fibers was inherent in the anelastic quality of the fiber and not in a velocity-dependent damping (residual gas). It was therefore a test of Kuroda's hypothesis.

A schematic of the apparatus appears in Fig. 1. The source masses are sintered tungsten spheres approximately four inches in diameter weighing 10.489 980 and 10.490 250 kg each, and have been used in previous experiments [10,11]. These spheres are situated upon a round table of Cervit glass held by a nonmagnetic aluminum air bearing. The table can be located at any angle by a stepper motor, and its position read out by an absolute shaft angle encoder. The separation of the spheres, to which the determination of G is sensitive, is determined by reference to an inner bearing race which fits between them, and this race is measured against gage blocks.

The pendulum consists of a tungsten "dumbbell" and a small mirror, connected by a rigid vertical shaft. The dumbbell is, in fact, composed of two disks of tungsten (diam = 7.1660, thickness = 2.5472 mm) mounted on the ends of a narrow tungsten shaft (diam = 1.0347 mm). The overall length of the small mass system is 28.5472 mm. The mirror serves as the reflector of an optical lever by which the angle of the pendulum is recorded over time. It is viewed by an autocollimator. The pendulum had a period of oscillation of 210 sec when suspended from either 12 μm tungsten wire.

An eddy current damper, whose purpose is to damp out the swing mode of the pendulum, is located roughly one-fifth of the way down from the suspension point of the wire. The top one-fifth of the suspending wire consists of a quartz optical communication fiber 140 μm in diameter which has been rendered electrically conductive by vacuum deposition of chromium and gold. This quartz fiber is flexible enough to allow the damping element to swing with the motion of the pendulum, but has a torsion constant greater by a factor of 10^4 or more, preventing the element from twisting with the pendulum and thereby

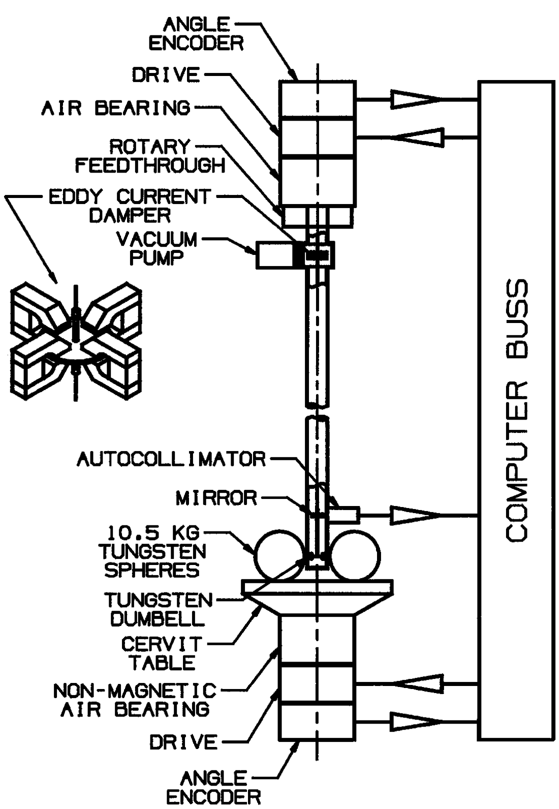

FIG. 1. A schematic of the apparatus used in the determination of G.

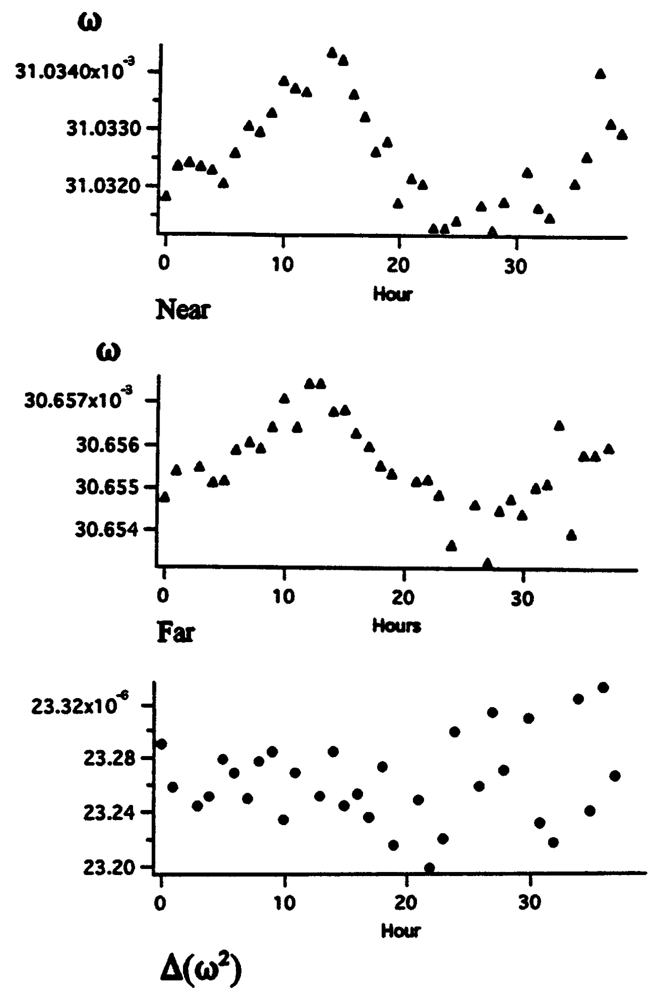

FIG. 2. Typical data record of the oscillation frequency in the "near" and "far" positions. Note that the diurnal drifts of ω^2 are similar in the two positions and nearly cancel when they are subtracted to obtain the value of $\Delta(\omega^2)$.

TABLE I. Error budgets for the two determinations in ppm of the final value of G.

		Expt. #1 (ppm)	Expt. #2 (ppm)
Data:	$\Delta(\omega^2)$	103	90
Metrology:	Separation of spheres	106	55
	Dimensions of bob	42	42
	Attraction of mirror	20	20
	Moment of inertia of mirror	12	12
	Moment of inertia of couple	10	10
	Masses of spheres	5	5
	Alignment error	54	36
TOTAL		165	122

selectively damping the swing mode without significantly reducing the torsion Q.

The pendulum is encased in a stainless vacuum system, maintained at an atmosphere of 10^{-7} torr by an ion pump near the suspension point of the wire.

The apparatus was located in a dedicated, isolated building (LANL Bldg. TA33-151) far from human activity atop a tall mesa composed of porous tuff, a soil incapable of supporting a water table. The room in which the instrument sat was sealed and insulated with 3 in. Styrofoam board, and several feet of loose earth piled atop the building, keeping its temperature constant to within 1 °C over the course of a day.

In the first determination, the internal damping of the uncoated tungsten wire allowed a Q of 950, while in the second the gold-coated wire allowed a Q of only 490. The separation of the centers of mass of the tungsten spheres was as follows:

$$\text{Expt. \#1 } (Q = 950): \quad 71.9092 \pm 0.0025 \text{ mm},$$

$$\text{Expt. \#2 } (Q = 490): \quad 69.8567 \pm 0.0013 \text{ mm}.$$

Data were taken for each determination by the following regimen: The large masses were positioned alternately in the near and far positions, the table rotating by 90° each half-hour, between which time the oscillations of the pendulum were observed by the autocollimator. Forty to ninety such half-hourly records were taken in a group, after which the site was visited in order to collect the data and initiate the next set. After several weeks of recording, the spheres were removed from the table, and a week of further data was recorded using the identical regimen to tare the measurement by investigating the attraction of the positioning table. No attraction was detected.

Figure 2 shows a typical forty-hour record of the oscillation frequencies in the near and far positions, the values being extracted from the angle-time data of the pendulum by fitting to sinusoid. Each record displays a diurnal variation and drift, but they are slow with respect to the hourly cycle of measurement, so the difference of squared frequencies, $\Delta(\omega^2)$, does not display as great a variation. The value of $\Delta(\omega^2)$ did not manifest long-term drift over the course of either experiment.

The gravitational torsion constants for the near and far orientations, as they appear in Eq. (4), are calculated from an expansion of the potential of the elements of which the dumbbell is composed, via Lowry *et al.* [12]. The attraction of the mirror is likewise calculated from a multipole expansion, and its contribution, roughly one one-thousandth of that of the dumbbell is included. The moment of inertia of the pendulum is determined from its geometry and densities.

Error budgets for each determination are given in Table I.

The values for G as determined by the two experiments are 6.6761 ± 0.0011 and 6.6784 ± 0.0008 $(\times 10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2})$, disagreeing by 345 ppm. The difference predicted by Kuroda is 315 ppm, and so our results agree quite well with him. Applying a correction as indicated by Kuroda [5], the values for G are 6.6739 ± 0.0011 and 6.6741 ± 0.0008 $(\times 10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2})$. Combining these two values, we obtain 6.6740 ± 0.0007 $(\times 10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2})$. This is in contrast with the previous work by Luther and Towler [10] which yielded a value of 6.6726 ± 0.0005, a difference of 2 standard deviations.

Work on the determination of G by this method is continuing at Los Alamos and will employ different, higher Q, suspensions and an improved small mass system.

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