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Three-Particle Entanglements from Two Entangled Pairs

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When a single particle from two independent entangled pairs is detected in a manner such that it is impossible to determine from which pair the single particle came, the remaining three particles become entangled in a GHZ state. This procedure can be realized with existing sources of entangled photons and with future sources of entangled atoms. [S0031-9007(97)02923-2]

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Entanglements of three or more particles are fascinating quantum systems, especially when the entanglement is maximal. For example, if the polarizations of three particles are maximally entangled, as in Greenberger-Horne-Zeilinger (GHZ) state [1], then, according to quantum mechanics, each of them is unpolarized. However, there are perfect correlations among the three: given the results of arbitrary polarization measurements on two of the particles one can predict with certainty the outcome of an appropriate measurement on the third particle. This feature seems to imply that each particle possesses many Einstein-Podolsky-Rosen (EPR) elements of reality [2]. However, introduction of these elements of reality implies a contradiction [1]. It would be interesting to experimentally exhibit the dance of correlations present in a three-particle entanglement. While there have been proposals for producing three- or four-particle entanglements, none of these has been achieved in the laboratory. Most of the earlier proposals [3] employ interaction between the particles to achieve entanglement. Here we propose a realizable method based entirely on the concept of quantum erasure [4].

In this Letter we present a general scheme and realizable procedures for generating three-particle entanglements out of just two pairs of entangled particles from independent emissions [5]. The basic idea is to set up an arrangement such that all information about the source of one of the four particles is erased. This entangles the other three particles as they propagate to their observation stations. Finally, we also propose a scheme for observing four-particle GHZ correlations.

Consider the arrangement of Fig. 1. Two independent sources each emit a pair of particles [6] in a beam-entangled state and, by chance, these emissions are nearly simultaneous. Suppose, for example, that the states of the pairs are

$$\frac{1}{\sqrt{2}}\left(|a\rangle\,|d\rangle + |a'\rangle\,|c'\rangle\right), \tag{1}$$

from source A, and

$$\frac{1}{\sqrt{2}}\left(|d'\rangle\,|b'\rangle + |c\rangle\,|b\rangle\right), \tag{2}$$

from source B (the letters represent beams taken by the particles in Fig. 1; all beams have the same polarization) [7]. The beams d and d' are mixed by a 50-50 beam splitter, behind which are two detectors D_T (*trigger*) and D'_T.

Suppose that one and only one of these four particles is detected by D_T, no particle is detected at D'_T, and the other six beams illuminate the three-particle interferometer [1] of Fig. 2. Because of the beam splitter, the trigger particle could have come from either source A or B. If it came from A, its companion must be in beam a, and the pair from B must be in beams b and c. Thus, the state of the triple of remaining particles is $|a\rangle\,|b\rangle\,|c\rangle$. If, on the other hand, the trigger particle came from source B, its companion must be in beam b' and the pair from A

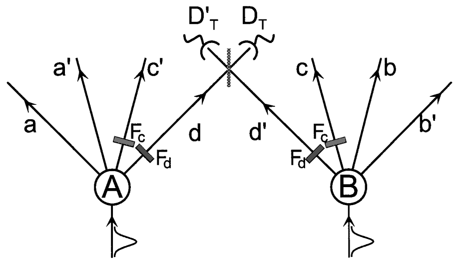

FIG. 1. A three-particle beam-entanglement source. Short pulses of duration ΔT stimulate two independent two-particle sources, A and B, to each emit a pair of beam-entangled particles. The state of the pair from A $[B]$ is given by Eq. (1) [(2)]. Suppose that the trigger detector, D_T, registers a single particle and the other three particles are eventually found to have been in beams a or a', b or b', and in c or c', respectively. If the trigger particle came from A via transmission at the beam splitter its sibling must be in beam a and the pair from B must be in beams b and c. If the trigger particle came from B via reflection at the beam splitter, its sibling must be in beam b' and the pair from A must be in beams a' and c'. Narrow filters, F_c and F_d, of widths much narrower than the pulse spectrum, $\sigma_p \approx 1/\Delta T$, make the source of the trigger particle essentially unknowable (see text). Consequently, the state of the other three particles is the entanglement of Eq. (3).

must be in beams a' and c'. Thus, if the trigger particle came from B, the state of the remaining triple is $|a'\rangle \times |b'\rangle |c'\rangle$.

Now, if the procedure of emission and selection of the four particles is such that one *cannot ever know, not even in principle,* which source produced the trigger event, then the other particles, as they enter the interferometer of Fig. 2, will be in a coherent superposition (rather than an incoherent mixture) of the two three-particle states mentioned above, i.e., in the GHZ state

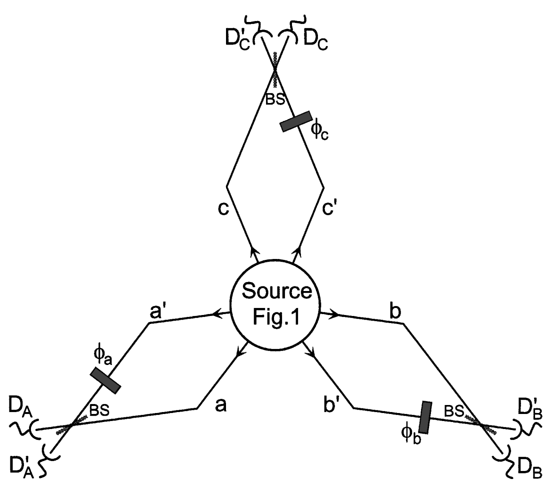

FIG. 2. A three-particle beam-entanglement interferometer. Three particles in state (3) enter the arrangement. Three adjustable phase shifters provide an additional contribution $\phi_A + \phi_B + \phi_C$ to the relative phase ϕ of the state. Consequently the threefold coincident count rate in, say, detectors D_A, D_B, and D_C will oscillate sinusoidally when the phase is varied linearly in time.

$$\frac{1}{\sqrt{2}}\left(|a\rangle |b\rangle |c\rangle + e^{i\phi}|a'\rangle |b'\rangle |c'\rangle\right), \tag{3}$$

where the relative phase ϕ depends on the positions of various elements of the full setup. Note that the use of D_T' instead of D_T as the trigger shifts ϕ by π.

For the coherent superposition of state (3) to form one must erase *all* ways by which one might in principle identify true pairs. Now, to emit beam-entangled pairs of particles, each source must initially contain a parent particle whose momentum is definite to some extent. That parent subsequently decays into a pair of daughters, with momentum conservation producing the desired entanglement. But pairs so produced will also, in general, carry correlations in polarization, energy, and time. Any of these may in principle be exploited to identify the true sibling and hence the source of the trigger particle, and thereby prevent the entanglement (3) from forming. However, polarization correlations can never be exploited if both particles from the two sources simply carry the same polarization. Energy correlations can never be exploited if all four particles carry the same energy or, more generally, if the energy correlations of true pairs (emitted by the same source) are indistinguishable from mixed pairs (one particle from each source). Similarly, temporal correlations can never be exploited if all four particles are produced or detected at the same time or, more generally, if the temporal correlations of true pairs and of mixed pairs are indistinguishable.

Can the scheme just outlined actually be realized with existing laboratory techniques? Clearly, there are two necessary requirements: (I) availability of two-particle four-beam entanglement sources, A and B, of sufficient intensity that, occasionally, both emit a pair, and (II) realizable techniques for ensuring that the source, A or B, of the trigger is unknowable. Two-particle four-beam entanglement has already been demonstrated in the laboratory [7] using a parametric down-conversion (PDC) source (type-I phase matching, the polarizations of the photons are identical). But unprocessed pairs of PDC photons possess almost perfect temporal and frequency correlations (for cw-monochromatic pumping). Thus, e.g., in the interferometer of Fig. 2, one could, in principle, determine that the trigger photon came from crystal A $[B]$ by noting the near simultaneity of its detection at D_T with the photon at D_A $[D_B]$. Alternatively, by measuring the frequencies one could deduce which two photons formed a single PDC pair, since the sum of their frequencies adds up to the pump frequency ν_p.

To erase these opportunities to identify the trigger source we place in the source beams c, c' and d, d' two pairs of narrow filters, centered at half of the pump frequency $\frac{1}{2}\nu_p$ (of widths σ_c for the first pair, and σ_d for the second). The original bandwidth of a single unprocessed PDC photon is very wide. But if such a photon passes through a filter F its coherence time dramatically increases to $1/\sigma_F$. Consequently, the perfect temporal correlation between photons of a single PDC pair gets blurred to the same extent.

In principle one could employ detectors of an extremely sharp time resolution τ, and *select* only *ultracoincident* pairs of counts [8] at D_T and D_C, i.e., those detection events which satisfy $|t_C - t_T| < \tau \ll 1/\sigma_d$ and $1/\sigma_c$, where t_C $[t_T]$ is the detection time at D_C $[D_T]$. Since these detection events at D_T and D_C are much closer in time than the temporal correlation between particles of a true pair (set by the filters), we can no longer associate the D_T event with either D_A or D_B detection events. And if all the filters are centered at half the pump frequency, energy correlations cannot reveal the true pairs. However, due to current technical limitations (the time resolution of detectors $\tau \approx 0.5$ ns $\gg 1/\sigma_F \approx 1$ ps) this method is unrealizable. An additional disadvantage is that here the GHZ triples would only be identified via a postselection procedure.

Alternatively, suppose the two crystals are pumped by very short pulses [9] of duration ΔT and spectral width $\sigma_p \approx 1/\Delta T$, and that the bandwidths of the filters are much narrower than σ_p. The unfiltered photons in D_A and D_B must appear within a coincidence window defined by the pulse duration; i.e., one has $|t_A - t_B| \leq \Delta T$. But the filtered photons in D_C and D_T rattle around in the filters for times of order $1/\sigma_c$ and $1/\sigma_d$ which greatly exceed ΔT and hence neither of these can be linked with either D_A or D_B. Thus the origin of the photon at D_T is erased. This is independent of the actual time resolution of the detectors. All detected triples are indeed in the GHZ state (3).

The visibility of the three-particle fringes in the interferometer of Fig. 2 measures how completely the source information has been erased. To estimate this parameter, assume, for simplicity, that the filters and the pump spectral profiles are Gaussians, $\exp\{-[(\nu - \nu_0)/2\sigma]^2\}$, where ν_0 is the mid frequency and σ the width. Calculation reveals the visibility is

$$V(3) = \frac{\sigma_p}{\sqrt{\sigma_p^2 + \frac{1}{2}\sigma_c^2 + \frac{1}{2}\sigma_d^2}}\,. \tag{4}$$

Currently realizable values of $\sigma_F \approx 1$ nm for filter widths and $\sigma_p \approx 5$ nm for pulse spectral width yield $V(3) \approx 97\%$ [10]. It is worthwhile to add that our current setup for pulsed down conversion give about 10^{-2} s^{-1} fourfold coincidences, but with wider filters.

Beam entanglements are not essential to prepare GHZ states. They can also be obtained by any type of entanglement, most easily with polarization entangled photon pairs [11]. The high stability of polarization experiments is a clear advantage over beam entanglements which are prone to phase drifts. The requirements on filter properties and pulse widths are the same as for beam entanglement.

Consider Fig. 3. Two type-II down-conversion crystals, A and B, each emit a pair of photons in the state $(1/\sqrt{2})(|H\rangle|V\rangle + |V\rangle|H\rangle)$, where V and H refer, respectively, to linear polarization perpendicular and parallel to the figure and the first [second] ket in each term specifies the left [right] going beam. In this scheme we first transform the polarization degree of freedom into the momentum degree of freedom by means of polarizing beam splitters [12], as shown in the figure. Overlapping the horizontally polarized components at a standard beam splitter can wipe out the possibility to infer (via polarizations) the origin of the trigger photon at D_T. Thus, if it came from crystal A, its V polarized companion must be in beam 1 and the pair from B must be in beams 2 and 3 with V and H polarizations, respectively. Then, the polarization state of the three photons in beams 1, 2, and 3 is $|V\rangle|V\rangle|H\rangle$. If, on the other hand, the trigger photon at D_T came from crystal B, its V-polarized companion is in beam 3 and the pair from crystal A must be in beams 1 and 2 both with polarization H. Then, the polarization state of the three photons in beams 1, 2, and 3 is $|H\rangle|H\rangle|V\rangle$. Again, the pulse-filter technique makes the source of the trigger unknowable and hence the state of the triple is $(1/\sqrt{2})(|V\rangle|V\rangle|H\rangle + e^{i\phi}|H\rangle|H\rangle|V\rangle)$.

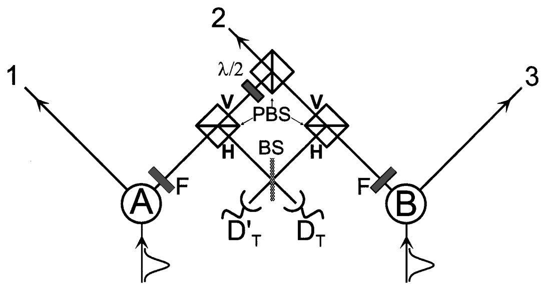

FIG. 3. A three-particle polarization-entanglement source. The two-particle sources, A and B, pumped by short pulses each emit a pair of photons in the superposition $|H\rangle|V\rangle + |V\rangle|H\rangle$, and subsequently four photons are detected, one in D_T and one in each of beams 1, 2, and 3. Because of the beam splitters, both polarizing (PBS) and regular (BS), the D_T photon could be from A or B and then the particles in the beams 1, 2, and 3 are in state $|V\rangle|V\rangle|H\rangle$ or $|H\rangle|H\rangle|V\rangle$, respectively. As in Fig. 1, narrow filters (F) make the source of photon D_T essentially unknowable, and consequently the other three photons are in a superposition $|V\rangle|V\rangle|H\rangle + |H\rangle|H\rangle|V\rangle$.

Other polarization entanglements can be prepared by either starting with different Bell states from the crystals, or by direct manipulations in beams 1, 2, and 3, or by using the detector D_T' instead of D_T as the trigger. For example, insertion of a $\lambda/2$ plate in beam 3 transforms the state into $(1/\sqrt{2})(|V\rangle|V\rangle|V\rangle + e^{i\phi}|H\rangle|H\rangle|H\rangle)$, a three photon realization of an earlier [13] gedanken three-spin state. The use of detector D_T' instead of D_T as trigger shifts the relative phase, ϕ, by π. With these techniques eight different mutually orthogonal two-term polarization entanglements (analogs of the two-particle Bell states) can easily be produced. And via polarizing beam splitters any

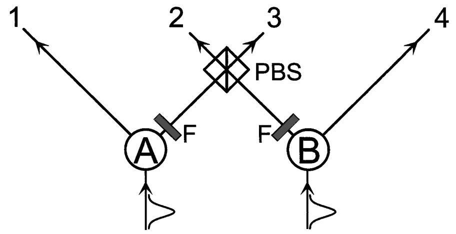

FIG. 4. A four-particle polarization-entanglement source. The two-photon sources, A and B, pumped by short pulses, each emit a pair of photons in the superposition $|H\rangle|H\rangle + |V\rangle|V\rangle$ and four photons are eventually detected, one in each of the beams 1, 2, 3, and 4. Because of the polarizing beam splitter, PBS, and the narrow filters, F, the sources of photons 2 and 3 are essentially unknowable. Consequently the four photons are in a superposition $|H\rangle|H\rangle|H\rangle|H\rangle + |V\rangle|V\rangle|V\rangle|V\rangle$.

of these can be converted [12] into beam entanglements for illuminating the interferometer in Fig. 2.

Finally, we note that an extension of these schemes would enable us to observe four-particle correlations specific to the four-particle GHZ state

$$\frac{1}{\sqrt{2}}(|H\rangle|H\rangle|H\rangle|H\rangle + |V\rangle|V\rangle|V\rangle|V\rangle). \qquad (5)$$

This can be achieved, for example, if both crystals emit the Bell state $(1/\sqrt{2})(|H\rangle|H\rangle + |V\rangle|V\rangle)$ and one observes coincident detections behind orientable polarizers placed in all four outgoing beams of Fig. 4. Note that here the state is not prepared but that the correlations are observed by destructive selection. Obviously this scheme can also provide a three-particle GHZ state if suitable measurements in either beam 2 or beam 3 are performed.

The laboratory realization of three-particle entanglement will open the door to many novel quantum phenomena and applications. These may include: (A) demonstration of GHZ correlations [1], (B) generalization of two-particle phenomena (e.g., illumination of a tritter [14] with three entangled particles), (C) demonstration of *entangled entanglement* [15], and (D) multiparticle quantum communication schemes (see, e.g., [16,17]).

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[10] The threshold fringe visibility for a three-particle interference pattern (for perfect collection efficiency) to exhibit violations of local realism is 50% [see, e.g., D. N. Klyshko, Phys. Lett. A **172**, 399 (1993)]. For any attempt to interpret experimental observations in a local realistic way, it will be significant to notice that while the filters select photons, one could in principle detect all of them by using a suitably sophisticated dispersive optical element. The other complications are less important. For example, the trigger detector may fire if (a) only one down conversion occurred, or (b) two down conversions occurred in one crystal. Case (a) can be rejected: two of the detector stations will show no counts. Case (b) can also be rejected: station D_A or D_B will exhibit no counts.
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