Universality of Transport through Dirty Interfaces

Kees M. Schep* and Gerrit E. W. Bauer

Faculty of Applied Physics and Delft Institute of Microelectronics and Submicrontechnology, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands

(Received 20 January 1997)

The distribution function of transmission matrix eigenvalues for an interface with a high density of randomly distributed scatterers is calculated from the Schrödinger equation. The distribution function is universal in the sense that it does not depend on the microscopic parameters. It differs, however, from the well-known universal distribution for diffusive bulk conductors. [S0031-9007(97)02960-8]

PACS numbers: 72.10.Fk, 73.40.Jn, 74.80.Fp

An important new concept in the theory of electronic transport coming from the study of phase-coherent conduction in disordered metals [1] is *universality*, i.e. the independence of transport properties on the microscopic parameters. The best known example is provided by the universal conductance fluctuations [2]: the sample to sample fluctuations of the conductance are of the order of e^2/h, independent of the sample size, the degree of disorder, and the dimensionality.

Here we focus on another property that may exhibit universality, namely, the distribution function of transmission matrix eigenvalues. The transmission matrix $\mathbf{t}\mathbf{t}^\dagger$ is the product of the transmission amplitude matrix $\mathbf{t}$ and its Hermitian conjugate. The matrix $\mathbf{t}$ collects the transmission amplitudes of the flux-normalized states on the left hand side of a scattering region to those on the right hand side. The distribution function of the eigenvalues T_n of the matrix $\mathbf{t}\mathbf{t}^\dagger$ is defined as

$$P(T) \equiv \left\langle \sum_n \delta(T - T_n) \right\rangle , \tag{1}$$

where the brackets indicate averaging over all possible realizations of disorder in a given Hamiltonian. This distribution function can be used (see, for example, Ref. [3]) to express the average value of any quantity a that is described by a linear statistic $a(T)$ as

$$\langle a \rangle = \left\langle \sum_n a(T_n) \right\rangle = \int dT\, a(T) P(T) . \tag{2}$$

The conductance g (in units e^2/h) is related to the transmission matrix by the Landauer formula

$$g = \mathrm{Tr}\, \mathbf{t}\mathbf{t}^\dagger = \sum_n T_n , \tag{3}$$

and is thus described by the linear statistic $g(T) = T$. The shot noise power p (in units $2e|V|e^2/h$ with V the applied voltage) is described by the linear statistic $p(T) = T(1 - T)$ [4]. For a normal metal/superconductor (NS) junction the conductance g_{NS} and the shot noise p_{NS} can also be expressed in terms of the transmission eigenvalues of the normal-metal region by the linear statistics $g_{\mathrm{NS}}(T) = 2T^2/(2 - T)^2$ [5] and $p_{\mathrm{NS}}(T) = 16T^2(1 - T)/(2 - T)^4$ [6], respectively.

The distribution function for disordered bulk conductors in the metallic regime (where $1 \ll g \ll N$ with N the number of conduction channels) has been shown to be universal [7–9]:

$$P(T) = \frac{\langle g \rangle}{2} \frac{1}{T\sqrt{1 - T}} , \tag{4}$$

independent of the shape of the conductor and of the spatial resistivity distribution. The distribution function is bimodal: most eigenvalues are either close to 1 ("open" channels) or close to 0 ("closed" channels). This is in contrast to the naive assumption that all eigenvalues are much smaller than 1 for $g \ll N$. It follows directly from Eq. (4) that $\langle p \rangle / \langle g \rangle = 1/3$ [10], which is only one-third of the classical value for a Poisson process for which all $T_n \ll 1$.

Universality, however, has its limits. Either close to the localization regime ($g \approx 1$) [8] or close to the ballistic regime ($g \lesssim N$) [11] Eq. (4) is no longer valid. Even in the metallic regime where $1 \ll g \ll N$ the universality can be broken by extended defects, such as tunnel barriers, grain boundaries, or interfaces [9].

In this Letter we consider transport through dirty interfaces for which Eq. (4) does not hold. An interface is a scattering region with a length L which is sufficiently smaller than the Fermi wavelength λ_F. The term "dirty" implies that $g \ll N$ and that the scattering is due to a random potential. In contrast, the calculations which lead to Eq. (4) are all in the weak scattering regime where $L \gg \lambda_F$.

The main result of the microscopic calculation presented below is a universal distribution function for a single dirty interface which differs from Eq. (4) for bulk systems. In other words, dirty interfaces ($L \ll \lambda_F$) belong to a different universality class than disordered bulk conductors ($L \gg \lambda_F$).

Besides the purely theoretical interest in the statistics of transport, our study of dirty interfaces is also motivated by experiments on transport through metallic interfaces in magnetic multilayers exhibiting giant magnetoresistance [12,13]. These interfaces strongly scatter electrons in a region with a length comparable to or smaller than λ_F [14].

Experiments of transport through a narrow disordered region in a two-dimensional electron gas are in progress [15]. With some modifications the present calculations are also applicable to other scattering problems, such as the transparency of a thin, yet strongly diffusing medium to light.

The scattering properties of an interface for states at the Fermi energy E_F follow directly from the Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x,\vec{\rho})\right]\psi(x,\vec{\rho}) = E_F\psi(x,\vec{\rho}), \quad (5)$$

where m is the electron mass. For arbitrary dimension d, $\vec{\rho}$ is the $d-1$ dimensional position vector orthogonal to the x direction. The scattering potential at the interface is modeled by short range scatterers with strength γ_α at position $\vec{\rho}_\alpha$ in the plane $x = 0$:

$$V(x,\vec{\rho}) = \sum_\alpha \gamma_\alpha \delta(x)\delta(\vec{\rho} - \vec{\rho}_\alpha). \quad (6)$$

The free electron states on both sides of the interface can be labeled by the $d-1$ dimensional parallel component of the wave vector $\vec{k}_\parallel$. The perpendicular part of the wave vector $k_\perp$ is then defined in terms of the Fermi wave vector $k_F = \sqrt{2mE_F}/\hbar$ as $k_\perp^2 = k_F^2 - k_\parallel^2$. From the Schrödinger equation (5) all transmission and reflection amplitudes corresponding to an incoming state $\vec{k}'_\parallel$ can be obtained [16]. The present calculations are in the regime where the transverse dimensions of the interface are much larger than λ_F. Scattering to evanescent states with imaginary $k_\perp$ is disregarded, since they affect only the conductance g and not the distribution function in the regime that is of interest here [17]. The transmission amplitudes can be expressed directly in terms of the scattering potential as $\mathbf{t} = [\mathbf{I} + i\mathbf{\Gamma}]^{-1}$. $\mathbf{I}$ is the unit matrix and the elements of the Hermitian matrix $\mathbf{\Gamma}$ are given by

$$\Gamma_{\vec{k}_\parallel,\vec{k}'_\parallel} = \frac{m}{\hbar^2}\sum_\alpha \frac{\gamma_\alpha}{A} e^{-i(\vec{k}_\parallel - \vec{k}'_\parallel)\vec{\rho}_\alpha} \frac{1}{\sqrt{k_\perp k'_\perp}}, \quad (7)$$

where A is the $d-1$ dimensional cross section of the interface.

From the expansion of $\mathbf{t}$ in powers of $\mathbf{\Gamma}$

$$\mathbf{t} = \sum_{M=0}^{\infty} (-i\mathbf{\Gamma})^M, \quad (8)$$

the relations $\mathbf{t}\mathbf{t}^\dagger = (\mathbf{t} + \mathbf{t}^\dagger)/2$ and $\mathbf{t}\mathbf{t} = (1 + m\partial/\partial m)\mathbf{t}$ can be derived. These so-called Ward identities relate the two-particle propagators to the single particle propagators and can be used to express higher powers of $\mathbf{t}\mathbf{t}^\dagger$ in terms of $\mathbf{t}$ and $\mathbf{t}^\dagger$ which simplifies the calculations enormously. The distribution function $P(T)$ can be rewritten in terms of a power series in the transmission matrix by expressing the δ function in Eq. (1) as a Fourier integral and subsequently expanding the exponent $\exp(iq\mathbf{t}\mathbf{t}^\dagger)$:

$$P(T) = \int \frac{dq}{2\pi} e^{-iqT} \sum_{n=0}^{\infty} \frac{(iq)^n}{n!} \mathrm{Tr}\langle(\mathbf{t}\mathbf{t}^\dagger)^n\rangle. \quad (9)$$

By using the Ward identities repeatedly and applying a slightly modified Stieltjes transform in the parameter $\eta = m^2$, this expression can be rewritten for $0 < T < 1$ as [11]

$$P(T) = \frac{1}{\pi}\frac{1}{T(1-T)}\mathrm{Im}\left[\left\langle g\left(\frac{\eta T}{T-1} - i0^+\right) - g(\infty)\right\rangle\right], \quad (10)$$

with $g(\eta) = \mathrm{Tr}\,\mathbf{t}\mathbf{t}^\dagger$ and 0^+ a positive infinitesimal. By calculating $\langle\mathbf{t}\rangle$ we obtain the conductance from $\langle g(\eta)\rangle = \mathrm{Re}[\mathrm{Tr}\langle\mathbf{t}\rangle]$. Average quantities are obtained by configurational averaging over the random impurity positions $\vec{\rho}_\alpha$ that are uniformly distributed.

The elements of the Green function matrix $\mathbf{G}^+$ are related to the transmission amplitudes by [18]

$$t_{\vec{k}_\parallel,\vec{k}'_\parallel} = i\frac{\hbar^2}{m}\sqrt{k_\perp k'_\perp}\, G^+_{\vec{k}_\parallel,\vec{k}'_\parallel}. \quad (11)$$

The elements of the unperturbed Green function matrix $\mathbf{G}^{+(0)}$ are given by

$$G^{+(0)}_{\vec{k}_\parallel,\vec{k}'_\parallel} = G^{+(0)}_{\vec{k}_\parallel}\delta_{\vec{k}_\parallel,\vec{k}'_\parallel} = -i\frac{m}{\hbar^2}\frac{1}{k_\perp}\delta_{\vec{k}_\parallel,\vec{k}'_\parallel}. \quad (12)$$

From Eqs. (8) and (11) an expansion for the Green function is obtained. The configurational averages of the different terms in this expansion can be calculated using diagrammatic perturbation theory [16]. The irreducible self-energy matrix $\mathbf{\Sigma}$ relates the configurationally averaged Green function to the unperturbed Green function by the Dyson equation $\langle\mathbf{G}^+\rangle = \mathbf{G}^{+(0)} + \mathbf{G}^{+(0)}\mathbf{\Sigma}\langle\mathbf{G}^+\rangle$. All internal Green functions in the complete perturbation expansion of the irreducible self-energy are fully renormalized.

Configurational averaging restores translational invariance parallel to the interface and $\mathbf{\Sigma}$ is diagonal in $\vec{k}_\parallel$. In the strong scattering regime ($g \ll N$) the self-energy is much larger than the inverse of the unperturbed Green function. In this limit the Dyson equation can be expanded as

$$\langle G^+_{\vec{k}_\parallel,\vec{k}'_\parallel}\rangle = -\frac{1}{\Sigma_{\vec{k}_\parallel}}\left[1 + \frac{1}{G^{+(0)}_{\vec{k}_\parallel}\Sigma_{\vec{k}_\parallel}} + \cdots\right]\delta_{\vec{k}_\parallel,\vec{k}'_\parallel}. \quad (13)$$

The leading term in the expansion of the Green function depends only on the self-energy and does not contain the unperturbed Green function or the effective mass explicitly. In the self-energy diagrams, all internal propagators must be renormalized and according to Eq. (13) lose all dependence on the electron mass in the strong scattering limit. It follows directly that the self-energy is independent of m. This general property for the self-energy in the strong scattering regime is what makes it possible to calculate $P(T)$ exactly in the present model.

The average conductance in the strong scattering regime can be calculated from Eqs. (3), (11), and (13) as

$$\langle g(\eta)\rangle = \frac{\hbar^2}{\sqrt{\eta}} \sum_{\vec{k}_\parallel} \frac{-\mathrm{Im}[\Sigma_{\vec{k}_\parallel}]k_\perp}{|\Sigma_{\vec{k}_\parallel}|^2}, \tag{14}$$

and is thus proportional to $1/\sqrt{\eta}$. Substitution of Eq. (14) into Eq. (10) and using $g(\infty) = 0$ yields

$$P(T) = \frac{\langle g\rangle}{\pi} \frac{1}{T^{3/2}\sqrt{1-T}}, \tag{15}$$

which is the main result of this Letter. The distribution function of the transmission matrix eigenvalues for dirty interfaces is universal in the sense that it does not depend on the microscopic parameters or on the dimension d, but only on the macroscopic conductance.

Also for dirty interfaces universality has its limits. If the self-energy has no imaginary part the conductance vanishes in lowest order. A real self-energy is, for example, characteristic for a tunnel barrier. To obtain a finite conductance in that case higher order terms in the expansion (13) should be taken into account, which gives rise to nonuniversal results. This loss of universality is analogous to the situation considered by Nazarov [9] who finds that universal behavior in bulk disordered conductors is destroyed by a nontransparent tunnel barrier.

To confirm the analytical results, we perform the configurational averaging numerically by calculating for many realizations of the impurity positions the eigenvalues of the matrix $\mathbf{I} + \mathbf{\Gamma\Gamma}$ which equal $1/T_n$. The numerical calculations are restricted to $d = 2$ and $\gamma_\alpha = \pm\gamma$. The distribution function is not a convenient function to calculate numerically because of the divergences at $T = 0$ and $T = 1$. Instead of $P(T)$ we calculate the well-behaved integrated quantity

$$Q(T) = \frac{1}{\langle g\rangle} \int_0^T dT' T' P(T'), \tag{16}$$

which is a smooth function of T. $Q(T)$ is the relative contribution to the conductance of all channels n with $T_n < T$. From Eqs. (4) and (15) it follows that $Q(T) = 1 - \sqrt{1-T}$ for a metallic bulk conductor and $Q(T) = \frac{1}{\pi}\arccos(1-2T)$ for a dirty interface. Figure 1 shows the numerically calculated $Q(T)$ for two different sets of microscopic parameters compared with the analytical results. Figure 1 shows excellent agreement between analytical and numerical configurational averaging. Note that the numerical results differ significantly from the analytical result for bulk systems. Numerically, universal behavior is confirmed for a broad range of microscopic parameters such as the number of scatterers, the scattering strength, and the number of evanescent modes [17]. Universality breaks down only when the average scattering strength $\sum_\alpha \gamma_\alpha$ becomes larger than some critical value, which can be ascribed to a vanishing imaginary part of the self-energy to lowest order in the expansion (13), as mentioned above.

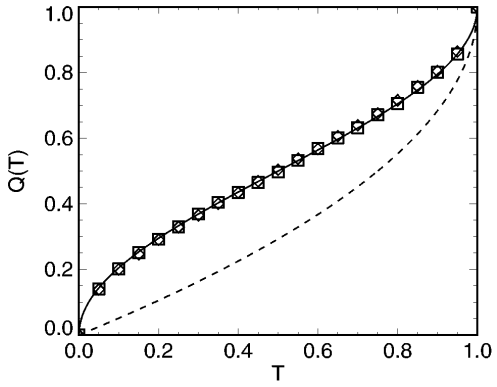

FIG. 1. The function $Q(T)$ given by Eq. (16), obtained from numerical calculations (symbols) and from the analytical results for a dirty interface (solid line) and for a disordered bulk conductor (dashed line). For the squares the microscopic parameters were 6000 impurities, $N = 40$, $\sum_\alpha \gamma_\alpha = 0$, and $m\gamma/\hbar^2\pi = 10$ which yielded $\langle g\rangle = 0.30$. For the diamonds we used 200 impurities, $N = 20$, $\sum_\alpha \gamma_\alpha = 0$, and $m\gamma/\hbar^2\pi = 30$ which yielded $\langle g\rangle = 0.20$. Configurational averaging was done using 5000 (squares) and 10 000 (diamonds) realizations of the disorder. Both calculations are clearly in the regime $g \ll N$.

Both distribution functions (4) and (15) are bimodal. For dirty interfaces there is, however, relatively more weight for small T_n. This is reflected in the values for the physical quantities. The ratio $\langle p\rangle/\langle g\rangle$, for example, equals $1/2$ for dirty interfaces compared to $1/3$ for disordered bulk conductors. In the case of a bulk disordered normal metal in contact with a superconductor $\langle g_{\mathrm{NS}}\rangle$ equals the conductance in the normal state and the ratio $\langle p_{\mathrm{NS}}\rangle/\langle g_{\mathrm{NS}}\rangle = 2/3$, which is twice the normal state result. In contrast, for a dirty interface in a normal metal in series with a superconductor $\langle g_{\mathrm{NS}}\rangle$ and $\langle g\rangle$ are no longer equal ($\langle g_{\mathrm{NS}}\rangle = \frac{1}{2}\sqrt{2}\,\langle g\rangle$) and the ratio $\langle p_{\mathrm{NS}}\rangle/\langle g_{\mathrm{NS}}\rangle = 3/4$, less than twice the normal state result. These differences should be observable experimentally.

For disordered bulk conductors, the bimodal distribution function (4) has been related to the occurrence of universal conductance fluctuations [19]. Despite the fact that $P(T)$ is bimodal for a dirty interface, numerical calculations [17] show that the fluctuations are not universal. Instead, the variance of g increases linearly with N as expected from classical arguments. Also close to the ballistic regime ($g \lesssim N$) the fluctuations display nonuniversal behavior [20].

In the above analytical treatment there is no distinction between the regimes $g > 1$ and $g < 1$, i.e., no localization transition is observed. This is confirmed by the numerical calculations [17]. The absence of localization follows from the well-behaved maximally crossed diagrams [11] and can be understood from the fact that for an interface the region of scattering has no spatial extent in the transport direction so there is no space available where the wave functions can localize.

The distribution function (15) for a dirty interface is identical to $P(T)$ for two identical tunnel barriers in series with the distance between the two tunnel barriers much larger than λ_F [21], despite the fact that the two physical systems are very different. At this moment we have no physical argument for this correspondence.

Because $P(T)$ for a dirty interface does not depend on the microscopic details, its calculation seems well suited for methods of random matrix theory [8]. Continuity of the wave functions on both sides of the scattering region gives rise to an additional constraint on the scattering matrix which we expect to be sufficient to describe the difference between interfaces and bulk systems.

In summary, based on an exact microscopic calculation we have shown that $P(T)$ for a dirty interface is universal but differs from $P(T)$ for disordered bulk conductors. Dirty interfaces belong to a different universality class than diffusive bulk conductors. It remains a challenge to test these results theoretically by random matrix theory and experimentally by transport studies of intentionally disordered metallic point contacts and wide quantum wires.

We thank Marc de Jong, Paul Kelly, and Yuli Nazarov for discussions. This work is part of the research program of the "Stichting voor Fundamenteel Onderzoek der Materie" (FOM), which is financially supported by the "Nederlandse Organisatie voor Wetenschappelijk Onderzoek" (NWO).

*Also at Philips Research Laboratories, Prof. Holstlaan 4, 5656 AA Eindhoven, The Netherlands.

[1] For review articles, see *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (North-Holland, Amsterdam, 1991).
[2] B. L. Altshuler, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 530 (1985) [JETP Lett. **41**, 648 (1985)]; P. A. Lee and A. D. Stone, Phys. Rev. Lett. **55**, 1622 (1985).
[3] C. W. J. Beenakker, Phys. Rev. B **47**, 15 763 (1993).
[4] M. Büttiker, Phys. Rev. Lett. **65**, 2901 (1990).
[5] C. W. J. Beenakker, Phys. Rev. B **46**, 12 841 (1992).
[6] M. J. M. de Jong and C. W. J. Beenakker, Phys. Rev. B **49**, 16 070 (1994).
[7] O. N. Dorokhov, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 259 (1982) [JETP Lett. **36**, 318 (1982)].
[8] A. D. Stone, P. A. Mello, K. A. Muttalib, and J.-L. Pichard, in *Mesoscopic Phenomena in Solids* (Ref. [1]); C. W. J. Beenakker, Rev. Mod. Phys. (to be published).
[9] Yu. V. Nazarov, Phys. Rev. Lett. **73**, 134 (1994).
[10] C. W. J. Beenakker and M. Büttiker, Phys. Rev. B **46**, 1889 (1992).
[11] G. E. W. Bauer, Phys. Rev. B **51**, 16 984 (1995).
[12] W. P. Pratt, Jr., S.-F. Lee, J. M. Slaughter, R. Loloee, P. A. Schroeder, and J. Bass, Phys. Rev. Lett. **66**, 3060 (1991); M. A. M. Gijs, S. K. J. Lenczowski, and J. B. Giesbers, Phys. Rev. Lett. **70**, 3343 (1993).
[13] For recent reviews, see P. M. Levy, Solid State Phys. **47**, 367 (1994); M. A. M. Gijs and G. E. W. Bauer, Adv. Phys. (to be published).
[14] It is, however, still a subject of controversy whether the scattering at these interfaces comes from diffuse scattering due to interface roughness or from specular scattering due to mismatch of the band structures.
[15] K.-J. Friedland, T. Fujikawa, and S. Tarucha (unpublished).
[16] A. Brataas and G. E. W. Bauer, Phys. Rev. B **49**, 14 684 (1994).
[17] K. M. Schep and G. E. W. Bauer (unpublished).
[18] D. S. Fisher and P. A. Lee, Phys. Rev. B **23**, 6851 (1981).
[19] Y. Imry, Europhys. Lett. **1**, 249 (1986).
[20] Y. Asano and G. E. W. Bauer, Phys. Rev. B **54**, 11 602 (1996).
[21] J. A. Melsen and C. W. J. Beenakker, Physica (Amsterdam) **203B**, 219 (1994). This correspondence was pointed out to us by M. J. M. de Jong.