Microfilamentation in Optical-Field-Induced Ionization Process

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A plasma-resonance field-ionization instability of uniform gas breakdown produced by intense laser fields via tunneling ionization of atoms is studied theoretically and by computer simulation. The field amplitude and produced plasma are found to be unstable relative to spatial modulation in the direction of electric field with the spatial period shorter than the wavelength. In a dense gas the process, at the nonlinear stage of instability, becomes explosive and leads to the formation of thin resonance layers and sharp peaks of the field amplitude. [S0031-9007(97)02954-2]

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It is well known that a powerful electromagnetic wave beam in a medium with positive (focusing) nonlinearity is subject to filamentation instability with a characteristic transverse scale large compared to the wavelength. This instability was predicted about thirty years ago based on the paraxial approximation for the scalar wave field [1]. Its manifestations were repeatedly observed in experiments with powerful laser and microwave pulses (see, for example, [2,3] and references therein).

A less known fact is the existence of "vector" small-scale instability of the wave in a transparent medium with a "defocusing" (ionization-type) nonlinearity. This instability was originally described based on the vector wave equation [4], and studied theoretically and experimentally [5–7] as one of a wider class of ionization-field (or electrodynamic) instabilities of high-frequency and microwave discharges in gases. Further, we will use the term "plasma-resonance ionization (PRI) instability," revealing the underlying physical mechanism of its generation. The PRI instability results in the filamentation of the wave and produced plasma with density gradients parallel to the wave electric field and with the spatial period shorter than the wavelength. So, it may be considered as an ionization analog of the known modulation instability of the field in a collisionless plasma with positive (ponderomotive-force-induced) nonlinearity. However, unlike the latter, it evolves not only in a narrow plasma resonance region (near the critical-density surface) but covers the entire transmission medium and affects significantly its electrodynamic characteristics.

The PRI instability, probably, has not yet been observed in experiments with powerful ionizing laser pulses, since the majority of the experiments realized the electron-impact (avalanche-type) mechanism of gas ionization; the characteristic time of instability for this mechanism turns out to be longer than the time of the avalanche itself or the time of gas heating. However, advances in the generation of powerful laser pulses with field amplitudes comparable to atomic fields have stimulated interest in studies on the dynamics of the laser breakdown determined by optical-field-induced (tunneling) ionization of gas atoms [8–14]. The growth rate of the PRI instability with this ionization mechanism, as we find below, can be high enough so that, at the final stage of the breakdown process in a dense gas, what is produced is not a homogeneous plasma but a lamellar plasma-field microstructure at scales much smaller than a wavelength (unlike the usual filamentation processes) and oriented with the lamella normals parallel to the oscillating electric field. Its appearance must be accompanied by a number of macroscopic manifestations (variation in the effective refraction index of the medium, excitation of higher harmonics, generation of fast electrons), thus changing drastically the conditions and possibilities of using laser plasma in applications widely discussed now, such as the creation of x-ray lasers, acceleration of particles in plasma, and efficient extreme ultraviolet (XUV) harmonic production. In this Letter, we find the characteristics for the initial (linear) stage of the PRI instability in the conditions of tunneling ionization and present the results of computer simulation of its nonlinear stage in the framework of the initial (temporal) evolution problem for spatially periodic perturbations in the field of a plane wave.

Our analysis will be based on the Maxwell equations for vectors of the electric $\mathcal{E}$ and magnetic $\mathcal{H}$ fields, the equation for the current of free electrons J in a plasma with variable density n,

$$\partial J/\partial t = (e^2 n/m)\mathcal{E}\,, \tag{1}$$

and the known static expression (used as the model one) for the rate of tunneling ionization of the hydrogen atom w [15],

$$\partial n/\partial t = w(\mathcal{E}, n) = 6\Omega(N_g - n)\frac{E_a}{|\mathcal{E}|}\exp\left(-\frac{E_a}{|\mathcal{E}|}\right). \tag{2}$$

Here e and m are electron charge and mass, respectively, $\Omega = me^2/h^3 = 4.16 \times 10^{16}\ \mathrm{s}^{-1}$ is the atomic frequency unit, h is Planck's constant, $E_a = (2/3)E_{a0}$, $E_{a0} = m^2e^5/h^4 = 5.14 \times 10^9$ V/cm is the field strength at the first Bohr radius, and N_g is the density of neutral atoms of the gas before the ionization process. Equation (1) is valid at any ionization rate w. It is easily deduced from the kinetic equation under the realistic assumption that the free electrons are born with zero (or isotropically distributed) velocity. Equation (2) is applicable to describe ionization in a laser field of frequency ω and amplitude E, when the following conditions are fulfilled: $\omega \ll \Omega$, $I \ll q$, and $E \ll E_a$, where $q = e^2E^2/2m\omega^2$ is the quiver energy of electrons and I is the ionization potential of the atom. Variations in the average plasma density, $N = \langle n \rangle$, are determined by the expression $\partial N/\partial t = \langle w(\mathcal{E}) \rangle$ (here and further the angle brackets denote averaging over the field period, $2\pi/\omega$).

We will deal with the solutions of equations for the field and plasma density in the following form:

$$\begin{Bmatrix} \mathcal{E} \\ \mathcal{H} \end{Bmatrix} = \frac{1}{2} \begin{Bmatrix} \mathbf{E}(x,t) \\ \mathbf{H}(x,t) \end{Bmatrix} \exp[ikz - i\varphi(t)] + \text{c.c.}, \quad N = N(x,t), \tag{3}$$

which describes the evolution of a quasimonochromatic p-polarized wave with a fixed longitudinal wave number k and a slow time-varying frequency $\omega(t) = \partial\varphi/\partial t$ and amplitudes $\mathbf{E}, \mathbf{H}$.

The wave propagates in the $+z$ direction and, in the general case, has transverse and longitudinal components of the electric field, $\mathbf{E} = \mathbf{x}_0E_x + \mathbf{z}_0E_z$, and only one (transverse) component of the magnetic field, $\mathbf{H} = \mathbf{y}_0H_y$. Field amplitudes and the plasma density are supposed to be a periodical function of the transverse coordinate x: $\{E_{x,z}, H_y, N\}(x,t) = \{E_{x,z}, H_y, N\}(x + L, t)$. The period of these functions, L, and the longitudinal wave number, k, which determines the wavelength in the longitudinal direction, $\lambda = 2\pi/k$, are constants preset at the initial instant of time, $t = 0$. The range of interest for these parameters is $L/\lambda < 1$. The wave frequency, ω, as well as transverse structures of the field and plasma, may change significantly in a long (on the scale of $1/\omega$) time.

At a sufficiently low ionization rate the stated problem can be solved within the adiabatic approximation based on the stationary wave equation for the magnetic field H_y and relations for the electric field components of a p-polarized wave (3):

$$\varepsilon \frac{\partial}{\partial x}\left(\frac{1}{\varepsilon}\frac{\partial H_y}{\partial x}\right) + \left(\frac{\omega^2}{c^2}\varepsilon - k^2\right)H_y = 0\,. \tag{4}$$

$$\omega\varepsilon E_x = ckH_y\,, \qquad \omega\varepsilon E_z = ic(\partial H_y/\partial x)\,. \tag{5}$$

Here $\varepsilon = 1 - (N/N_c) = 1 - (\omega_L^2/\omega^2)$ is the permittivity of the plasma, $N_c = m\omega^2/4\pi e^2$ is the critical density, and $\omega_L = (4\pi e^2N/m)^{1/2}$ is the Langmuir frequency. Using the stationary equation (4) without the terms containing temporal derivatives of the amplitude and the density, we actually neglect group lagging in the transverse direction and excitation of natural plasma oscillations at frequency ω_L in time-varying plasma. This is valid for perturbations with the transverse scale $L < \lambda$, if a characteristic time of variation in the complex amplitude, $\tau_e = |\mathbf{E}||\partial\mathbf{E}/\partial t|^{-1}$, is great compared to the period corresponding to the beat frequency, $\tau_d = (\omega - \omega_L)^{-1}$. With the given x-periodicity condition, the solution of Eq. (4) determines, at each instant of time t [for known distribution of the density $N(x,t)$], the eigenfunction $H_y(x,t)$ and eigenvalue $\omega(t)$. The time-dependent normalization factor in the determination of eigenfunction H_y must be calculated by means of the evolution equation that describes the variation of the wave energy in a plasma with growing density:

$$\frac{\partial}{\partial t}\int_0^L |\mathbf{E}|^2\,dx = -\frac{4\pi e^2}{m\omega^4}\int_0^L \langle Q \rangle dx\,, \quad Q = \frac{\partial n}{\partial t}\left(\frac{\partial \mathcal{E}}{\partial t}\right)^2. \tag{6}$$

This equation is a generalization of the intensity transport equation obtained earlier for a homogeneous quasimonochromatic wave [10,12], with transverse modulations of the field and density taken into account. It may be derived directly from Poynting's theorem for the quasimonochromatic field by using the equation for the electron current (1) and the x-periodicity condition for the fields. The right-hand side of the equation determines energy losses caused by energy transfer to newly born electrons [8,10,12]. Direct energy losses by the electrons' detachment, as well as collision losses, can be neglected in the validity region of the static formula (2) ($I \ll q$) and at small collision frequency of electrons ($\nu \ll wN_c \ll \omega$).

The dynamics of a homogeneous breakdown (in the absence of space modulations) produced by the field of a linearly polarized transverse wave $\mathcal{E} = \mathbf{x}_0E_0(t)\cos[kz - \varphi(t)]$ in the case of $|E| \ll E_a$ is described by smooth functions $E_0(t)$, $\omega(t)$, and $\omega_L(t)$ [14]. The average rates of ionization, $\langle w \rangle$, and dissipation, $\langle Q \rangle$, in Eq. (6) and the long-time asymptotic solution are determined in this case by the following expressions: $\langle w \rangle = w(E_0)\sqrt{E_0/E_a}$, $\langle Q \rangle = \langle w(\partial\mathcal{E}/\partial t)^2 \rangle = \langle w \rangle \omega^2E_0^3/E_a$, $\omega_L^2(t) = \omega^2(t) - \omega^2(0) \sim t^{2/3}$, and $E_0(t) \sim (\ln\omega)^{-1}$. Let us study the stability of this process with respect to small perturbations that modulate the wave amplitude and plasma density in the x direction. Representing the expressions for field components and plasma density as $E_x = E_0(t) + E_1(x,t)$ and $N = N_0(t) + N_1(x,t)$, and linearizing Eqs. (2), (4), and (5) against the homogeneous (but nonstationary) background E_0 and N_0, we obtain the following equations for the small perturbations E_1 and $\varepsilon_1 = -N_1/N_c$:

$$\frac{\partial^2E_1}{\partial x^2} + \frac{E_0}{\varepsilon_0}\frac{\partial^2\varepsilon_1}{\partial x^2} + \left(\frac{\omega}{c}\right)^2\varepsilon_1E_0 = 0\,, \quad \frac{\partial\varepsilon_1}{\partial t} + \gamma_0\varepsilon_1 + \alpha E_1 = 0\,, \tag{7}$$

where $\varepsilon_0 = 1 - (N_0/N_c)$, $\alpha = N_c^{-1}(d\langle w \rangle/dE_0)$, and $\gamma_0 = \langle w \rangle (N_g - N_0)^{-1}$. For perturbations having the form of $E_1, \varepsilon_1 \sim \cos(2\pi x/L)\exp\int\gamma(t)dt$, we find, based on the system (7), the following expression for the growth rate of the instability γ:

$$\gamma = \gamma_0 \left[\frac{N_g - N_0}{N_c - N_0} \frac{E_a}{E_0} \left(1 - \frac{L^2}{\lambda^2} \right) - 1 \right]. \qquad (8)$$

It follows from Eq. (8) that perturbations with the spatial period L shorter than the length of the electromagnetic wave, $\lambda = 2\pi/k$, can be unstable ($\gamma > 0$). The instability is caused, in fact, by a plasma resonance phenomenon and can be understood in the framework of a simple quasistatic ("plane capacitor") model which is valid for the small scale $L \ll \lambda$. As the undercritical plasma density $N < N_c$ increases in a thin plane layer, the normal field component increases according to the quasistatic relation $E_x = \text{const}/\varepsilon$. It results in the growth of the ionization rate $\langle w \rangle$ and the further growth of the density. At the same time, defocusing of the wave by the dense plasma is negligible on this small scale (but it suppresses the instability with the scale $L > \lambda$). When the condition of $(N_g - N_0)E_a \gg (N_c - N_0)E_0$ is fulfilled, the maximum of γ reached in the limit of $L/\lambda \to 0$ is $\gamma_{\max} = \langle w \rangle E_a / N_c E_0 \varepsilon_0$. The lower limit of the instability scale, $L_{\min}$, cannot be found in the framework of the local relations (1) and (2), and, probably, must be determined either by the amplitude of electron oscillations in the optical field or by the Debye lengths. In a gas with high density ($N_g > N_c$) the value of γ grows infinitely as the density N_0 approaches the critical value N_c. However, it begins to exceed the growth rate of the homogeneous background ($\gamma > \langle w \rangle / N_0$) even at the values of the background density N_{0S} which are much smaller than the critical one: $N_{0S}/N_c \simeq E_0/E_a \ll 1$. If, at such values of N_0, the electron or neutral density fluctuations or an external random source of ionization produce a "seed" small-scale perturbation with a sufficiently great value N_{1S}, then the spatial modulation of the plasma density $N(x)$ may become significant even in the region of $\varepsilon \sim 1$, i.e., much earlier than the point of plasma resonance is achieved. For example, in the case of $N_{1S}/N_{0S} = E_0/E_a = 0.1$, the value of $N_1(0, t)$ calculated by means of the above linear theory becomes of the order of $N_0(t)$ at $N_0/N_c \simeq 0.3$.

The dynamics of the field and plasma at the nonlinear stage of instability (at large ε_1 and E_1) was studied by computer simulation. The system of Eqs. (2), (4)–(6) was integrated numerically in the interval $0 < x < L$ with the periodic boundary conditions and the following initial conditions: $N(x, 0) = N_{00} + N_{10}\cos(2\pi x/L)$, $H_y(0, 0) = H_0$, and $\partial H_y/\partial x(0, 0) = 0$; at small N_{00} and N_{10} the field at the initial instant $t = 0$ is an almost transverse wave with amplitudes $E_x \simeq H_y \simeq H_0$ and frequency $\omega_0 \simeq kc$. The results of numerical calculations for the values of dimensionless parameters $N_{c0}/N_g = 0.8$, $\lambda/L = 2\pi/kL = 3$, $H_0/E_a = 8.25 \times 10^{-2}$, $N_{00}/N_{c0} = 0.1$, and $N_{10}/N_{c0} = 0.01$ $[N_{c0} = N_c(\omega_0)]$ are shown in Figs. 1 and 2.

Figures 1(a) and 1(b) present spatial distributions of the electric field components $E_x(x)$ and $E_z(x)$, the plasma density $N(x)$, and permittivity $\varepsilon(x)$ at various time moments t. We see that the nonlinear stage is characterized by the formation of sharp maxima in the transverse field and density; the electric field in a thin layer with $\varepsilon \ll 1$ increases significantly, whereas outside this layer it significantly decreases as compared to the initial unperturbed value, $E_0(0)$. At a certain point, this process acquires the explosion character and goes on until the gas is almost completely ionized at the maximum point, where the difference $\omega - \omega_L$ reduces to the minimum (positive) value. Thereafter, the produced layer with increased density extends slowly, and the field maximum becomes lower.

Figure 2 shows, as time functions, the frequency $\omega(t)$, field energy $K(t)$, and average energy flux $P(t)$ (related to their initial values), and the real part of the effective permittivity, $\varepsilon_{\text{eff}}(t) = (ck/\omega)^2$, which determines the characteristics of average (over the x coordinate) fields. These dependencies are rather steep at the stage of fast growth of the field and density, and slow down sharply after ionization is saturated at the maximum point.

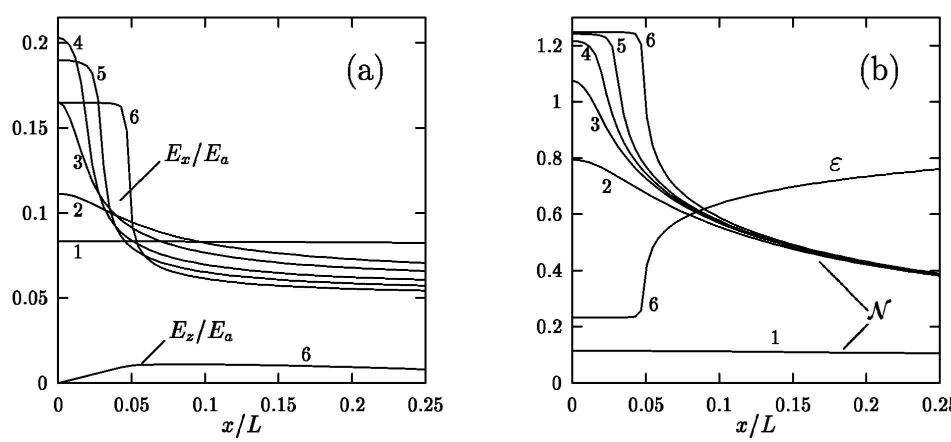

FIG. 1. Spatiotemporal evolution of the field and plasma at the nonlinear stage of PRI instability: (a) Transverse, $E_x(x)$, and longitudinal, $E_z(x)$, field components; (b) dimensionless plasma density, $\mathcal{N}(x) = N/N_{c0}$, and permittivity, $\varepsilon(x)$, at various instants of time; the curves 1–6 correspond to the values of $\Omega t \times 10^{-3} = 0$, 7.39, 7.96, 8.45, 10.2, and 25, respectively. The period of initial transverse modulation is $L = \lambda/3$.

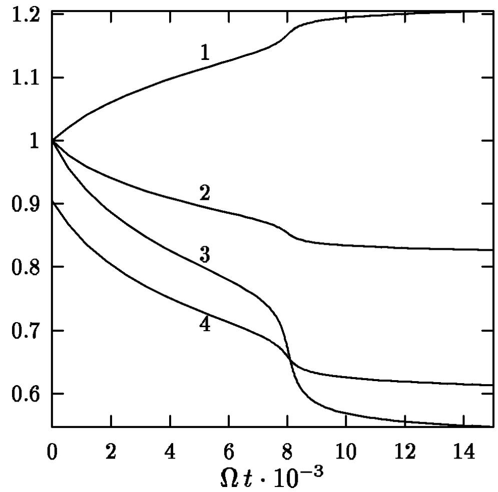

FIG. 2. Wave frequency ω/ω_0, energy K/K_0, energy flux P/P_0, and effective permittivity $\varepsilon_{\rm eff}$ (curves 1–4, respectively) as time functions.

The studied numerical example may correspond to the ionization of H_2 gas by the laser pulses with the intensity $S \simeq 10^{14}$ W/cm^2, duration $\tau_p > 200$ fs, and wave band $\lambda \sim 3\text{–}10$ μm, conforming to the approximations used above. Gas pressures corresponding to these band boundaries (at the given relation $N_{c0}/N_g = 0.8$) are, respectively, 6 and 0.6 atm. An analysis of the calculation results shows that the condition of validity of adiabatic approximation (4)–(6) is fulfilled approximately in this case ($\tau_d/\tau_e \simeq 0.1 - 0.3$). Note, however, that the maximum rate of the explosion process that is achieved near $N = N_g$ grows as the parameter N_g/N_{c0} increases; thus at a sufficiently high gas density (actually, starting from $N_g/N_{c0} \simeq 2$) the adiabaticity condition, $\tau_e \gg \tau_d$, must be inevitably violated near the sharpening point. By that, probably, the shock excitation of a reflected wave and the generation of intense plasma oscillations will take place. In this case, the theory developed makes it possible only to conclude that the explosion process is inevitable and to describe the first (adiabatic) stage of the process culminating in the formation of contrast plasma-field structures.

The important feature of the resulting structures is a long lifetime. After filamentation, the ionization process is actually stopped both in the dense layers (where the gas is fully ionized) and in the spacing between them (where the electric field and ionization probability decrease drastically). This makes possible the realization and detection of the filamentation in a sufficiently wide ranges of the pulse intensity ($\sim 10^{14}\text{–}10^{16}$ W/cm^2) and duration ($\sim$100 fs–10 ns). In the above example, the structure arises in 200 fs and lives (until full ionization of the spacing) about 10 ns. In the case of double pressure (1.2 atm for $\lambda = 10$ μm), the electric field decreases in the spacing by an order of magnitude. If this process occurs, for example, at a smooth leading edge of the Gaussian shaped ps laser pulse, the contrast structure (forming at a field $E_0/E_a \sim 0.05\text{–}0.1$) is conserved throughout the pulse, even though the field E_0 increases by several times (with the corresponding increase in intensity up to $10^{15}\text{–}10^{16}$ W/cm^2) in the pulse maximum. As for the preferable way for detection of structures, it is probably a scattering diagnostic with a wavelength far shorter than that of the driving laser.

In conclusion, we have shown that in the process of optical-field-induced ionization the field and plasma cannot stay homogeneous even on scales small compared to the wavelength. Because of the effect of PRI instability, on such scales the development of the periodic plasma-field microstructure occurs with gradients parallel to the wave electric field. At a sufficiently high gas density, the sharpening regime is realized at the nonlinear stage of the instability, the process acquires the explosion character and leads to the formation of thin resonance layers, in which the electric field of the wave concentrates. In this regime the processes of scattering and the generation of higher radiation harmonics must be strongly intensified, and this is highly important for problems of production and diagnostics of laser plasmas with high density. In future work, attention should be paid to the effect of realistic pulse shapes rather than the constant amplitude (in the z direction) oscillating laser field considered here.

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