Inertial- and Dissipation-Range Asymptotics in Fluid Turbulence

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We propose and verify a wave-vector-space version of generalized extended self-similarity [R. Benzi *et al.*, Europhys. Lett. **32,** 709 (1995)] and broaden its applicability to uncover intriguing, universal scaling in the far dissipation range by computing high-order (≤ 20) structure functions numerically for (1) the three-dimensional, incompressible Navier-Stokes equation (with and without hyperviscosity) and (2) the Gledzer-Ohkitani-Yamada shell model for turbulence. Also, in case (2), with Taylor-microscale Reynolds numbers $4 \times 10^4 \leq \mathrm{Re}_\lambda \leq 3 \times 10^6$, we find that the inertial-range exponents (ζ_p) of the order-p structure functions do not approach their Kolmogorov value $p/3$ as Re_λ increases. [S0031-9007(97)02862-7]

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Kolmogorov's pioneering work (K41) [1] on homogeneous isotropic turbulence used the cascade picture to predict simple scaling forms for velocity structure functions (see below). These forms hold for distances r in the *inertial range* that lies between L, the forcing scale, and η_d, the dissipation scale at which viscosity starts modifying the invariant energy cascade. Subsequent studies [2–11] have refined K41, as we outline below, but have concentrated principally on the inertial range. In this Letter we use recently developed generalizations of such scaling [2,5] to elucidate the crossover from inertial- to dissipation-range behaviors in fluid turbulence.

The order-p velocity structure functions $S_p(r) \equiv \langle |\mathbf{v}_i(\mathbf{x} + \mathbf{r}) - \mathbf{v}_i(\mathbf{x})|^p \rangle$, where i (= 1, 2, or 3) denotes components, scale as $S_p(r) \sim r^{\zeta_p}$ at high Reynolds numbers Re_λ and for the inertial range $20\eta_d \lesssim r \ll L$ (where λ is the Taylor microscale). The K41 result $\zeta_p = p/3$ works well for $p \lesssim 4$; but for large p, most studies [2–11] find multiscaling, i.e., $\zeta_p = p/3 - \delta\zeta_p$, a nonlinear increasing function with $\delta\zeta_p > 0$. Also, a procedure called extended self-similarity (ESS) [5], in which ζ_p is obtained from $S_p \sim S_3^{\zeta_p}$, *extends* the apparent inertial range down to $r \simeq 5\eta_d$. A more recent technique, generalized extended self-similarity (GESS) [2], uses the dimensionless structure functions $\mathcal{G}_p(r) \equiv \mathcal{S}_p(r)/[\mathcal{S}_3(r)]^{p/3}$ and suggests that the form $\mathcal{G}_p \sim [\mathcal{G}_q]^{\rho_{pq}}$, with $\rho_{p,q} = [\zeta_p - p\zeta_3/3]/[\zeta_q - q\zeta_3/3]$, holds down to the lowest resolvable values of r. GESS has been tested [2] to some extent ($p, q \leq 6$). We show that ESS and GESS provide us with sensitive ways of studying the crossover of structure functions from their inertial- to dissipation-range forms.

Specifically, we show how GESS is modified at sufficiently small r by computing wave-vector-space (k-space) analogs of high-order (≤ 20) structure functions for (1) the three-dimensional, incompressible Navier Stokes equation (3D NS), with and without hyperviscosity, and (2) the Gledzer-Ohkitani-Yamada (GOY) shell model for turbulence [9–12] (where we attain both large Re_λ and $k \gg k_d \equiv \eta_d^{-1}$). We further propose a k-space GESS [2], show that it holds for $L^{-1} \ll k \lesssim 1.5k_d$, but then *crosses over to another form in the far dissipation range*. To study this we postulate k-space ESS [for real-space structure functions we use the symbols $\mathcal{S}$ and $\mathcal{G}$ and for their k-space analogs (*not* Fourier transforms) the symbols S and G]:

$$S_p \equiv \langle |\mathbf{v}(\mathbf{k})|^p \rangle \approx A_{Ip}(S_3)^{\zeta'_p}, \quad L^{-1} \ll k \lesssim 1.5k_d\,,$$
$$S_p \equiv \langle |\mathbf{v}(\mathbf{k})|^p \rangle \approx A_{Dp}(S_3)^{\alpha_p}, \quad 1.5k_d \lesssim k \ll \Lambda\,, \quad (1)$$

where A_{Ip} and A_{Dp} are, respectively, nonuniversal amplitudes for inertial and dissipation ranges and Λ^{-1} the (molecular) length at which hydrodynamics fails (see [5,6] for real-space analogs). Our study shows (Figs. 1 and 2)

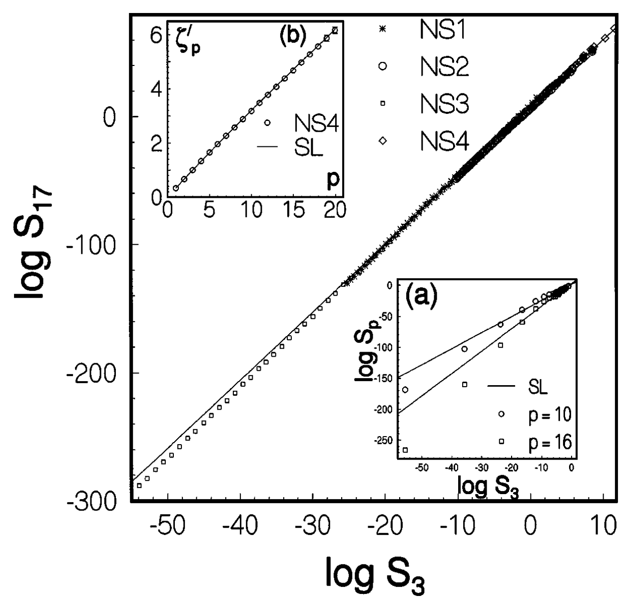

FIG. 1. Log-log plots (base 10) of S_p versus S_3 for 3D NS ($p = 17$ for runs NS1-4) and GOY [run G1 in inset (a)] models showing our k-space ESS [Eq. (1)]; full lines are the SL prediction [4]. Inset (b): ζ'_p (circles) from run NS4; the line is $\zeta'_p = 2(\zeta_p + 3p/2)/11$, with the $\zeta_p = \zeta_p^{\mathrm{SL}}$. Note the deviation of our data points from SL lines at small S_3, i.e., in the dissipation range; this shows clearly only for NS3 on this scale, but is also present in runs NS1 and NS2 (Fig. 3).

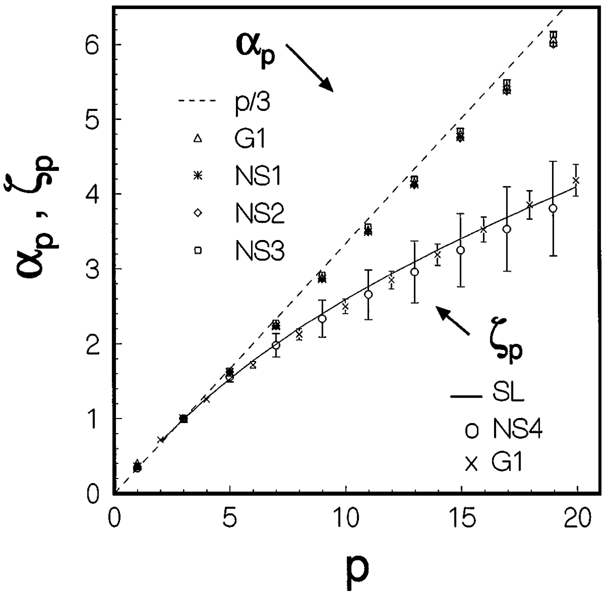

FIG. 2. Inertial- and dissipation-range exponents ζ_p and α_p (extracted from plots like Fig. 1) versus p for GOY and NS runs and their comparison with the SL formula [4] and $p/3$. We obtain ζ_p from our measured ζ'_p and the formula $\zeta_p = 11\zeta'_p/2 - 3p/2$; this amplifies the error bars relative to Fig. 1(b). Error bars for α_p are shown but not apparent since they are comparable to the symbol sizes.

that Eq. (1) holds with two different exponents α_p and ζ'_p. In the GOY model $\zeta'_p = \zeta_p$, but we find explicitly [Fig. 1(b)] that, for the 3D NS case, $\zeta'_p = 2(\zeta_p + 3p/2)/11$ [i.e., $S_p(k) \sim k^{-(\zeta_p+3p/2)}$ in the inertial range [13]]; the difference between the two arises because of phase-space factors. *Both* ζ_p and α_p (Fig. 2) seem universal [the same for all GOY and 3D NS runs (Table I) [14]]. ζ_p agrees fairly with the She-Leveque (SL) [4] formula $\zeta_p^{\rm SL} = p/9 + 2[1 - (2/3)^{p/3}]$ for the ranges of p and Re_λ in Fig. 2; and α_p is close to, but *systematically less* than, $p/3$. The k dependences of the inertial- and dissipation-range asymptotic behaviors follow now from the dependence of S_3 on k: We find

$$S_3 \approx B_I k^{-\zeta_3 - 9/2}, \quad L^{-1} \ll k \lesssim 1.5k_d\,, \tag{2}$$

$$S_3 \approx B_D k^{\delta} \exp(-ck/k_d), \quad 1.5k_d \lesssim k \ll \Lambda\,, \tag{3}$$

where B_I and B_D are, respectively, nonuniversal amplitudes [Eq. (2) holds [13] for 3D NS; for GOY the factor $9/2$ is absent]. Thus, in the far dissipation range, *all* $S_p \sim k^{\theta_p} \exp(-c\alpha_p k/k_d)$ for $1.5k_d \lesssim k \ll \Lambda$, with $\theta_p = \alpha_p \delta$, a form not easy to verify numerically for large p, given the rapid decay at large k, and suggested hitherto [15] only for S_2. In Eq. (3), δ, c, k_d are not universal, but we extract the universal part of the crossover via our k-space GESS: Define $G_p \equiv S_p/(S_3)^{p/3}$; log-log plots of G_p versus G_q yield curves [Figs. 3(a) and 3(b)] with asymptotes which have *universal*, *but different*, slopes in inertial and dissipation ranges. The inertial-range asymptote has a slope $\rho(p,q)$ (as in real-space GESS [2] which follows from the formulas above); the resulting ζ_p are in fair agreement with the SL formula [4]. The dissipation-range asymptote has a slope $\omega(p,q) \equiv [\alpha_p - p/3]/[\alpha_q - q/3]$. The slopes of these asymptotes are universal, but the point at which the curve veers off from the inertial-range asymptote depends on the model (GOY, NS, etc.). However, a simple transformation yields a *universal crossover scaling function* [different for each (p,q) pair because of multiscaling]: Define $\log_{10}(H_{pq}) \equiv D_{pq} \log_{10}(G_p)$ and $\log_{10}(H_{qp}) \equiv D_{qp} \log_{10}(G_q)$; the scale factors $D_{pq} = D_{qp}$ are *nonuniversal*, but plots of $\log_{10}(H_{pq})$ versus $\log_{10}(H_{qp})$ show data from *all* GOY and 3D NS runs collapsing onto *one universal curve* within our error bars [Fig. 3(c) for $p = 6$ and $q = 9$] for *all* k and Re_λ. (This transformation holds the G1-8 GESS plots fixed and stretches the NS plots, without changing their slopes, until the asymptotes match.) Both ESS (Fig. 1) and GESS (Fig. 3) remove the exponential *controlling factor* [16] from the *leading asymptotic behavior* of S_p in the far dissipation range and expose the remaining power-law dependence on k. Also, it is easy to see analytically that GESS plots (Fig. 3) amplify slope differences between inertial- and dissipation-range asymptotes relative to ESS plots (Fig. 1).

How robust is the fair agreement of ζ_p (Fig. 2) with the SL formula? Some studies [17–19] suggest that, as $\mathrm{Re}_\lambda \to \infty$, $\delta\zeta_p \equiv (p/3 - \zeta_p) \to 0$. Numerical solutions of the 3D NS equation can at best achieve [7–20] $\mathrm{Re}_\lambda \lesssim 220$, too small, by far, to resolve this issue, so we address it for the GOY model by studying the range $4 \times 10^4 \lesssim \mathrm{Re}_\lambda \lesssim 3 \times 10^6$. We find (Fig. 4) that $\delta\zeta_p$ does not vanish with increasing Re_λ, but rises marginally [21]. Systematic experiments at high Re_λ can check if the trends of Fig. 4 obtain in the NS case.

We remark that if we *assume* the hierarchy $[G_{p+1}/G_p] = [G_p/G_{p-1}]^\gamma [\lim_{p\to\infty} G_{p+1}/G_p]^{1-\gamma}$ with $\gamma^3 = 2/3$ (whose real-space analog is equivalent [2] to the SL moment hierarchy for the energy dissipation [4]) and use [22] $G_p(k) \approx C_p k^{\beta_p}$, we get a difference equation for β_p *identical* to the SL one (our β_p is their $-\tau_{p/3}$). This, when solved with the boundary conditions $\beta_0 = \beta_3 = 0$ and $\lim_{p\to\infty}(\beta_{p+1} - \beta_p) = 2/9$, yields the SL formula (via $\zeta_p = -\beta_p + p\zeta_3/3$). However, our GESS yields $[G_{p+1}/G_p] \approx C'_p [G_p/G_{p-1}]^{\Upsilon_p}$ with $\Upsilon_p = (\zeta_{p+1} - \zeta_p - 1/3)/(\zeta_p - \zeta_{p-1} - 1/3)$. Superficially, this might seem to violate the hierarchy assumed above, but it turns out to be consistent with our GESS form, if $\Upsilon_p = \gamma - 2(1 - \gamma)/[9(\zeta_p - \zeta_{p-1} - \zeta_3/3)]$, which is precisely the SL difference equation. Of course, our GESS form can hold with $\zeta_p \neq \zeta_p^{\rm SL}$; Fig. 2 shows the quality of agreement between our measured ζ_p and $\zeta_p^{\rm SL}$.

We use a pseudospectral method [7] to solve the incompressible 3D NS equation. We force the first two k shells, use a box with side $L_B = \pi$ and 64^3 modes. Our dissipation term $-(\nu + \nu_H k^2)k^2$ allows for both viscosity ν and hyperviscosity ν_H. For time integration

TABLE I. Parameters ν (viscosity), ν_H (hyperviscosity), Re_λ (Taylor-microscale Reynolds number), τ_e (box-size eddy-turnover time), $\tau_{\rm av}$ (averaging time), τ_t (transient time), and k_d (dissipation-scale wave number) for our 3D NS runs NS1-4 ($k_{\max} = 64$) and GOY-model runs G1-8 ($k_{\max} = 2^{22}k_0$). The step size (δt) used is 0.02 for NS1-4, 10^{-4} for G1-4, and 2×10^{-5} for G5-8. Note $\tau_e \simeq 8\tau_I$, the integral time for our NS runs.

Run	ν	ν_H	Re_λ	$\tau_e/\delta t$	τ_t/τ_e	$\tau_{\rm av}/\tau_e$	$k_{\max}/k_d$
NS1	5×10^{-4}	0	$\simeq 3.5$	$\simeq 3 \times 10^4$	$\simeq 1$	2	$\simeq 4$
NS2	2×10^{-4}	0	$\simeq 8$	$\simeq 3 \times 10^4$	$\simeq 1$	$\simeq 2.5$	$\simeq 2.3$
NS3	5×10^{-4}	5×10^{-6}	$\simeq 3.5$	$\simeq 3 \times 10^4$	$\simeq 1$	$\simeq 1$	$\simeq 6.5$
NS4	5×10^{-4}	10^{-6}	$\simeq 22$	$\simeq 3 \times 10^3$	$\simeq 10$	$\simeq 7$	$\simeq 2$
G1-4	5×10^{-6}–10^{-7}	0	4×10^4–3×10^5	$\simeq (1.5\text{–}2.0) \times 10^4$	$\simeq 500$	$\simeq 2500$	$\simeq 2^5$–2^3
G5-8	5×10^{-8}–10^{-9}	0	3.5×10^5–3×10^6	$\simeq (0.7\text{–}1) \times 10^5$	$\simeq 500$	$\simeq 2500$	$\simeq 2^3$–1

we use an Adams-Bashforth scheme (step size δt) [7]. Parameters for our 3D NS runs NS1-4 are given in Table I, where $\tau_e \equiv L_B/v_{\rm rms}$ is the box-size eddy-turnover time and $\tau_{\rm av}$ the averaging time, after initial transients have decayed over a period τ_t. We use $\mathrm{Re}_\lambda = v_{\rm rms}\lambda/\nu$, where $\lambda = [\int_0^\infty E(k)\,dk/\int_0^\infty k^2E(k)\,dk]^{1/2}$, $v_{\rm rms} = [(2/3L_B^3)\int_0^\infty E(k)dk]^{1/2}$, and $E(k) \sim S_2(k)k^2$. All $S_p(k)$ are averaged over shells of radius k. Care must be exercised in choosing δt and the forcing amplitude, otherwise there is a slow but systematic stretching of the points along the asymptotes in Figs. 1 and 3 with increasing $\tau_{\rm av}$ (over the time scales of our low-Re_λ NS runs). Fortunately, this hardly affects our exponents: Any attendant systematic errors in Fig. 2 are certainly less than the random errors indicated. All our NS runs use quadruple-precision arithmetic and we have checked that halving our integration time step does not affect our results perceptibly. Note also that sample fluctuations over even a few orders of magnitude are unimportant, given the range of our log-log plots like Fig. 1. Also, the agreement between our GOY and NS runs confirms our results. Our GOY-model data are, of course, of much better quality. Here Fourier components of the velocity are labeled by a discrete set of wave vectors $k_n = k_0q^n$. The dynamical variables are the *complex, scalar* velocities v_n for each shell n; v_n is affected directly only by the velocities in nearest and next-nearest shells. This model yields scaling properties [9–12] akin to experimental ones. The GOY-model equations are

$$\frac{d}{dt}v_n = iC_n - \nu k_n^2 v_n + f_n\,, \qquad (4)$$

where ν is the kinematic viscosity, f_n the external force on shell n, $C_n = (ak_nv_{n+1}v_{n+2} + bk_{n-1}v_{n-1}v_{n+1} + ck_{n-2}v_{n-1}v_{n-2})^*$, and a, b, and c can be fixed up to a constant by demanding [11], for ν, $f_n = 0$, that $v_n \sim k_n^{-1/3}$ be a stationary solution of Eq. (4), and the GOY-model kinetic energy and helicity be conserved. We adopt the conventional parameters [10,11] $k_0 = 2^{-4}$, $q = 2$, $a = 1$, $b = c = -1/2$, and use $f_n = 5 \times 10^{-3}(1 + i)\delta_{n,1}$, i.e., we force the first shell [23]. The GOY-model structure functions are $S_{n,p} \equiv \langle |v_n|^p\rangle \sim k_n^{-\zeta_p}$ [9–11]; reliable values of ζ_p obtain [11] if we use $\Sigma_{n,p} = \langle |\mathrm{Im}[v_nv_{n+1}v_{n+2} + v_{n-1}v_nv_{n+1}/4]|^{p/3}\rangle$ since this removes an underlying three cycle. We have used $\Sigma_{n,p}$ to obtain Fig. 4 [24], but $S_{n,p}$ in Figs. 1–3 for consistency with 3D NS. We use an Adams-Bashforth scheme [10] (step size δt) to integrate Eq. (4). The average of the time scale associated with the smallest wave number $(|v_1|k_1)^{-1}$ gives the "box-size" eddy turnover time. Table I lists other parameters for our 8 GOY-model runs G1–8, for which we use (cf. [10]) $E(k) = S_{n,2}/k_n$, $\lambda = 2\pi/k_0[\Sigma_nS_{n,2}/\Sigma_nk_n^2S_{n,2}]^{1/2}$, and $v_{\rm rms} = [k_0\Sigma_nS_{n,2}/\pi]^{1/2}$. This yields $\mathrm{Re}_\lambda \sim \nu^{-0.5}$, as expected [25] at large Re_λ. Our GOY model runs are done using double-precision arithmetic, but we have repeated run Gl in quadruple precision and checked that our results are unchanged.

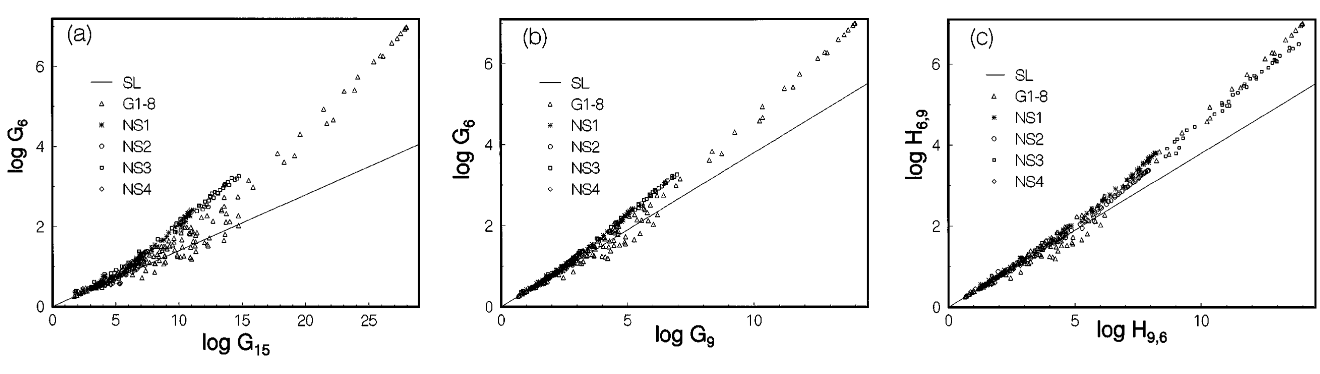

FIG. 3. Log-log (base 10) plots of G_6 versus (a) G_{15} and (b) G_9 illustrating our k-space GESS; (c) $H_{6,9}$ versus $H_{9,6}$ showing the universal inertial- to dissipation-range crossover (see text). The line shows the SL, inertial-range prediction.

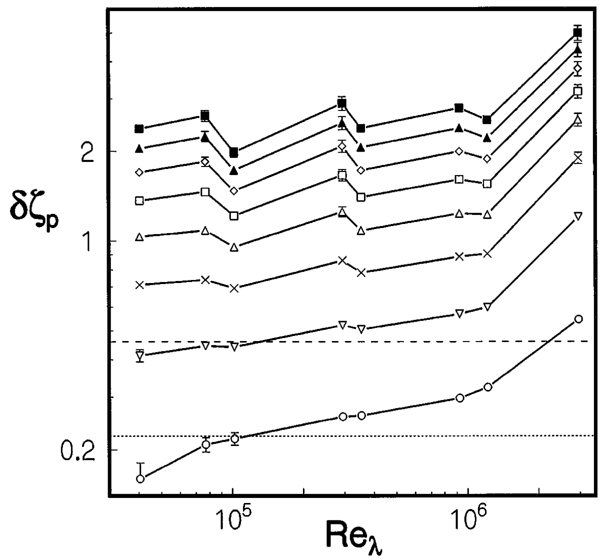

FIG. 4. Log-log plot (base 10) of $\delta\zeta_p$ versus the Taylor-microscale Reynolds number Re_λ for our GOY runs (G1-8) with $p = 6, 8 \ldots, 20$ (from bottom to top). The dotted ($p = 6$) and dashed ($p = 8$) lines show the SL results [4]. Error bars are shown but are often smaller than the symbol sizes.

Experimental evidence for the slope change in the dissipation range in real-space analogs of Fig. 1 was given by Stolovitzky and Sreenivasan [6], who postulated $S_p \sim S_3^{\alpha'_p}$ in the dissipation range and suggested $\alpha'_p \simeq (\zeta_{3p/2} + p/2)/(\zeta_{9/2} + 3/2)$. We have not been able to obtain a simple relation between our α_p and their α'_p (unlike [13] that between ζ_p and ζ'_p) since S_p does not have a power-law dependence on k in the dissipation range.

In conclusion, then, we have used our k-space ESS and GESS to obtain universal inertial-to-dissipation-range crossover in structure functions. It would be interesting to test this novel *universality* of dissipation-range asymptotics in diferent flows. The multiscaling we find in the far dissipation range might, at first sight, seem surprising because dissipation dominates here, but, as has been noted earlier [15], the intermittency seen in the far dissipation range can plausibly enhance mean nonlinear transfer even at low Re_λ. Our dissipation-range multiscaling is a manifestation of such intermittency. Preliminary studies [26] yield similar phenomena in MHD turbulence.

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