Exploring a Metal-Insulator Transition with Ultracold Atoms in Standing Light Waves?

Klaus Drese and Martin Holthaus
Fachbereich Physik der Philipps-Universität, Renthof 6, D-35032 Marburg, Germany
(Received 5 November 1996)

We suggest the possibility to realize an optical quasicrystal with ultracold atoms in far-detuned, bichromatic standing light waves. If the optical potentials created by the individual light waves have sufficiently different strength, one obtains an atom-optical realization of Harper's model. This model exhibits a metal-insulator transition at a certain value of the site-to-site hopping integral. Since this hopping integral is effectively renormalized by an additional oscillating force, one can switch from the regime of extended states to the regime of localized states by varying the amplitude of that force. [S0031-9007(97)02994-3]

PACS numbers: 32.80.Pj, 42.50.Vk, 71.30.+h

An atom with mass M in a monochromatic, one-dimensional standing light wave detuned sufficiently far from a dipole-allowed transition and oriented in the x direction can be described by an effective Hamiltonian

$$H(x) = \frac{p^2}{2M} + \frac{V_0}{2}\cos(2k_L x), \tag{1}$$

where p is the x component of the center-of-mass momentum and k_L is the light wave number. The depth V_0 of the optical potential is proportional to the intensity of the light field and inversely proportional to the detuning [1]. The atomic de Broglie wavelength $2\pi\hbar/p$ becomes large compared to the lattice constant $d = \pi/k_L$ when

$$\frac{p^2}{2M} \ll 4\,\frac{\hbar^2 k_L^2}{2M} \equiv 4E_R, \tag{2}$$

i.e., when the kinetic energy associated with the motion in the direction of the standing wave is smaller than or at most comparable to the single-photon recoil energy E_R. Such ultracold atoms in standing light waves constitute ideal test systems for studying fundamental phenomena of solid state physics [2–4]: the initial momentum distribution of a dilute atom cloud can be predetermined, there are no lattice defects and, hence, no unwanted scattering processes, and at the end of the experiment the optical potential can simply be turned off, thus allowing a clean measurement of the momentum distribution of the final states. Moreover, the atom-atom interaction remains negligible, so that one can probe single-particle effects by performing measurements on an ensemble.

In two recent landmark experiments, constant forces were exerted on atoms in optical lattices in order to explore Bloch oscillations [3] and Wannier-Stark ladders [4]. In this Letter we suggest to employ ultracold atoms in *periodically* forced optical lattices to study the effect of a strong, oscillating force on the Bloch dynamics. Experiments aiming in this direction are presently being performed with electrons in semiconductor superlattices that interact with strong terahertz fields [5,6]. But whereas these experiments are inevitably plagued by dissipation, lattice defects, and the non-negligible Coulomb interaction, ultracold atoms in standing light waves can be expected to show basic effects with paradigmatic clarity.

We will proceed in three steps. First we will show that periodically forced optical lattices exhibit dynamic localization [7,8] as a consequence of the collapse of the ground state quasienergy band [9]. In a second step we suggest to realize Harper's model [10,11] with ultracold atoms in bichromatic standing light waves. This model originally appeared in the description of Bloch electrons in a magnetic field; it serves as a paradigm for the physics of incommensurate structures. The most striking feature of this model is that it shows a metal-insulator transition, i.e., a transition from extended to localized states. The possibility of an atom-optical realization of Harper's model thus implies the possibility to investigate such a transition with atomic matter waves. In the third step we combine the results of the previous two: Since the quasienergy band collapse stems from the renormalization of the site-to-site hopping integrals by the oscillating force, and since the ratio of the on-site energies' modulation amplitude and the hopping integrals is the crucial parameter that governs the metal-insulator transition, one can *control* the transition by varying the strength of the force. An experimental confirmation of this scenario, although certainly quite ambitious, does not seem to be entirely out of reach.

(i) Forces can be exerted on atoms in standing light waves by introducing a small, time-dependent frequency difference $\delta\nu(t)$ between the two counterpropagating laser beams that generate the optical potential [2–4]. A periodic frequency modulation $\delta\nu(t) = k_L K_1 \sin(\omega t)/M\omega\pi$ results in an effective Hamiltonian

$$H(x,t) = \frac{p^2}{2M} + \frac{V_0}{2}\cos\left(2k_L\left[x + \frac{K_1}{M\omega^2}\cos(\omega t)\right]\right), \tag{3}$$

which is unitarily equivalent to

$$\tilde{H}(x,t) = \frac{p^2}{2M} + \frac{V_0}{2}\cos(2k_L x) + K_1 x\cos(\omega t). \tag{4}$$

Let us now fix the potential depth V_0/E_R such that the Wannier states $|\ell\rangle$ building up the lowest band of the optical lattice have appreciable overlap with their nearest neighbors $|\ell \pm 1\rangle$ only. Next, let $\hbar\omega$ be large compared to the width Δ of this band, but still much smaller than

 0031-9007/97/78(15)/2932(4)$10.00

the gap between the lowest two bands. As long as the amplitude K_1 is weak, the dynamics then remain restricted to the lowest band, so that we can invoke the single-band tight-binding approximation: the Hamiltonian

$$H_0 = -\frac{\Delta}{4} \sum_\ell (|\ell + 1\rangle\langle\ell| + |\ell\rangle\langle\ell + 1|) \qquad (5)$$

governs the dynamics in the unperturbed band, and

$$H_{\rm tb}(t) = H_0 + K_1 \cos(\omega t) \sum_\ell |\ell\rangle \ell d \langle\ell| \qquad (6)$$

approximates (4). The properties of this model system are well known. Because it is T-periodic in time, with $T = 2\pi/\omega$, there is a complete set of Floquet states $\psi_k(t) = u_k(t)\exp[-i\varepsilon(k)t/\hbar]$ with T-periodic functions $u_k(t) = u_k(t + T)$ and quasienergies $\varepsilon(k)$. These states are uniformly extended over the lattice and characterized by a quasimomentum k; the dispersion relation is [9,12]

$$\varepsilon(k) = -J_0\left(\frac{K_1 d}{\hbar\omega}\right)\frac{\Delta}{2}\cos(kd) \bmod \hbar\omega\,. \qquad (7)$$

Hence, the quasienergy band collapses when the ratio $K_1 d/\hbar\omega$ equals a zero of the ordinary Bessel function J_0. Then there is no dispersion at all: *any* initially localized wave packet will stay localized; that is, we have dynamic localization [7,8].

The crucial question now is whether the driving strength required for a band collapse in an optical lattice can still be compatible with the assumption of single-band dynamics. An example shows that it can: for $V_0 = 5E_R$ the width of the unforced lattice's lowest band becomes $\Delta = 0.264E_R$; the gap between this band and the next is $2.44E_R$. We then fix the frequency $\omega = E_R/2\hbar$, so that $\hbar\omega$ exceeds Δ by a factor of almost 2, but is still roughly 5 times smaller than the gap. For cesium atoms in optical lattices generated by 852 nm laser light, as employed in the recent Bloch-oscillation experiment [3], this corresponds to $\omega/2\pi \approx 1$ kHz, which has to be compared to a spontaneous emission rate of less than 4 s^{-1}. It should then be possible to study the time evolution over several 100 cycles without having to consider the detrimental effect of spontaneous emission [13].

Figure 1 shows quasienergies computed from the *full* Hamiltonian (4) with the above parameters for the band edges $k/k_L = 0$ and 1 [14]. Arrows in the left margin indicate the quasienergy band that originates from the ground state energy band. This band is very well described by the approximation (7): there is a band collapse at $K_1 d/\hbar\omega \approx 2.35$, quite close to the first zero $j_{0,1} \approx 2.405$ of J_0. Only for even higher amplitudes an avoided crossing signals the onset of strong interband effects, i.e., the breakdown of the single-band approximation (6). We conclude that it should be possible to observe dynamic localization with ultracold atoms in periodically driven optical lattices. Signatures of dynamic localization have recently been detected in experiments with semiconductor superlattices [6]. But whereas a proper theory of dynamic localization in these systems requires a consistent treatment of the Coulomb interaction [15], dynamic localization in optical lattices would be a genuine single-particle phenomenon, and probably the cleanest possible realization of the original proposal [7].

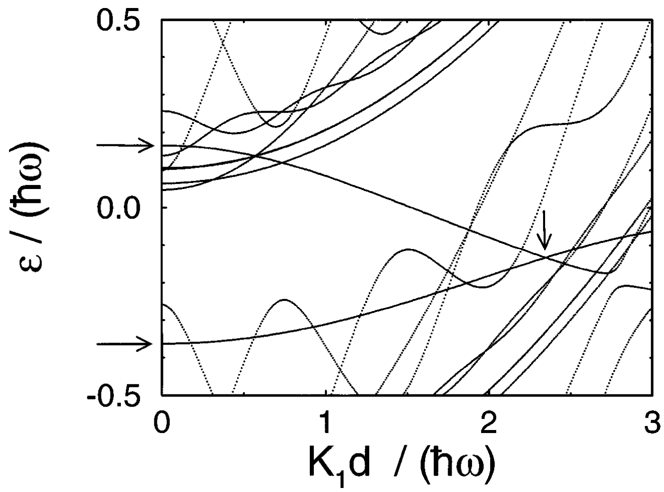

FIG. 1. Quasienergies for the Hamiltonian (4) with $V_0/E_R = 5.0$ and $\hbar\omega/E_R = 1/2$, at the band edges $k/k_L = 0$ and 1. Arrows in the left margin indicate the quasienergies at the edges of the ground state band; a further arrow indicates the band collapse at $K_1 d/\hbar\omega \approx 2.35$.

(ii) Next, we consider ultracold atoms in a *bichromatic* standing light wave generated by superimposing two standing waves with wave numbers $k_L^{(1)}$ and $k_L^{(2)}$ along the x axis. We assume that each of the two laser frequencies is appropriately detuned from a dipole-allowed transition $|g\rangle \to |e_1\rangle$ or $|g\rangle \to |e_2\rangle$, respectively. Employing the rotating wave approximation, and adiabatically eliminating the amplitudes of the excited states $|e_1\rangle$ and $|e_2\rangle$ as usual [16], one obtains an effective Hamiltonian

$$H(x) = \frac{p^2}{2M} + \frac{V_0^{(1)}}{2}\cos(2k_L^{(1)}x) + \frac{V_0^{(2)}}{2}\cos(2k_L^{(2)}x + \eta)\,, \qquad (8)$$

where η is a phase. If the number

$$g = k_L^{(2)}/k_L^{(1)} \qquad (9)$$

is irrational, this provides an atom-optical realization of a one-dimensional quasicrystal. Let us now fix the parameters such that the first light wave creates a tight-binding system (5) as before, and that the additional potential generated by the second light wave is merely a weak perturbation. If we approximate the Wannier states $|\ell\rangle$ by harmonic-oscillator ground states, we can estimate that the second potential effectuates a change

$$\nu(\ell) = \nu_0 \cos(2\pi g\ell + \eta) \qquad (10)$$

of the on-site energies, with

$$\nu_0 = \frac{V_0^{(2)}}{2}\exp\left(-g^2/\sqrt{V_0^{(1)}/E_R}\right), \qquad (11)$$

so that the single-band approximation to (8) becomes

$$H_{\rm Harper} = H_0 + \sum_\ell |\ell\rangle\nu(\ell)\langle\ell|\,. \qquad (12)$$

This is exactly Harper's model, which is usually encountered in the theory of two-dimensional Bloch electrons in a magnetic field. In that case g is the ratio between the magnetic flux per unit cell and the flux quantum; in our case it is simply the ratio (9) between the two wave numbers. When g is irrational, this model exhibits a metal-insulator transition: all eigenstates are extended if $\nu_0/\Delta < 1/2$, but localized if $\nu_0/\Delta > 1/2$ [11].

If we start again from an optical potential with $V_0^{(1)}/E_R = 5$, so that $\Delta/E_R = 0.264$, and if we arbitrarily set $g = (\sqrt{5} + 1)/2$, then the condition $\nu_0/\Delta = 1/2$ yields $V_0^{(2)}/V_0^{(1)} \approx 0.17$: at the transition, the strength of the second potential should be about $1/5$ of that of the first. This is not small, and the approximation (12), which takes into account only the on-site changes of the optical potential, might not be sufficient. We therefore also change the hopping integral connecting $|\ell\rangle$ and $|\ell + 1\rangle$ from $\Delta/4$ to $\Delta/4 + \delta(\ell)$, where

$$\delta(\ell) = f\,\nu_0 \cos[2\pi g(\ell + 1/2) + \eta]. \qquad (13)$$

The parameter f is a dimensionless perturbation strength. We then compute the eigenstates $\varphi_n = \sum_\ell a_\ell^{(n)}|\ell\rangle$ numerically for a lattice with 1500 sites, and calculate for each state the standard deviation $\sigma^{(n)}$ of the site-occupation probabilities $|a_\ell^{(n)}|^2$. The average value $\bar{\sigma}$, normalized by the standard deviation σ_0 for a uniformly extended state, provides a measure for the degree of localization. Figure 2 shows $\bar{\sigma}/\sigma_0$ as a function of ν_0/Δ, for $f = 0.0, 0.2, 0.49$, and 0.5. The curves for $f < 0.5$ almost coincide; a violent change of behavior occurs only for $f = 0.5$, which is by far larger than realistic. This result is quite important: the modulation of the hopping integrals does not destroy the self-duality of the Harper model, which is a crucial ingredient for the explanation of its metal-insulator transition [11]. Hence this transition remains clearly visible even

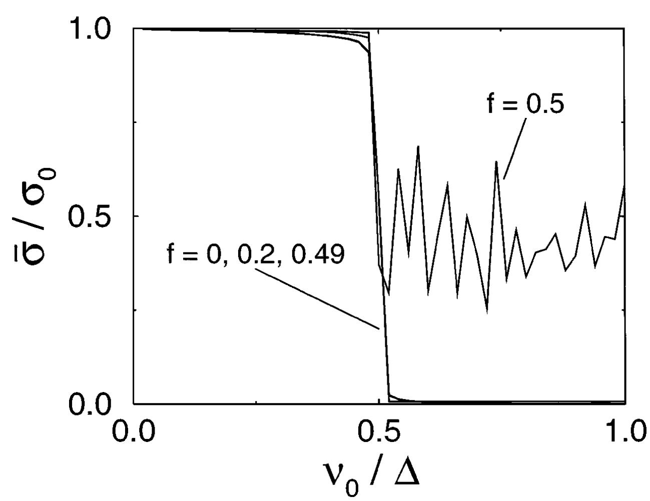

FIG. 2. Average, normalized standard deviation for the squared expansion coefficients of eigenstates of Harper's model with hopping integrals modified according to (13); $\bar{\sigma}/\sigma_0 = 1$ indicates completely delocalized states. Note that curves for $f < 0.5$ fall almost on top of each other.

under conditions where the modulation of the hopping integrals is not negligible.

(iii) One could, in principle, reduce the strength $V_0^{(2)}$ required for the transition by increasing $V_0^{(1)}$ (and thus reducing Δ). However, this means employing stronger lasers and hence increasing the spontaneous emission rate, which should be avoided. But there is another possibility. According to (7), the effect of the oscillating force $K_1\cos(\omega t)$ on the tight-binding model (5) is just a renormalization of the hopping integrals: the driven lattice with hopping integrals $\Delta/4$ behaves as an undriven lattice with hopping integrals $\Delta J_0(K_1 d/\hbar\omega)/4$—the effective bandwidth thus becomes smaller than the original Δ [17]. But since the ratio ν_0/Δ governs the metal-insulator transition, it must be possible to drive the system through the transition by varying the amplitude K_1 of the oscillating force. Suppose that $V_0^{(2)}$ is quite small compared to $V_0^{(1)}$, so that (12) is a good approximation to (8), and all states are extended, since ν_0/Δ is much smaller than $1/2$. If then a force $K_1\cos(\omega t)$ acts on the system, the transition should occur when

$$\frac{\nu_0}{\Delta} \approx \frac{1}{2}\left|J_0\left(\frac{K_1 d}{\hbar\omega}\right)\right|. \qquad (14)$$

This reasoning is confirmed in Fig. 3. For a lattice of 1001 sites ($\ell_{\min} = -500$, $\ell_{\max} = +500$) we compute wave functions $\psi(t) = \sum_\ell c_\ell(t)\,|\ell\rangle$ for the periodically driven Harper model, and compare them to wave functions for undriven models with renormalized hopping integrals. We set $\Delta/\hbar\omega = 0.385$, $\nu_0/\hbar\omega = 0.1$, $g = (\sqrt{5} + 1)/2$, and $\eta = 0$; the initial condition is $c_\ell(0) = \delta_{\ell,0}$ for each run. Figure 3(a) shows the evolution of the fourth moments $M_4(t) = \sum_\ell \ell^4 |c_\ell(t)|^2$ for the driven system; $K_1 d/\hbar\omega$ varies in steps of 0.1 from 1.3 to 1.9 (top to bottom). Figure 3(b) depicts the moments for the corresponding undriven, renormalized systems. The agreement speaks for itself. Since $\nu_0/\Delta \approx J_0(1.5)/2$, we find the expected change of behavior for $K_1 d/\hbar\omega \approx 1.5$: a wave packet made up of localized states only must stay localized, so that $M_4(t)$ remains bounded; a packet made up of extended states delocalizes, so that $M_4(t)$ grows indefinitely.

Figure 4 shows the wave functions for the driven system after 2000 cycles, for $K_1 d/\hbar\omega = 1.4$ (thin line) and 1.6 (heavy line). In the first case the wave function spreads over all the lattice, but in the second case it stays localized. The almost perfect exponential decay, clearly developed over no less than 25 orders of magnitude, leaves no doubt that we are dealing with a genuine, amplitude-controlled localization effect.

In general, the time-evolution operator $U(t, 0)$ for a periodically driven quantum system can be written as $P(t)\exp(-iGt/\hbar)$ with $P(t) = P(t + T)$; the eigenvalues of the time-independent operator G are the quasienergies. For the driven tight-binding system (6), this operator G is just the undriven Hamiltonian (5) with renormalized hopping integrals. But for the driven Harper model,

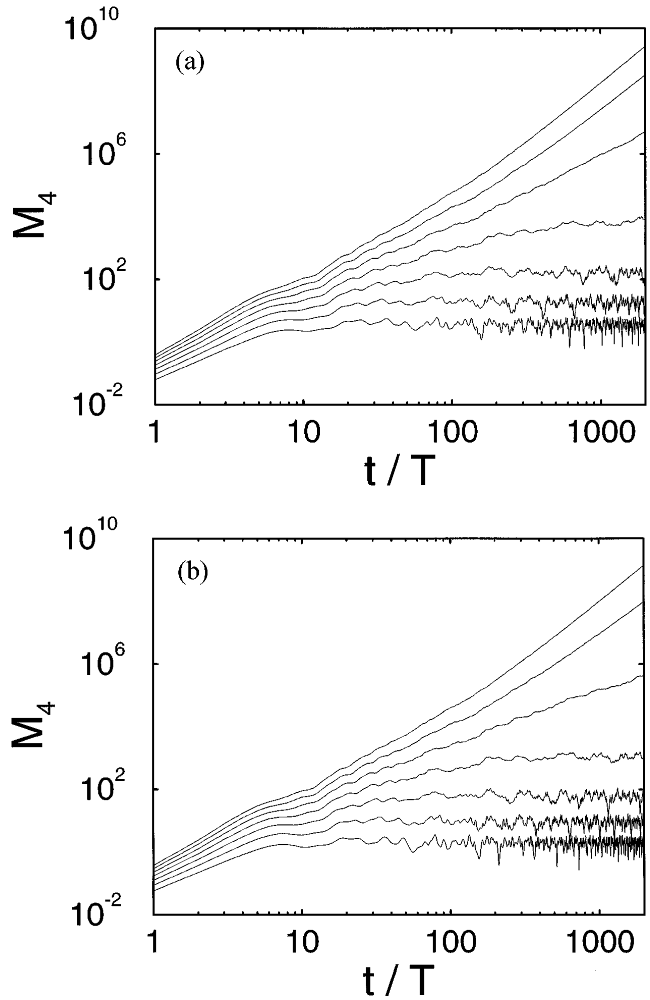

FIG. 3. Evolution of the moments $M_4(t) = \sum_\ell \ell^4 |c_\ell(t)|^2$ of wave functions $\psi(t) = \sum_\ell c_\ell(t)|\ell\rangle$ for the driven Harper model (a) and the undriven model with renormalized hopping integrals (b). Parameters are $\Delta/\hbar\omega = 0.385$ and $\nu_0/\hbar\omega = 0.1$; initially $c_\ell(0) = \delta_{\ell,0}$. $K_1 d/\hbar\omega$ varies from 1.3 to 1.9 (top to bottom), in steps of 0.1.

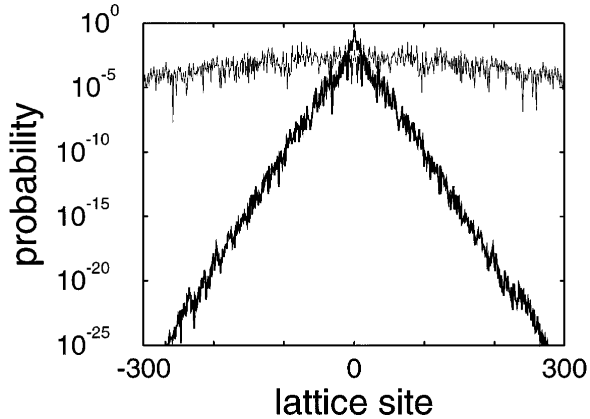

FIG. 4. Occupation probabilities $|c_\ell(t_0)|^2$ of lattice sites for the periodically driven Harper model [cf. Fig. 3(a)] with $K_1 d/\hbar\omega = 1.4$ (thin line) and 1.6 (heavy), at $t_0 = 2000\,T$.

the renormalized, undriven Harper Hamiltonian appears only as the dominant term in a high-frequency expansion of G; the leading corrections are of second order in $\Delta/\hbar\omega$. Therefore, (14) can be valid only when $\hbar\omega \gg \Delta$. This places an experiment right between Skylla and Charybdis: too small a driving frequency might not work as desired, since G would not be close enough to Harper's Hamiltonian, and too large a frequency will entail undesired interband effects. But our numerical results indicate that there is viable territory in between.

To summarize: (i) ultracold atoms in far-detuned, periodically forced optical lattices exhibit dynamic localization (in the sense of [7]), (ii) Harper's model can be realized approximately with atomic de Broglie waves in "optical quasicrystals," and (iii) this system can be driven through a metal-insulator transition by varying the amplitude of an oscillating force. An experimental confirmation—possibly by measuring the mean-square position spread [18] of light atoms such as lithium [19] for different amplitudes of the force—would certainly not be easy. But in view of the rapid development of atom optics, there is room for cautious optimism.

[1] See, e.g., A.P. Kazantsev, G.I. Surdutovich, and V.P. Yakovlev, *Mechanical Action of Light on Atoms* (World Scientific, Singapore, 1990).
[2] Q. Niu, X.-G. Zhao, G.A. Georgakis, and M.G. Raizen, Phys. Rev. Lett. **76**, 4504 (1996).
[3] M. Ben Dahan, E. Peik, J. Reichel, Y. Castin, and C. Salomon, Phys. Rev. Lett. **76**, 4508 (1996).
[4] S.R. Wilkinson, C.F. Bharucha, K.W. Madison, Q. Niu, and M.G. Raizen, Phys. Rev. Lett. **76**, 4512 (1996).
[5] A.A. Ignatov *et al.*, Ann. Phys. **3**, 137 (1994).
[6] B.J. Keay *et al.*, Phys. Rev. Lett. **75**, 4098 (1995); **75**, 4102 (1995); K. Unterrainer *et al.*, Phys. Rev. Lett. **76**, 2973 (1996).
[7] D.H. Dunlap and V.M. Kenkre, Phys. Rev. B **34**, 3625 (1986). See also S. Raghavan *et al.*, Phys. Rev. A **54**, R1781 (1996).
[8] Note that this dynamic localization [7] is unrelated to the quantum suppression of classical diffusive motion studied by F.L. Moore *et al.*, Phys. Rev. Lett. **73**, 2974 (1994).
[9] M. Holthaus, Phys. Rev. Lett. **69**, 351 (1992).
[10] P.G. Harper, Proc. Phys. Soc. London Sect. A **68**, 874 (1955).
[11] J.B. Sokoloff, Phys. Rep. **126**, 189 (1985).
[12] J. Zak, Phys. Rev. Lett. **71**, 2623 (1993).
[13] R. Graham and S. Miyazaki, Phys. Rev. A **53**, 2683 (1996).
[14] K. Drese and M. Holthaus, Chem. Phys. (to be published).
[15] T. Meier *et al.*, Phys. Rev. Lett. **73**, 902 (1994); **75**, 2558 (1995); Phys. Rev. B **51**, 14 490 (1995).
[16] R. Graham, M. Schlautmann, and P. Zoller, Phys. Rev. A **45**, R19 (1992).
[17] From the mathematical viewpoint, this renormalization of the hopping integrals resembles the renormalization of atomic g-factors by oscillating magnetic fields as studied by S. Haroche *et al.*, Phys. Rev. Lett. **24**, 861 (1970).
[18] G. Raithel *et al.*, Phys. Rev. Lett. **78**, 630 (1997).
[19] B.P. Anderson, T.L. Gustavson, and M.A. Kasevich, Phys. Rev. A **53**, R3727 (1996).