Dense Nuclear Matter in a Strong Magnetic Field

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We investigate in a relativistic Hartree theory the gross properties of cold symmetric nuclear matter and nuclear matter in beta equilibrium under the influence of strong magnetic fields. If the field strengths are above the critical values for electrons and protons, the respective phase spaces are strongly modified. This results in additional binding of the systems with distinctively softer equations of state compared to the field free cases. For magnetic field $\sim 10^{20}$ G and beyond, the nuclear matter in beta equilibrium practically converts into a stable proton-rich matter. [S0031-9007(97)02880-9]

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Large magnetic fields $B_m = 10^{12}-10^{14}$ G have been associated with the surfaces of supernovas [1] and neutron stars [2,3]. On the other hand, extremely large fields could exist in the interior of a star. It is presumed from the scalar virial theorem [4] that the interior field in neutron stars could be as high as $\sim 10^{18}$ G. Besides, the matter density in the neutron star core could exceed up to a few times the nuclear matter density. At such high fields and/or matter density, constituents of matter are relativistic. Moreover, the energy of a charged particle changes significantly in the quantum limit if the magnetic field is comparable to or above a critical value. The critical field is defined as that value where the cyclotron quantum is equal to or above the rest energy of the charged particle, which for electrons is $B_m^{(e)(c)} = 4.4 \times 10^{13}$ G, and for protons it is $B_m^{(p)(c)} \sim 10^{20}$ G. Theoretical studies of free electron gas in intense magnetic fields relevant to the neutron star crust have been carried out by several authors using the Dirac theory [4] as well as Thomas-Fermi and Thomas-Fermi-Dirac models [5]. The intense fields were shown to drastically reduce photon opacities and greatly accelerate the cooling rates in neutron stars [6]. It has been also demonstrated [7,8] that the magnetic fields have significant effects on the weak interaction rates and the abundances of light elements in the early Universe. The influence of extremely large fields on neutron matter [9] relevant to the neutron star interior and on the thermodynamic properties of strange quark matter and baryon inhomogeneity in the early Universe [10] have been also investigated.

Motivated by the existence of strong magnetic fields which quantize the motion of the electrons, we investigate in this Letter its influence on the gross properties of dense nuclear matter appropriate to the interior of a neutron star. This may have profound implications on cooling rates, mass-radius relationship of neutron stars. It is also instructive to extend the calculations to values of $B_m \geq 10^{20}$ G where along with the electron, the proton motion is strongly quantized. Fields of such magnitude, appropriate to the neutron star interior, could largely modify the proton phase space in the quantum limit. Though such a high field is hitherto unestimated, it may possibly exist in the core of the neutron star.

We therefore consider strong magnetic field effects on nuclear matter and a system composed of neutrons, protons, and electrons (*n*-*p*-*e* system) in beta equilibrium within a relativistic Hartree approach in the linear σ-ω-ρ model [11]. In the beta equilibrium case, the electrons are assumed to move freely in the strong magnetic field, whereas the produced neutrinos/antineutrinos escape from the system. In a uniform magnetic field B_m along the z axis, the Lagrangian is given by

$$\mathcal{L} = \overline{\psi}\Big[i\gamma_\mu D^\mu - m - g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - \frac{1}{2} g_\rho \gamma_\mu \tau \cdot \rho^\mu \Big]\psi + \frac{1}{2}(\partial^\mu \sigma)^2 - \frac{1}{2} m_\sigma^2 \sigma^2 - \sum_{k=\omega,\rho} \Big[\frac{1}{4}(\partial_\mu V_\nu^k - \partial_\nu V_\mu^k)^2 - \frac{1}{2} m_k^2 (V_\mu^k)^2 \Big], \tag{1}$$

in the usual notation [11]. Here, $D^\mu = \partial^\mu + iqA^\mu$, where the choice of gauge corresponding to the constant B_m along the z axis is $A_0 = 0$, $\mathbf{A} \equiv (0, xB_m, 0)$. The general solution for protons is $\psi(\mathbf{r}) \propto e^{-i\epsilon^H t + ip_y y + ip_z z} f_{p_y,p_z}(x)$, where $f_{p_y,p_z}(x)$ is the four-component spinor solution. The Dirac-Hartree equation for protons in a magnetic field is then given by

$$[-i\alpha_x \partial/\partial x + \alpha_y(p_y - qB_m x) + \alpha_z p_z + \beta m^* + U_{0,p}^H] f_{p_y,p_z}^{(r)}(x) = \epsilon^H f_{p_y,p_z}^{(r)}(x). \tag{2}$$

The equation of motion for neutrons is obtained by setting the charge $q = 0$ and replacing $U_{0;p}^H$ by $U_{0;n}^H$ in Eqs. (1) and (2); the corresponding solution is a plane wave. It may be mentioned that the Dirac theory for free electrons in a homogeneous magnetic field was first studied by Rabi [12] and can be obtained by putting $U^H_{0;p} = 0$ in Eq. (2). Since we confine to cold systems ($T = 0$), only positive energy spinors are considered. These in the chiral representation [13] are of the forms

$$f^{(1)}_{p_y,p_z}(x) = N_\nu \begin{bmatrix} (\epsilon^H_\nu + p_z) I_{\nu;p_y}(x) \\ -i\sqrt{2\nu q B_m}\, I_{\nu-1;p_y}(x) \\ -m^* I_{\nu;p_y}(x) \\ 0 \end{bmatrix}, \qquad (3)$$

$$f^{(2)}_{p_y,p_z}(x) = N_\nu \begin{bmatrix} 0 \\ -m^* I_{\nu-1;p_y}(x) \\ -i\sqrt{2\nu q B_m}\, I_{\nu;p_y}(x) \\ (\epsilon^H_\nu + p_z) I_{\nu-1;p_y}(x) \end{bmatrix}, \qquad (4)$$

where $N_\nu = 1/\sqrt{2\epsilon^H_\nu(\epsilon^H_\nu + p_z)}$, and $\epsilon^H_\nu = \epsilon^H - U^H_{0;p} = (p_z^2 + m^{*2} + 2\nu q B_m)^{1/2}$ is the effective Hartree energy, with ν the Landau principal quantum number which can take all possible positive integer values including zero. The function $I_{\nu;p_y}(x)$ is similar in form as in Ref. [13]. The effective nucleon mass $m^* = m + U^H_S$, where the nucleon rest mass is taken as $m = m_n = m_p = 939$ MeV, and $U^H_S = -(g_\sigma/m_\sigma)^2 n_S$. The total scalar density is $n_S = n_S^{(n)} + n_S^{(p)}$, with

$$n_S^{(n)} = \frac{m^*}{2\pi^2}\left[\mu^*_n \mathcal{O}_n^{1/2} - m^{*2}\,\ln\left\{\frac{\mu^*_n + \mathcal{O}_n^{1/2}}{m^*}\right\}\right], \qquad (5)$$

$$n_S^{(p)} = \frac{m^* q B_m}{2\pi^2}\sum_{\nu=0}^{\nu_{\max}^{(p)}} g_\nu \,\ln\left[\frac{\mu^*_p + \mathcal{O}_{p,\nu}^{1/2}}{(m^{*2} + 2\nu q B_m)^{1/2}}\right], \qquad (6)$$

where $\mathcal{O}_n = \mu_n^{*2} - m^{*2}$ and $\mathcal{O}_{p,\nu} = \mu_p^{*2} - m^{*2} - 2\nu q B_m$. The interaction energy density U^H_0 for protons and neutrons is given by $U^H_{0;p} = (g_\omega/m_\omega)^2 n_B + (g_\rho/m_\rho)^2 \rho_3/4$ and $U^H_{0;n} = (g_\omega/m_\omega)^2 n_B - (g_\rho/m_\rho)^2 \rho_3/4$, where $\rho_3 = n_p - n_n$. The total baryon number density is $n_B = n_n + n_p$, with $n_n = \mathcal{O}_n^{3/2}/3\pi^2$ and $n_p = \frac{qB_m}{2\pi^2}\sum_{\nu=0}^{\nu_{\max}^{(p)}} g_\nu \mathcal{O}_{p,\nu}^{1/2}$. Here $\nu_{\max}$ is the largest integer not exceeding $(\mu_p^{*2} - m^{*2})/(2qB_m)$, and the effective chemical potential μ^*_p is ϵ^H_ν at the Fermi surface. The Landau level degeneracy factor g_ν is 1 for $\nu = 0$ and 2 for $\nu > 0$. The total energy density of the system is given by

$$\epsilon = \frac{g_\sigma^2}{2m_\sigma^2} n_S^2 + \frac{g_\omega^2}{2m_\omega^2} n_B^2 + \frac{g_\rho^2}{8m_\rho^2}\rho_3^2 + \frac{1}{8\pi^2}\left[2\mu_n^{*3}\mathcal{O}_n^{1/2} - m^{*2}\mu^*_n\mathcal{O}_n^{1/2} - m^{*4}\,\ln\left\{\frac{\mu^*_n + \mathcal{O}_n^{1/2}}{m^*}\right\}\right]$$
$$+ \frac{qB_m}{4\pi^2}\sum_{\nu=0}^{\nu_{\max}^{(p)}} g_\nu\left[\mu^*_p\mathcal{O}_{p,\nu}^{1/2} + m^{*2}_{p,\nu}\,\ln\left\{\frac{\mu^*_p + \mathcal{O}_{p,\nu}^{1/2}}{m^*_{p,\nu}}\right\}\right] + \frac{qB_m}{4\pi^2}\sum_{\nu=0}^{\nu_{\max}^{(e)}} g_\nu\left[\mu_e\mathcal{O}_{e,\nu}^{1/2} + m^2_{e,\nu}\,\ln\left\{\frac{\mu_e + \mathcal{O}_{e,\nu}^{1/2}}{m_{e,\nu}}\right\}\right]. \qquad (7)$$

Here $\mathcal{O}_{e,\nu} = \mu_e^2 - m_e^2 - 2\nu q B_m$ and $m^{*2}_{i,\nu} = m_i^{*2} + 2q\nu B_m$, where m^*_is denote m^*s(m_es) for $i = p(e)$. The first three terms of Eq. (7) correspond to the interaction energy densities for σ, ω, and ρ mesons. The last three terms are the expressions for kinetic energy densities for *n*, *p*, *e*. The total pressure is given by $P = n_B^2 \partial(E/A)/\partial n_B$, where E/A is the total energy per baryon. For symmetric nuclear matter (SNM), where $n_n = n_p = n_B/2$, m^* is evaluated self-consistently for a given n_B and B_m. On the other hand, the *n*-*p*-*e* system under the beta equilibrium and the charge neutrality conditions is in particular important for the neutron star. For these two cases, when $B_m \geq B_m^{(e)(c)}$, the charge neutrality condition $n_p = n_e$ gives

$$\sum_{\nu=0}^{\nu_{\max}^{(p)}} g_\nu \mathcal{O}_{p,\nu}^{1/2} = \sum_{\nu=0}^{\nu_{\max}^{(e)}} g_\nu \mathcal{O}_{e,\nu}^{1/2}. \qquad (8)$$

When $B_m \geq B_m^{(e)(c)}$, but appreciably smaller than $B_m^{(p)(c)}$, a large number of Landau levels are populated and the relations are almost similar to the field-free case. However, when B_m significantly affects the electrons so that $\nu_{\max}$ is small (≈ 0), the protons are also affected. Employing Eq. (8) in conjunction with the β-equilibrium condition $\mu_n = \mu_p + \mu_e$, one can obtain m^* self-consistently for a given n_B and B_m. The proton and neutron chemical potentials μ_p and μ_n are related to their respective Fermi momenta $k_F^{(p)}$ and $k_F^{(n)}$ by $\mu_p = U^H_{0;p} + [k_F^{(p)2} + m^{*2}_{p,\nu}]^{1/2}$ and $\mu_n = U^H_{0;n} + [k_F^{(n)2} + m^{*2}]^{1/2}$. Therefore, the neutral ρ meson field affects the chemical composition inside the neutron star through the different proton and neutron vector potential U^H_0 in the asymmetric *n*-*p*-*e* system.

In the present calculation the parameters for the coupling constants and mesons masses are taken from Horowitz and Serot [14] to be $g_\sigma^2(m/m_\sigma)^2 = 357.47$, $g_\omega^2(m/m_\omega)^2 = 273.87$, and $g_\rho^2(m/m_\rho)^2 = 97$. This yields nuclear matter saturation density at $n_0 = 0.1484$ fm^{-3} with a binding energy of 15.75 MeV and a bulk symmetry energy of 35 MeV. In the top panel of Fig. 1, the variation of effective nucleon mass m^*/m with baryon density n_B/n_0 is displayed. Curves *a* and *b* represent the SNM case for $B_m = 0$ and $B_m^{(p)(c)}$, respectively. It is found that for $B_m = 0$, m^* decreases gradually with n_B while for $B_m^{(p)(c)}$, the decrease is relatively much faster beyond $n_B \approx n_0$. This is attributed to the drastic reduction in the proton Fermi momentum $k_F^{(p)}$, whereas the neutron Fermi momentum $k_F^{(n)}$ is unaffected by B_m and is identical to the Fermi momentum k_F for $B_m = 0$. Consequently, at any n_B, μ^*_p is smaller than μ^*_n. These are reflected in the larger value of n_S and hence in the magnitude of U_S^H for nonzero magnetic field, in contrast to the field free case. By further increasing B_m to $10B_m^{(p)(c)}$, m^*/m for SNM (curve c) undergoes a further reduction beyond the density $\sim 3n_0$.

For a n-p-e system, curve d in the top panel of Fig. 1 shows the variation of m^*/m at $B_m = 0$. If $B_m < B_m^{(p)(c)}$, the variation of m^* remains virtually unaltered from the field free case (not shown in the figure). If the field is further increased to $B_m^{(p)(c)}$ and $10B_m^{(p)(c)}$, the m^*/m values are significantly reduced as evident from curves e and f of the figure. Furthermore, in the presence of B_m, the m^* values here are found to be much smaller than those for SNM. This may be attributed to the neutron-proton asymmetry in the system.

The energy per baryon E/A with varying baryon density n_B/n_0 is exhibited in Fig. 2. Curves a and b, respectively, correspond to $B_m = 0$ and $10B_m^{(p)(c)}$ for SNM. It is observed that the strong magnetic field $B_m \geq B_m^{(p)(c)}$ causes additional binding of the nuclear matter with $E/A \approx -41$ MeV for $B_m = 10B_m^{(p)(c)}$. The usual binding energy curve for n-p-e system in absence of magnetic field (curve c) shows no binding. When the field is slightly quantizing, the system is still unbound as indicated by curve d for $B_m = 10^4 B_m^{(e)(c)}$; it is only sightly softer than c. However, when both protons and electrons are strongly quantized by B_m, the n-p-e system is strongly bound, and the binding increases with B_m as exhibited by curves e and f for $B_m = B_m^{(p)(c)}$ and $10B_m^{(p)(c)}$, respectively. In contrast to the $B_m = 0$ case, for nonzero B_m, even though the contribution from the scalar density is increased, the relatively larger decrease in kinetic energy density especially for protons results in the excess binding. Furthermore, it is observed that with increasing B_m, the minima of the binding energy curves, where the pressure $P = 0$, shift towards higher densities. This is clearly seen in the inset of Fig. 2 where the pressure P is displayed as a function of energy density ε; curves a to f correspond to the same values of B_m as in Fig. 2. The causality condition $\partial P/\partial\varepsilon \leq 1$ is fulfilled by all the cases considered here. It is evident from Eq. (7) that the kinetic energy density for protons is strongly suppressed, and the σ meson term is strongly enhanced in the magnetic field. The latter term has a negative contribution to the pressure. On the other hand, ω and ρ meson terms ($\rho_3 = 0$ for SNM) which increase with n_B, compensate the reduction in the kinetic energy and the scalar meson terms in the pressure at higher density to produce zero pressure compared to the $B_m = 0$ cases. For the n-p-e system, considerable suppression of $k_F^{(p)}$ and m^* in the magnetic field accentuates the above effect; as a consequence, it is more bound with the minimum occurring at a higher density than the SNM.

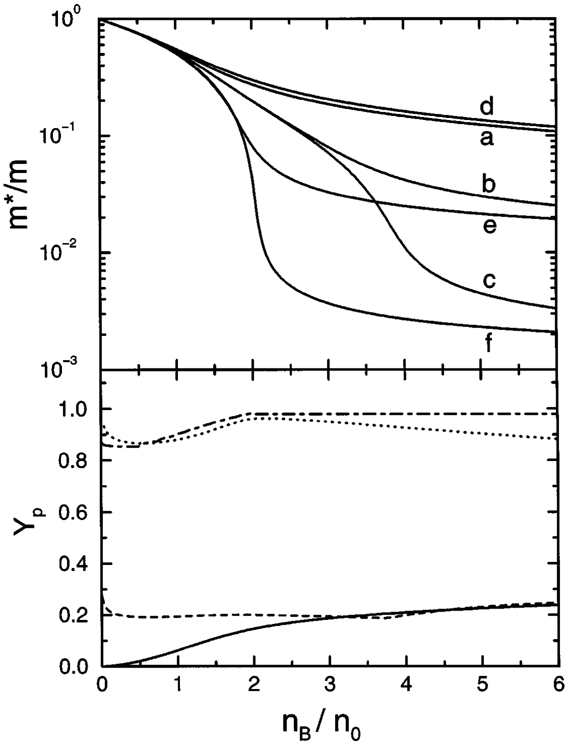

FIG. 1. The variations of effective nucleon mass m^*/m (top panel) and proton fraction Y_p (bottom panel) with baryon density n_B/n_0 for different values of magnetic field B_m.

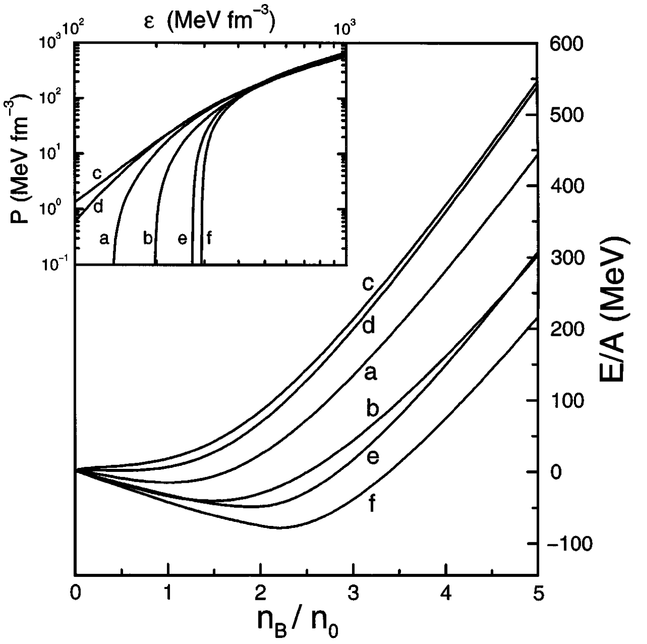

FIG. 2. The energy per baryon E/A as a function of n_B/n_0 for different values of B_m. In the inset, the pressure P is shown as a function of energy density ε for different values of B_m.

The proton fraction in neutron star matter is crucial in determining the direct URCA process which leads to the cooling of neutron stars [15,16]. In the bottom panel of Fig. 1, the proton fraction $Y_p = n_p/n_B$ is shown for the n-p-e system for $B_m = 0$ (solid line) and for $10^4 B_m^{(e)(c)}$ (dashed line). The proton fraction is observed to be enhanced in the latter case. For the direct URCA process, the inequality $k_F^{(e)} + k_F^{(p)} \geq k_F^{(n)}$, which corresponds to $Y_p \geq 0.11$ for $B_m = 0$ [16], should be satisfied. In the linear σ-ω-ρ model with $B_m = 0$, this condition is satisfied at $n_B \geq 1.5n_0$ and thus rapid cooling by direct URCA process can occur. On the other hand, for $B_m = B_m^{(p)(c)}$, the proton fraction shown by the dotted line in the figure is found to be considerably enhanced. The drastic fall in the proton Fermi momentum entails a substantial $n \rightarrow p$ conversion; as a result, the system is converted to a highly proton-rich matter. Moreover, it has been demonstrated in Fig. 2 that such systems are energetically more favorable. Therefore, if the magnetic field is strong enough $\sim 10^{20}$ G, possible existence of stable "proton matter" may be envisaged. If the field is further increased to $10B_m^{(p)(c)}$, the proton fraction (shown by the dash-dotted line) saturates to a value of 0.98 at $n_B \geq 2n_0$.

When $B_m \geq B_m^{(e)(c)}$, $\nu_{\max}^{(e)}$ for various values of n_B and B_m is found to follow the relationship $\nu_{\max}^{(e)} \approx [1/(B_m/B_m^{(e)(c)})][\mathcal{I}(n_B/n_0) - \mathcal{J}(n_B/n_0)^2]$, where for symmetric nuclear matter, $\mathcal{I} = 101\,217.12$ and $\mathcal{J} = 5458.64$, and for the n-p-e system in beta equilibrium $\mathcal{I} = 46\,571.24$ and $\mathcal{J} = 562.35$. Thus for a fixed n_B, $\nu_{\max}^{(e)}$ decreases monotonically with increasing B_m. For all n_B values of interest, $\nu_{\max}^{(e)} = 0$ when $B_m \gtrsim 10^6 B_m^{(e)(c)}$, and this is found to be in conformity with the values $B_m \geq B_m^{(e)(c)}(\mu_e/m_e)^2/2$ predicted in Ref. [17]. As a consequence of charge neutrality, $\nu_{\max}^{(p)} = \nu_{\max}^{(e)} = 0$ and $k_F^{(p)} = k_F^{(e)}$ [see Eq. (8) and Ref. [4]]. Therefore in such strong fields, the direct URCA process in stars would occur if $Y_p \geq (X^{1/3} - X^{-1/3}/3d)$, which corresponds to the real positive root of the above mentioned inequality condition. In this expression $X = \{1 + (1 + 4/27d)^{1/2}\}/2d$ and $d = 64\pi^4 n_B^2/3(qB_m)^3$. Interestingly, Y_p depends not only on n_B but also on B_m. Comparing the values of the proton fraction obtained from the model calculations (Fig. 1) and the inequality condition, we find the threshold for direct URCA process is not reached for $B_m \geq B_m^{(p)(c)}$.

The effect of intense fields on the neutron star profiles is obtained by applying the equation of state to solve the Tolman-Oppenheimer-Volkoff equation [18]. For magnetic fields $B_m = 0$, $10^4 B_m^{(e)(c)}$, $B_m^{(p)(c)}$, and $10B_m^{(p)(c)}$, the maximum masses of the stars are found to be $M_{\max} = 3.10M_\odot$, $2.99M_\odot$, $2.91M_\odot$, and $2.86M_\odot$, respectively. The corresponding radii are $R_{M_{\max}} = 15.02$, 14.95, 12.25, and 12.00 km. These values suggest that the neutron star masses are practically insensitive to the effects of the magnetic fields, whereas the radii decrease in intense fields, leading to their compactness.

In this Letter, we primarily focused on the new qualitative features that arise out of nuclear matter in a strong magnetic field within a relativistic Hartree approach in a simple linear σ-ω-ρ model. We believe that these features will survive even in more sophisticated calculations with a more refined equation of state. It will be worth investigating the influence of a quantizing field on the quark matter in a relativistic Hartree-Fock model.

[1] V. Ginzburg, *High Energy Gamma Ray Astrophysics* (North-Holland, Amsterdam, 1991).

[2] I. Fushiki, E. H. Gudmundsson, and C. J. Pethick, Astrophys. J. **342**, 958 (1989).

[3] C. Woltjer, Astrophys. J. **140**, 1309 (1964); J. Trümper *et al.*, Astrophys. J. Lett. **219**, L105 (1978); W. A. Wheaton *et al.*, Nature (London) **289**, 240 (1979); D. E. Gruber *et al.*, Astrophys. J. Lett. **240**, L127 (1980); T. A. Mihara *et al.*, Nature (London) **346**, 250 (1990); G. Chanmugam, Annu. Rev. Astron. Astrophys. **30**, 143 (1992).

[4] D. Lai and S. L. Shapiro, Astrophys. J. **383**, 745 (1991), and references therein.

[5] I. Fushiki, E. H. Gudmundsson, C. J. Pethick, and J. Yngvason, Ann. Phys. (N.Y.) **216**, 29 (1992); D. G. Yakovlev and A. D. Kaminker, *The Equation of States in Astrophysics,* edited by G. Chabrier and E. Schatzman (Cambridge University Press, Cambridge, 1994), p. 214.

[6] S. Tsuruta, V. Canuto, J. Lodenquai, and M. A. Ruderman, Astrophys. J. **176**, 739 (1972); S. Tsuruta, *Physics of Dense Matter,* edited by C. J. Hansen (D. Reidel Publishing Company, Boston, 1974), p. 209; G. Baym, *Statistical Mechanics of Quarks and Hadrons,* edited by H. Satz (North-Holland, Amsterdam, 1981), p. 17.

[7] B. Cheng, D. N. Schramm, and J. W. Truran, Phys. Lett. B **316**, 521 (1993); Phys. Rev. D **49**, 5006 (1994).

[8] D. Grasso and H. R. Rubinstein, Astropart. Phys. **3**, 95 (1995); Report No. Astro-ph/9602055.

[9] G. A. Shulman, Sov. Astron. **35**, 50 (1991); **35**, 492 (1991); A. S. Vshivtsev and D. V. Serebryakova, JETP **79**, 17 (1994).

[10] S. Chakrabarty, Phys. Rev. D **54**, 1306 (1996).

[11] B. D. Serot, Phys. Lett. B **86**, 146 (1979); Y. C. Leung, *Physics of Dense Matter* (World Scientific, Singapore, 1984); B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).

[12] I. I. Rabi, Z. Phys. **49**, 507 (1928).

[13] M. Kobayashi and M. Sakamoto, Prog. Theor. Phys. **70**, 1375 (1983); A. Das and M. Hott, Phys. Rev. D **53**, 2252 (1996).

[14] C. J. Horowitz and B. D. Serot, Nucl. Phys. **A368**, 503 (1981).

[15] J. Boguta, Phys. Lett. B **106**, 255 (1981).

[16] J. M. Lattimer, C. J. Pethick, M. Prakash, and P. Haensel, Phys. Rev. Lett. **66**, 2701 (1991); C. J. Pethick, Rev. Mod. Phys. **64**, 1133 (1992).

[17] V. Canuto and J. Ventura, Fundam. Cosm. Phys. **2**, 203 (1977).

[18] S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars* (Wiley, New York, 1983), p. 125.