Inhibition of Coherence in Trapped Bose-Einstein Condensates

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We analyze the dependence of the collapse and revival of many-atom coherence of a trapped Bose-Einstein condensate on the trap potential, dimensionality of the gas, and atom number fluctuations. We show that in a class of experimentally relevant systems the collapse time vanishes in the limit of a large number of atoms, implying that the trapped Bose gas cannot sustain a well-defined quantum phase. [S0031-9007(97)02828-7]

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The recent experimental realization of Bose-Einstein condensates (BEC) of trapped cold rubidium [1], lithium [2], and sodium [3] atoms have stimulated research on the fundamental properties of *confined coherent matter*. The manifestation of many interesting physical phenomena in this context, such as superfluidity and the Josephson effect, is closely related to the existence of the phase of the condensate [4]. In addition, the assumption of a well-defined condensate phase is implicit in theoretical analysis carried out using the mean-field theory [5,6]. Several authors have already analyzed the quantum phase dynamics in trapped BEC atoms and reached the common conclusion that due to the nonlinearities arising from the atom-atom interactions and the discreteness of the spectrum of the many-body system, there will be collapses and revivals in the many-body coherence [7–10]. In Ref. [10] we have related the phenomenon of collapse (termed as quantum phase diffusion) to quantum fluctuations of the atom number N using the Bogoliubov-Hartree (BH) approach. In a Bose gas trapped in a harmonic potential in 3D, both the collapse and the revival times increase with N, implying that the mean-field approximation remains valid for long time scales. However, contrary to the case of homogeneous systems in the thermodynamical limit [11], the collapse time τ_c in the 3D harmonic trap grows as $N^{1/10}$ instead of $N^{1/2}$, i.e., much slower.

In this Letter, we analyze the dependence of the collapse and revival times on the specific trap potential $V_t(r) = ar^\eta$, the dimensionality D of the Bose gas, and the initial dispersion $\sigma(N)$ of the atom number N, which is determined by the way the system is prepared initially. In the continuous spectrum approximation [12], such a trap supports condensation for $\eta < 2$ in 1D, and arbitrary $0 < \eta < \infty$ in $D \geq 2$. We find that if $\eta > D$ and the dispersion is normal [i.e., $\sigma(N) \propto N^{1/2}$], the collapse time goes to zero in the limit $N \to \infty$. This fact sets fundamental limits on the existence of *coherent* condensates. Any *well-defined condensate* phase that may be artificially created using a quantum measurement process [8,13,14] will *collapse* in an arbitrarily short time scale and will never *revive*. Even in the absence of coupling to the environment, the system will inhibit coherence. This result has important consequences for experiments with the trapped BEC gases [1,3], especially for the studies of the Josephson effect [9,11] between two condensates.

Two trapped condensates that are brought to contact establish a relative phase after a preparation time [8,11,14]. If the contact is interrupted, the phase memory lasts a time of the order of τ_c. In the case of $\tau_c \to 0$, the phase memory will be lost practically immediately, and the condensates will not exhibit any signatures of the Josephson effect. In the opposite case, the coherence will last a *finite time* τ_c during which the signatures of the Josephson effect (or beatnote between the condensates [8]) are observable. We show that the combination of intracondensate interactions and Josephson coupling between the two condensates A and B establish, in general, *sub-Poissonian fluctuations* $\sigma(N) \propto N^{1/2-\beta}$ of the number of A, or equivalently B atoms, with $\beta > 0$ determined by the trap potential and the coupling parameters. The initial phase fluctuations are enhanced in such a situation, while τ_c is prolonged. Increasing the strength of the Josephson coupling decreases β, but above some critical scaling law between the coupling and N, this effect saturates, and $\sigma(N) = \sqrt{N/2}$. The Josephson coupling thus cannot establish broader fluctuations of N than the normal ones, i.e., of the order of $O(N)$.

The essential physics of the phase collapse and revival is captured by a toy model for the "zero-mode" dynamics. The model is described by the U(1)-gauge symmetric Hamiltonian,

$$\mathcal{H} = \frac{\tilde{u}}{2}\,\hat{g}^\dagger\hat{g}^\dagger\hat{g}\hat{g} - \mu\hat{g}^\dagger\hat{g}\,, \tag{1}$$

where $\hat{g}$, $\hat{g}^\dagger$ are, respectively, annihilation and creation operators for atoms occupying the condensate (zero) mode, $[\hat{g}, \hat{g}^\dagger] = 1$, and μ is a "chemical potential," which fixes either the exact value of the atom number N, or its mean $\langle \hat{N} \rangle = \langle \hat{g}^\dagger \hat{g} \rangle$. The coupling constant $\tilde{u}$ is assumed to scale as $\tilde{u} = u_0/V$, where V is the volume of the condensate mode. Such scaling mimics the real behavior of the full second quantized Hamiltonian describing a Bose gas in the trap.

Obviously, Fock states $|n\rangle$ are eigenstates of $\mathcal{H}$ [Eq. (1)] with energies $E_n = \frac{\tilde{u}}{2} n(n-1) - \mu n$. The exact ground state is thus the Fock state

$$|N\rangle_F = \frac{(\hat{g}^\dagger)^N}{(N!)^{1/2}} |\mathrm{vac}\rangle , \tag{2}$$

for which $N = (2\mu + \tilde{u})/\tilde{u}$. The latter expression can be regarded either as determining N for a given μ, or determining μ in order to fix N. The ground state is symmetric with respect to the phase change, i.e., its phase is not definite.

It is worth observing that a coherent state, $\hat{g}|z\rangle = z|z\rangle$, minimizes the mean energy $\langle z|\mathcal{H}|z\rangle$ for $|z| = \sqrt{(N - 1/2)} \simeq \sqrt{N}$. This state provides a very good variational approximation of the true ground state energy, and has a definite phase.

The coherent state, however, is not an eigenstate of $\mathcal{H}$. It undergoes dynamics determined by the Hamiltonian (1). In particular, the mean value of the atomic field operator $\hat{g}$ evolves as [7]

$$\langle z|\hat{g}(t)|z\rangle = \sqrt{N} \sum_{n=0}^{\infty} \frac{N^n e^{-N}}{n!} \exp[i(E_n - E_{n+1})t]$$

$$= \sqrt{N}\, e^{i\mu t} \exp[N(e^{-i\tilde{u}t} - 1)] . \tag{3}$$

For short times

$$\langle z|\hat{g}(t)|z\rangle \simeq \sqrt{N}\, e^{i\mu t - i\tilde{u}Nt} \exp(-N\tilde{u}^2 t^2/2) , \tag{4}$$

so that the collapse time of the coherent state is $\tau_c = 1/\sqrt{N}\,\tilde{u}$. On a longer time scale, in contrast, the function (3) is periodic, and revives at every period. The time between the two subsequent revivals ($\mathrm{mod}\, 2\pi$) is thus $\tau_r = 1/\tilde{u} = \sqrt{N}\,\tau_c$.

The phenomenon of collapse and revival is generic for nonlinear quantum dynamics [15]. Note that in the standard thermodynamic limit $\tau_c \propto \sqrt{N}/u_0$ since V $\propto N$. The fact that the ratio $\tau_r/\tau_c = \sqrt{N}$ is a consequence of the Poissonian fluctuations of the atom number in the coherent state. Had we used another initial state $\sum_{n=0}^{\infty} \alpha_n |n\rangle$, with α_n's well peaked around $n = N$, the result (3) would read

$$\langle z|\hat{g}(t)|z\rangle = \sum_n \sqrt{n}\, \alpha_n^* \alpha_{n+1} e^{i(E_n - E_{n+1})t} . \tag{5}$$

Approximating $\alpha_n \propto \exp[-(n-N)^2/4\sigma^2(N)]$, we obtain the same τ_r as before, with $\tau_c = \tau_r/\sigma(N)$.

The calculation is only a little more complex for the real condensate in the Bogoliubov-Hartree approximation [10]. At zero temperature $T = 0$ we linearize the second quantized Hamiltonian $\mathcal{H}$ for a system of N atoms in a trap by setting $\hat{\Psi}(\vec{r}) = \sqrt{N}\,\psi_0(\vec{r}) + \delta\hat{\Psi}(\vec{r})$, where $\hat{\Psi}(\vec{r})$ is the atomic field annihilation operator, $\psi_0(\vec{r})$ is the c-number condensate wave function normalized as $\int d\vec{r}\,|\psi_0(\vec{r})|^2 = 1$, whereas $\delta\hat{\Psi}(\vec{r})$ is the quantum fluctuation part; $\psi_0(\vec{r})$ is the lowest energy (μ) solution of the nonlinear Schrödinger equation (NLSE)

$$[\mathcal{L} + u_0\rho(\vec{r})]\psi_0(\vec{r}) = 0 , \tag{6}$$

with $\mathcal{L} \equiv -\frac{\hbar^2}{2M}\nabla^2 + V_t(\vec{r}) - \mu$, and $\rho(\vec{r}) = N\psi_0^2(\vec{r})$.

The Hamiltonian becomes a bilinear form of $\delta\hat{\Psi}(\vec{r})$ and $\delta\hat{\Psi}^\dagger(\vec{r})$, and can be transformed to a canonical form

$$\mathcal{H} = N\tilde{u}\hat{P}^2 + \sum_{k\neq 0} \hbar\omega_k g_k^\dagger g_k , \tag{7}$$

where g_k, $g_k^\dagger$ are, respectively, the annihilation and creation operators of the elementary excitations, whereas the "momentum" operator is

$$\hat{P} = \int d\vec{r}\,\psi_0(\vec{r})[\delta\hat{\Psi}(\vec{r}) + \delta\hat{\Psi}^\dagger(\vec{r})] . \tag{8}$$

The "position" operator canonically conjugated to $\hat{P}$,

$$\hat{Q} = i\int d\vec{r}\,\Phi_0(\vec{r})[\delta\hat{\Psi}(\vec{r}) - \delta\hat{\Psi}(\vec{r})] , \tag{9}$$

has to fulfill $[\hat{Q}, \hat{P}] = i$, $[Q, g_k] = 0$ (for $k \neq 0$) by definition, and $[\hat{Q}, \mathcal{H}] = 2iN\tilde{u}\hat{P}$ because of (7). These commutation relations give

$$[\mathcal{L} + 3u_0\rho(\vec{r})]\Phi_0(\vec{r}) = 2N\tilde{u}\psi_0(\vec{r}) , \tag{10}$$

which has a unique solution since the operator $\mathcal{L} + 3u_0\rho(\vec{r})$ is positive definite [10]. We can now express $\hat{\Psi}(\vec{r})$ in terms of $\hat{P}$, $\hat{Q}$, g_k, and $g_k^\dagger$; as we have shown elsewhere [10], $\langle\hat{\Psi}(\vec{r})\rangle$ collapses within the time τ_c, due to quantum phase spreading.

In the limit of large N we use the Thomas-Fermi approximation [16,17], i.e., neglect the kinetic energy term in NLSE. For a D-dimensional isotropic trap with the potential $V_t(r) = ar^\eta$ we obtain $\psi_0(\vec{r}) = \{[\mu - V_t(r)]/u_0 N\}^{1/2}$, $\Phi_0(\vec{r}) = \frac{1}{[2\upsilon_D r_0^D \psi_0(\vec{r})]}$, for $r \leq r_0$ and zero otherwise; $\mu = V_t(r_0)$, whereas $\tilde{u} = u_0/(2\upsilon_D r_0^D)$ with $r_0 = [(\eta + D)Nu_0/\upsilon_D \eta a]^{1/(\eta+D)}$ and with υ_D denoting the volume of a D-dimensional sphere with unit radius. As expected $N\tilde{u}$ is proportional to the condensate peak density, and $\tau_c = 1/\sigma(N)\tilde{u}$.

Consider a Gedanken experiment where the relative phase $\theta = \theta_0$ between the condensate of interest (system) and a reference condensate is established. As is known, this can be achieved by allowing for Josephson-type coupling between the two condensates. Because of such a process, the system enters a state characterized by some dispersion of the number of particles, $\sigma(N) \propto N^{1/2-\beta}$,

and some distribution of the relative phase peaked around θ_0. Alternatively, this can be achieved by performing a beatnote measurement [8,14], or by carrying out nonlocal light scattering between the condensates that undergo particle exchange [18,19]; we shall not discuss, however, these alternative methods in this Letter. Assuming that the contact between the condensates is interrupted, and that both condensates are trapped in the potentials of the same form, the relative phase will collapse and revive after times:

$$\tau_c \propto N^{\frac{D-\eta}{2(D+\eta)}+\beta}, \qquad \tau_r \propto N^{\frac{D}{(D+\eta)}}. \tag{11}$$

If the phase measurement is performed again before time τ_c, the same value of θ_0 will be recovered. If the measurement is performed after τ_c, but before τ_r, it will lead to the establishment of a fixed, but randomly picked value of θ. Finally, if the measurement is performed within the time interval $[n\tau_r - \tau_c, n\tau_r + \tau_c]$, with $n = 0, 1, \ldots$, the same θ_0 will be measured.

The principal result of our Letter is the scaling law of τ_c and τ_r with N as given by Eq. (11): If the initial atom number fluctuations are normal ($\beta = 0$), the standard mean-field result where both $\tau_c \to \infty$ and $\tau_r \to \infty$ holds only when $D > \eta$. If the opposite is true (i.e., $D < \eta$), then increasing the number of atoms in the trap results in shorter collapse and longer revival times as $N \to \infty$. Any coherence prepared in the system will collapse in an arbitrarily short time and will never revive.

In an actual experiment, however, the exponent β is established by the interplay of Josephson coupling and condensate interactions. We illustrate this point using a toy model for the two symmetric condensates [9]. For the moment, we leave a more technical discussion of the general asymmetric case in the framework of quantum field theory aside. We consider two condensates A and B interacting via Josephson coupling that allow for coherent exchange of particles,

$$\mathcal{H} = \frac{\tilde{u}}{2}(\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} + \hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b}) - \mu(\hat{a}^\dagger\hat{a} + b^\dagger\hat{b}) - \lambda(\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\hat{a}), \tag{12}$$

where $\hat{a}$, $\hat{a}^\dagger$, $\hat{b}$, $\hat{b}^\dagger$ are the annihilation and creation operators of the A and B atoms, respectively. The coupling constant λ is assumed to be positive. Such a coupling at $T = 0$ selects the relative phase. In order to estimate the collapse time of the relative phase τ_c after the interruption of the coupling, we have to evaluate dispersion of the A-atom number $\sigma_A(N)$ in the ground state of Eq. (12). This can be done either using the BH approach, or directly, from the Hamiltonian. In the latter case, we represent the exact ground state for a fixed total number of atoms $2N$ in a form $|G\rangle = \sum_{n=0} c_n|n\rangle_A|2N - n\rangle_B$, where n is the number of A atoms. The amplitudes c_n, which can be chosen to be real, are strongly peaked at $n \simeq N$. By shifting $n = N + k$, making an ansatz

$$|c_n|^2 \propto \exp\left(-\frac{(n-N)^2}{2\sigma_A^2(N)}\right) = \exp\left(-\frac{k^2}{2\sigma_A^2(N)}\right), \tag{13}$$

and expanding the Schrödinger equation for small k's, we obtain

$$\sigma_A^2(N) = N\sqrt{\lambda/[4(N\tilde{u} + \lambda)]}. \tag{14}$$

The collapse time can be now associated with the short time decay of the correlation

$$\langle G|\hat{b}^\dagger(t)\hat{a}(t)|G\rangle = \sum_{n=0}\sqrt{(n+1)(2N-n)} \times c_n c_{n+1}\exp[i(E_n - E_{n+1})t], \tag{15}$$

evaluated after the switch-off of the contact, i.e., in the absence of the Josephson interaction. In such a case $E_n = (\tilde{u}/2)[n(n-1) + (2N-n)(2N-n-1)] - 2\mu N$. Making use of Eq. (13), and replacing the discrete sum over n by an integral, we get

$$\langle G|\hat{b}^\dagger(t)\hat{a}(t)|G\rangle \propto N\exp[-2\tilde{u}^2\sigma_A^2(N)t^2] \tag{16}$$

so that $\tau_c = 1/[2\tilde{u}\sigma_A(N)]$. From Eq. (14) we obtain $\beta = \eta/[4(\eta + D)]$ in the limit $N \to \infty$ and $\lambda = \text{const}$. Since β is positive, the atom number fluctuations are sub-Poissonian, whereas the initial phase distribution is broader than in the case of Poissonian fluctuations. Physically it means that when λ does not vary with N, Josephson coupling becomes relatively weak compared to the internal nonlinear interactions ($N\tilde{u}$), and the precision of fixing of the relative phase decreases.

In fact, if $\lambda \propto N^\zeta$, with $\zeta_{\text{crit}} = \eta/(\eta + D)$, we get $\beta = \eta/[4(\eta + D)] - \zeta/4 > 0$, and the corresponding dependence of the collapse time on the atom number

$$\tau_c \propto N^{\frac{D-\eta/2}{2(\eta+D)}-\zeta/4}. \tag{17}$$

For $\zeta = 0$, the collapse time is the longest. The critical value of η is then $2D$, i.e., inhibition of coherence occurs for $\eta > 2D$.

For the values of ζ greater than the critical value ζ_{crit}, the scaling behavior changes completely, since as clearly seen from Eq. (14) β is strictly equal to zero in this case, independent of the value of ζ. We deal here with the critical behavior with respect to the exponent ζ. Below the critical value ζ_{crit}, the Josephson interactions may be termed as *weak*, and their effect on establishing the phase varies continuously with ζ. Above the critical value, the Josephson interactions become *strong*, and their effect on establishing the phase saturates. The atom number fluctuations remain sub-Poissonian, but become normal with $\sigma_A^2(N) = N/2$, whereas the phase is established with the corresponding maximal possible precision. The collapse time is given in this limit as in Eq. (11) by $\tau_c \propto N^{D-\eta/2(\eta+D)}$, and the critical value of η is D.

The predicted scaling laws of collapse and revival times could in principle be measured in lower dimensional trap

TABLE I. Exponents of the atom number N dependence of various quantities for a condensate in the D-dimensional trap with $V_t(r) = ar^\eta$ and with normal atom number fluctuations.

	r_0	μ	$\tilde{u}$	τ_r	τ_c
N's exponent	$\frac{1}{\eta + D}$	$-\frac{D}{\eta + D}$	$\frac{\eta}{\eta + D}$	$\frac{D}{\eta + D}$	$-\frac{D - \eta}{2(\eta + D)}$

structures. In the discussion presented above, we have assumed for simplicity that the measurements of the relative phase between the two condensates is an abrupt strong measurement and that in between the measurements the two condensates are uncoupled. Another possible scheme would involve a weak Josephson-type coupling that would make a continuous phase measurement [18]. The existence of collapse and revivals in the presence of Josephson coupling has been shown numerically in Ref. [9]; we have not, however, examined the dependence of atom-number scaling laws under a continuous coupling scheme. Finally, we point out that it is in principle possible to use *weak measurements* of the expectation value of the relative phase without having actual transfer of atoms [20].

In summary, we have analyzed the dependence of the collapse and revival times of many-atom coherence in a trapped BEC on the trap potential, dimensionality of the system, and atom number dispersion. We have shown that in a class of experimentally relevant systems, the collapse time tends to zero, as the number of atoms increases. This sets fundamental limits on the existence of a coherent BEC. Inhibition of coherence can be observed in an experiment in which (i) the relative phase between the two condensates is established via Josephson interactions, (ii) the contact is then interrupted, and (iii) the relative phase is measured after some delay time. We have discussed in detail what kind of initial states can be prepared in phase (i) of the experiment, and analyzed a critical behavior controlled by the scaling exponent of the Josephson coupling with N. As this exponent grows, the atom number dispersion increases gradually, until the critical value of the exponent is reached, above which the atom number dispersion becomes strictly constant, and equal to $\sqrt{N/2}$. A summary of our results is presented in Tables I and II.

TABLE II. Exponents of the atom number N dependence of the collapse time and the atom number dispersion $\sigma(N) \propto N^{1/2-\beta}$ for a condensate coupled via a Josephson-type coupling $\lambda(N) \propto N^\zeta$ to another equivalent one; $\zeta_{\text{crit}} = \frac{\eta}{\eta+D}$.

	$\zeta < \zeta_{\text{crit}}$	$\zeta \geq \zeta_{\text{crit}}$
β	$\frac{\eta}{4(\eta + D)} - \frac{\zeta}{4}$	0
N's exponent for τ_c	$\frac{D - \eta/2}{2(\eta + D)} - \frac{\zeta}{4}$	$\frac{D - \eta}{2(\eta + D)}$

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