Photon Landau Damping

R. Bingham,* J. T. Mendonça,† and J. M. Dawson‡
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX England
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We present a general description describing the nonlinear dispersion relation of electron plasma waves or electromagnetic waves propagating in a plasma with intense radiation or turbulence. We find that a spectrum of photons behaves similarly to particles and can Landau damp electron plasma waves. We derive a linear Landau damping coefficient and a quasilinear equation for this process. [S0031-9007(96)02073-X]

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The dispersion relation of electron plasma waves in an unmagnetized plasma in the presence of an intense radiation field (or turbulence) is obtained using a kinetic equation of the Klimontovich type for the photons (or plasmons). A fluid description is used for the electron plasma waves and thus normal Landau damping of the electron plasma wave is absent. The photon "Landau damping" will still be present since the kinetic effects of the photons are retained in the Klimontovich description. The process we describe is equivalent to photon acceleration described by Wilks *et al.* [1], who did numerical calculations of photon acceleration in a large amplitude plasma wave. More recent analytical and numerical studies of the same process were based on single photon trajectories [2].

In this Letter we present an analytical description of photon acceleration (or deceleration) based on a kinetic description of the photons, and a fluid description of the plasma wave thus enabling collective photon effects to be described.

We study the dispersion relation of electron plasma waves in an unmagnetized plasma, in the presence of radiation (or turbulence). If the spectral width of this radiation is larger than the characteristic time scales of the dominant wave processes, the wave phase effects can be neglected and a photon description of the radiation field can be assumed [3].

Using this dispersion relation we can show that a new kind of kinetic instability will eventually occur, where the source of free energy is not the population of plasma electrons, but the radiation field itself. This can be also seen as a new kind of modulational instability [4]. Apart from the linear version of the plasma wave dispersion relation, the quasilinear saturation mechanisms will also be discussed.

In equilibrium, we have a mean electron density n_0, and some distribution of photons (or plasmons) N_{k0}. We then perturb the plasma and the photon spectrum:

$$n = n_0 + \tilde{n} \qquad N_k = N_{k0} + \tilde{N}_k . \tag{1}$$

From the electron fluid equations and Poisson's equation we derive the evolution equation for $\tilde{n}$:

$$\frac{\partial^2 \tilde{n}}{\partial t^2} + \omega_{p0}^2 \tilde{n} - v_{Te}^2 \nabla^2 \tilde{n} = -n_0 \nabla \cdot \left\langle \frac{\partial \vec{v}}{\partial t} \right\rangle . \tag{2}$$

The term on the right-hand side is due to ponderomotive force effects due to the electromagnetic waves, and v_{Te}^2 is the average random velocity (thermal) of the electrons. We also have from the momentum equation the expression for the ponderomotive force

$$\left\langle \frac{\partial \vec{v}}{\partial t} \right\rangle = -\frac{1}{2} \nabla |v_{\mathrm{EM}}|^2 = -\frac{1}{2}\left(\frac{e}{m}\right)^2 \nabla \int \frac{|E_k|^2}{\omega_k^2} \frac{d\vec{k}}{(2\pi)^3} . \tag{3}$$

By definition, the number of photons is

$$N_k = \frac{\epsilon_0}{4}\left(\frac{\partial D}{\partial \omega}\right)_{\omega_k} |E_k|^2 , \tag{4}$$

where $D \equiv D(\omega, k) = 0$ is the photon dispersion relation. Using Eqs. (3) and (4) in Eq. (2), we get

$$\frac{\partial^2 \tilde{n}}{\partial t^2} + \omega_{p0}^2 \tilde{n} - v_{Te}^2 \nabla^2 \tilde{n} = 2 \frac{\omega_{p0}}{n} \nabla^2 \int \frac{\tilde{N}_k}{\omega_k^2 (\frac{\partial D}{\partial \omega})_{\omega_k}} \times \frac{d\vec{k}}{(2\pi)^3} . \tag{5}$$

This equation is now coupled with the kinetic equation for the photons, which gives $\tilde{N}_k$ as a function of N_{k0} and $\tilde{n}$:

$$\frac{\partial \tilde{N}_k}{\partial t} + \vec{v} \cdot \frac{\partial \tilde{N}_k}{\partial \vec{r}} + \vec{F} \cdot \frac{\partial N_{k0}}{\partial \vec{k}} = 0 . \tag{6}$$

The equivalent force F acting on the photons is determined by $\vec{F} = -\nabla \omega_k$, where

$$\omega_k = \begin{cases} \sqrt{\omega_p^2 + k^2 v_{Te}^2}, & \text{for plasmons}, \\ \sqrt{\omega_p^2 + k^2 c^2}, & \text{for photons}, \end{cases} \tag{7}$$

In this Letter we will consider only the kinetic description for photons and consider the plasma wave only from a fluid approximation. The expression for the force F in Eq. (6) is due to the gradient of the plasma wave frequency.

In both cases we can write

$$F = -\frac{1}{2\omega_k}\frac{e^2}{\epsilon_0 m}\nabla\tilde{n}\,. \tag{8}$$

Let us assume the perturbations $\tilde{n}$ and $\tilde{N}_k$ are written as

$$\tilde{n}(\vec{r},t) = \tilde{n}(t)e^{i\vec{k}\cdot\vec{r}}; \qquad \tilde{N}_{k'}(\vec{r},t) = \tilde{N}_{k'}(t)e^{i\vec{k}\cdot\vec{r}}. \tag{9}$$

From Eqs. (5) and (6) we obtain

$$\begin{cases} \frac{\partial^2\tilde{n}}{\partial t^2} + (\omega_{p0}^2 + k^2 v_{Te}^2)\tilde{n} = -2k^2\frac{\omega_{p0}^2}{m}\int \frac{\tilde{N}_{k'}}{\omega_{k'}^2(\partial D/\partial\omega)_{\omega_{k'}}}\frac{d\vec{k}'}{(2\pi)^3}\,, & \text{(10a)} \\ \frac{\partial\tilde{N}_{k'}}{\partial t} + i\vec{k}\cdot\vec{v}(\vec{k}')\tilde{N}_{k'} = \frac{i}{2\omega_{k'}}\frac{e^2}{\epsilon_0 m}\tilde{n}\vec{k}\cdot\frac{\partial\tilde{N}_{k_0'}}{\partial\vec{k}'}\,. & \text{(10b)} \end{cases}$$

Now we follow the usual Landau approach and use a Laplace transformation in time. From Eq. (10b) we then get

$$\tilde{N}_{k'} = -\frac{\tilde{n}}{2n_0}\frac{\omega_{p0}^2}{\omega_{k'}}\frac{\vec{k}(\partial N_{k_0'}/\partial\vec{k}')}{\omega - \vec{k}\cdot\vec{v}(\vec{k}')}\,, \tag{11}$$

where ω is complex, and from Eq. (10a)

$$(\omega_{p0}^2 + k^2 v_{Te}^2 - \omega^2)\tilde{n} = \frac{k^2}{m}\omega_{p0}^r\frac{\tilde{n}}{n_0}\int\frac{1}{\omega_{k'}^3(\partial D/\partial\omega)_{\omega_{k'}}} \times \frac{(\vec{k}\cdot\partial N_{k_0'}/\partial\vec{k}')}{\omega - \vec{k}\cdot\vec{v}(\vec{k}')}\frac{d\vec{k}'}{(2\pi)^3}\,. \tag{12}$$

We can use $(\frac{\partial D}{\partial\omega})_{\omega_{k'}} \simeq \frac{2}{\omega_{k'}}$ which leads to

$$\omega^2 = k^2 v_{Te}^2 + \omega_{p0}^2\left\{1 - \frac{2k^2}{mn_0}\omega_{p0}^2\int\frac{1}{\omega_{k'}^2} \times \frac{(\vec{k}\cdot\partial N_{k_0'}/\partial\vec{k}')}{\omega - \vec{k}\cdot\vec{v}(\vec{k}')}\frac{d\vec{k}'}{(2\pi)^3}\right\}. \tag{13}$$

In order to develop this integral we consider the parallel and the perpendicular photon motion:

$$\begin{cases} \vec{k}' = p\frac{\vec{k}}{k} + \vec{k}_\perp'\,, \\ \vec{v}(\vec{k}') = u(p,\vec{k}_\perp')\frac{\vec{k}}{k} + \vec{v}_\perp\,. \end{cases} \tag{14}$$

The integral in Eq. (13) becomes

$$\int\frac{1}{\omega_{k'}^2}\frac{(\vec{k}\cdot\partial N_{k_0'}/\partial\vec{k}')}{[\omega - \vec{k}\cdot\vec{v}(\vec{k}')]}\frac{d\vec{k}'}{(2\pi)^3} = -\int\frac{d\vec{k}'_\perp}{(2\pi)^3} \times \int\frac{\partial N_{k_0'}/\partial p}{[u - \omega/k]} \times \frac{dp}{\omega_{k'}^2}\,. \tag{15}$$

Developing the parallel photon velocity u around the resonant value $u(p_0) = \omega/k$:

$$u(p,\vec{k}'_\perp) \simeq u(p_0,\vec{k}'_\perp) + (p - p_0)\left(\frac{\partial u}{\partial p}\right)_{p_0}\,,$$

we get

$$\int\frac{\partial N_{k_0'}/\partial p}{(u - \omega/k)}\frac{dp}{\omega_{k'}^2} \simeq \frac{1}{(\partial u/\partial p)_{p_0}}\int\frac{\partial N_{k_0'}/\partial p}{(p - p_0)}\frac{dp}{\omega_{k'}^2}\,. \tag{16}$$

This last integral takes the standard form

$$I(z) \equiv \int\frac{h(z)}{z - z_0}dz = P\oint\frac{h(z)}{z - z_0}dz + i\pi h(z_0)\,,$$

which means that

$$\int\frac{1}{\omega_{k'}^2}\frac{\vec{k}\cdot\partial N_{k_0'}/\partial\vec{k}'}{\omega - \vec{k}\cdot\vec{v}(\vec{k}')}\frac{d\vec{k}'}{(2\pi)^3} = -\int\frac{d\vec{k}'_\perp}{(2\pi)^3}\frac{1}{(\omega_{k'}^2)_{p_0}}\frac{1}{(\partial u/\partial p)_{p_0}}\left\{P\oint\frac{\partial N_{k_0'}/\partial p}{p - p_0}dp + i\pi\left(\frac{\partial N_{k_0'}}{\partial p}\right)_{p_0}\right\}. \tag{17}$$

Replacing this result in Eq. (13) and using $\omega = \omega_r + i\gamma$, we get from the real part of the resulting equation the following dispersion relation:

$$\omega_r^2 = \omega_{p0}^2 + k^2 v_{Te}^2 + \frac{2k^2}{mn_0}\omega_{p0}^4\int\frac{d\vec{k}'_\perp}{(2\pi)^3}(\omega_{k'}^{-1})_{p0} \times \int\frac{(\partial N_{k_0'}/\partial p)}{u - \omega/k}dp\,. \tag{18}$$

The imaginary part gives the photon Landau damping

$$\gamma = \pi\frac{k^2\omega_{p0}^3}{mn_0}\left(\frac{\partial G_p}{\partial p}\right)_{p_0}\,, \tag{19}$$

where G_p is a kind of reduced distribution function for photons:

$$G_p = \int\frac{1}{(\omega_{k'}^2)_{p0}}\frac{N_{k_0'}}{(\partial u/\partial p)_{p_0}}\frac{d\vec{k}'_\perp}{(2\pi)^3}\,. \tag{20}$$

From this result some conclusions can be drawn.

Plasma waves with high (relativistic) phase velocities with negligible electron Landau damping can still be attenuated by photon Landau damping. For thermal radiation at a temperature T we should use in Eqs. (19), (20) the Planck distribution:

$$N_{k0} = \frac{\omega_k^2}{\pi^2 c^3}\frac{1}{\exp(\hbar\omega_k/k_B T) - 1}\,, \tag{21}$$

where $\omega_k = \sqrt{\omega_{p0}^2 + k^2c^2}$.

If a photon beam is used instead of this equilibrium distribution, relativistic plasma waves can be destabilized by inverse photon Landau damping.

Let us now study the quasilinear saturation of a photon (or plasmon) beam by this mechanism of photon Landau damping. We start with the evolution equation for N_{k_0}, which for uniform turbulence $(\partial N_{k_0'}/\partial\vec{r} = 0)$ can be written as

$$\frac{\partial N_{k_0'}}{\partial t} = -\left\langle \vec{F}_{k'} \cdot \frac{\partial \tilde{N}_{k'}}{\partial \vec{k}'} \right\rangle = -\int \vec{F}^{*}_{k'} \cdot \frac{\partial \tilde{N}_{k'}}{\partial \vec{k}'} \frac{d\vec{k}}{(2\pi)^3} . \tag{22}$$

Using Eq. (8) for $\vec{F}$ and Eq. (11) for $\tilde{N}_k$ we can derive from this equation a quasilinear diffusion equation for photons:

$$\frac{\partial N_{k_0'}}{\partial t} = \frac{\partial}{\partial \vec{k}'} \cdot \overline{\overline{D}}(\vec{k}', t) \cdot \frac{\partial}{\partial \vec{k}'} N_{k_0'} , \tag{23}$$

where the diffusion coefficient is

$$\overline{\overline{D}}(\vec{k}', t) = -\frac{i}{4n_0^2} \frac{\omega_{p0}^2}{\omega_{k'}^2} \int \vec{k}\vec{k} \frac{|\tilde{n}_k|^2}{\omega - \vec{k} \cdot \vec{v}(\vec{k}')} \frac{d\vec{k}}{(2\pi)^3} . \tag{24}$$

These equations are coupled with the equation for the evolution of the intensity of the electron plasma waves:

$$\frac{\partial}{\partial t} |\tilde{n}_k|^2 = 2\gamma_k |\tilde{n}_k|^2, \tag{25}$$

where γ_k is determined by (19).

With Eqs. (19) and (23)–(25) we can study the quasilinear relaxation of a photon beam. The inverse problem of photon acceleration by a spectrum of electron plasma waves $|\tilde{n}_k|^2$ can also be studied.

In conclusion we have obtained the Landau damping coefficient of plasma waves by photons and derived a quasilinear treatment of a photon spectrum interacting with plasma waves.

*Permanent address: Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX England.

†Permanent address: Instituto Superior Técnico, University of Lisbon, 1096 Lisbon Codex, Portugal.

‡Permanent address: Department of Physics, University of California, Los Angeles, California 90024-1547.

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