Inelastic Quantum Magnetotransport in a Highly Correlated Two-Dimensional Electron Liquid

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We present new low temperature magnetoconductivity (σ_{xx}) measurements for the two-dimensional electron system on a liquid helium surface and the theoretical concept of the inelastic quantum magnetotransport that explains the data. It is shown that predictions for σ_{xx} made in the elastic approximation are valid only within a limited temperature range which narrows with the magnetic field increase. The temperature and magnetic field dependencies of σ_{xx} observed can be perfectly described as the interplay of the inelastic and many-electron effects. [S0031-9007(97)02776-2]

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At low temperatures, $T \leq 0.5$ K, the surface electrons (SE) on superfluid helium form a unique two-dimensional (2D) electron liquid of which the mean potential energy is usually a hundred times larger than the mean kinetic energy. In the presence of a strong magnetic field ($\hbar\omega_c \gg k_B T$, ω_c is the cyclotron frequency) oriented normally the system can be used for studying quantum transport phenomena in highly correlated 2D electron liquids.

The scattering of SE arises from helium vapor atoms and capillary wave quanta (ripplons). For vapor atom scattering ($T > 1$ K) the magnetoconductivity is well understood in terms of the extended self-consistent Born approximation (SCBA) [1,2]. The deviations from the theory observed at weak magnetic fields were explained as many-electron effects [3]. As for the electron-ripplon scattering, which is analogous to the electron-phonon scattering in solids, the quantum magnetotransport still challenges both theory and experiment.

The singular nature of a 2D electron system in a magnetic field B makes the usual Born approximation for the longitudinal conductivity unsatisfactory: It results in $\sigma_{xx} = \infty$ for the elastic scattering from impurities and in $\sigma_{xx} = 0$ for the inelastic scattering within the ground level. The generally accepted way of treating the system developed mostly for elastic scattering is the SCBA theory [4] which makes σ_{xx} finite by taking into account the Landau level broadening Γ in a self-consistent way.

In semiconductor 2D electron systems, the acoustic phonon scattering is usually treated as quasielastic [5], since the typical phonon energies $\hbar\omega_q$ involved are much smaller than Γ. The single-electron [6] and many-electron [7] theories of the low temperature (LT) quantum magnetotransport of SE organized in a different way actually treated the electron-ripplon scattering as quasielastic as well ($\hbar\omega_q \ll \Gamma$). In this approximation the wave vectors q involved are limited by the magnetic length [$q \leq 1/\ell$, here $\ell = (\hbar c/eB)^{1/2}$]. If the inelastic effect becomes important, it additionally restricts q ($\hbar\omega_q \leq \Gamma$), reducing both Γ and σ_{xx}. For SE on helium, as against the semiconductor electron-phonon systems, the inelastic parameter $\delta = \hbar\omega_q/\Gamma$ increases with the magnetic field due to unusual ripplon dispersion $\omega_q \propto q^{3/2} \propto B^{3/4}$ and the quantum magnetotransport becomes of pure inelastic nature.

The temperature dependence of the longitudinal conductivity σ_{xx} is the most decisive for the electron-ripplon scattering. Theories [6,7] give $\sigma_{xx} \propto 1/\sqrt{T}$. We will show that this behavior changes drastically at LT and strong magnetic fields due to the inelastic effect.

First, measurements of σ_{xx} [8,9] show that the conductivity had a minimum at $T \approx 1$ K and increased slowly with lowering T down to 0.5 K. Still, it was necessary to reduce Γ substantially to explain the data. From the data performed by another experimental group [10], it is clearly seen that the resistivity ρ_{xx}, and consequently σ_{xx}, is decreasing with lowering T, at least down to 0.4 K. This T dependence is *opposite* to the prediction of the Dykman-Khazan (D-K) theory [7] applied there to explain the data. Recent data and qualitative analyses of the Einstein relation reported in Ref. [11] were done for fixed temperatures $T > 0.6$ K, where the decrease of the vapor atom density dominates the dependence $\sigma_{xx}(T)$, and the elastic concept is approximately valid.

Previous experimental studies of the quantum magnetotransport of SE on helium were based on measuring the electron response to the ac voltage. At LT and strong magnetic fields it is very difficult to avoid the excitation of low frequency edge magnetoplasmons (EMP), if small deviations from axial symmetry are present, which affects the electron response and spoils the analysis of the data. Since the experimental data [8–10] contradict each other, we propose an alternative approach for studying the quantum magnetotransport.

The EMP waves which were a hindrance in a previously used technique can be a tool for studying σ_{xx} at LT. Indeed, it can be shown in a rather general way [12,13] that damping of the EMP is proportional to the longitudinal conductivity, if the magnetic field is strong enough (usually it is valid at $B > 0.5$ T). In this limit the proportionality constant can be considered as a geometrical factor independent of B and T. Therefore at fixed electron density, n, the EMP-damping data can be used for determining temperature and magnetic field dependencies of the SE magnetoconductivity.

In this Letter we report the first magnetoconductivity data obtained from the damping coefficient of EMP and the theoretical concept of the inelastic ripplon-induced quantum magnetotransport of SE on liquid helium. Following the general idea of Fukuyama, Kuramoto, and Platzman [14], the many-electron effect is taken into account as the Coulomb correction to the single-electron density of states, Γ_C, which additionally broadened due to interactions with scatterers. In this concept, the results of the Saitoh theory [6] are reproduced, if $\Gamma_C \rightarrow 0$ and $\delta \rightarrow 0$. As for the many-electron D-K theory, we show that in terms of the Landau level broadening it corresponds to the limiting case $\Gamma \rightarrow \Gamma_C$, in which the ripplon-induced broadening Γ_r is neglected. Since Γ_r increases with B while $\Gamma_C \propto 1/\sqrt{B}$ decreases, the D-K theory cannot be used for describing the magnetoconductivity of SE in the ultraquantum limit, even if $\delta = 0$. The new magnetoconductivity data, as a function of T and B without any adjusting parameter, are in good agreement with the concept of the inelastic quantum magnetotransport presented.

To study EMP damping, we use the conventional experimental technique similar to the one described in Ref. [15]. The electrode assembly consisted of a circular disk of 20 mm in diameter and four surrounding arc-shaped outer electrodes (see the inset in Fig. 1). The total diameter of the assembly was 30 mm, which was immersed in liquid helium (1.0 mm under the surface). The resonance curve was obtained by sweeping the frequency of the ac excitation voltage, which was applied to one of the four surrounding electrodes. The magnetic field and temperature have been kept constant during the measurement of each resonance curve. The electron density was fixed to 3.5×10^7 cm^{-2}.

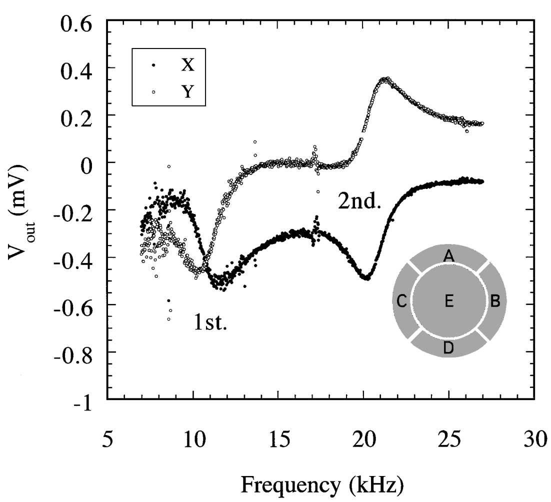

FIG. 1. The experimental signal from biphase lock-in amplifier (X: in-phase, Y: out-of-phase) as a function of frequency: $T = 0.81$ K, $n = 3.5 \times 10^7$ cm^{-2}, $B = 1.47$ T. The excitation voltage on electrode A (inset) is 10 mV (peak to peak), and the signal is registered on the electrode D.

The experimental signal of two resonance modes is shown in Fig. 1. The first mode is contaminated with a low frequency noise. Therefore the second resonance mode with a higher wave number was chosen to obtain the damping coefficient by fitting to the Lorentzian. It should be pointed out that this choice does not affect the conductivity results, since practically the same damping coefficient was found for the third and fourth resonance modes, in accordance with the basic concept of EMP waves. Above 1 K the magnetic field dependence of the damping coefficient was found to be the same as the B dependence of the previously studied longitudinal conductivity of SE. Therefore we have determined the geometrical factor which gives the relation between the damping coefficient and σ_{xx} at 1.1 K, where the electron-ripplon scattering can be neglected and the SE magnetoconductivity is well understood both experimentally and theoretically [3]. This factor was then used in the LT ripplon scattering regime. We additionally checked this factor by extrapolating our data to the zero-field limit, according to the Drude model, and found zero-field mobilities consistent with previously established results.

It should be noted that the driving amplitude was kept low enough to make sure that the electron transport is in the linear regime. The nonlinear distortion of the EMP line shape appears at $V_{\mathrm{in}} > 20$ mV (peak to peak).

In strong magnetic fields, the charge density oscillations of the EMP wave are smoothed within the density profile strip at the edge. It might be concluded that the many-electron effect, which is important for weak magnetic fields, would be reduced or averaged over the density profile. Still, the conductivity data, which will be discussed later, show the many-electron effect of the "bulk" area. We assume that the maximum of the dissipation is rather in the bulk area, since the perpendicular current is zero at the edge [$j_x(x \rightarrow 0) = 0$], and the whole current penetrates much deeper into the electron liquid than density perturbations. The last statement follows from the fact that the current is mixed with the incompressible motion of the liquid [the general solution of the equation $\mathrm{div}(\mathbf{j}) = 0$] to satisfy boundary conditions.

For a highly correlated system such as SE on liquid helium, the conventional theoretical approaches developed for electrons with weak mutual interaction cannot be used. At the same time, the substantial simplification of the mathematical formalism appear to be possible, since the conductivity of such a system can be expressed in terms of the equilibrium dynamic structure factor $S_0(k,\omega) = N_e^{-1} \int e^{i\omega t} \langle n_{\mathbf{k}}(t) n_{-\mathbf{k}}(0) \rangle dt$ [1,2], where $n_{\mathbf{k}} = \sum_e \exp(-i\mathbf{k} \cdot \mathbf{r}_e)$; N_e is the total number of SE. In this case, the quantum magnetotransport can be described by elementary expressions for the conductivity tensor components with the field-dependent effective collision frequency $\nu(B)$. Contributions from all scattering mechanisms should be added to get $\nu(B)$. The ripplon contribution can be written as

$$\nu_r(B) = \frac{1}{8\pi m\alpha} \int_0^\infty dqqV_q^2 S_0(q, \omega_q), \tag{1}$$

where α is the surface tension and V_q is the electron-ripplon interaction.

The evaluation of $S_0(k, \omega)$ mathematically is equivalent to the evaluation of the imaginary part of the density-density correlation function widely used in semiconductor systems [5,16]. The level broadening is defined as $\Gamma_N = -2\,\mathrm{Im}\Sigma_N(E_N^*)$, where E_N^* is the central position of the Landau level and $\Sigma_N(E)$ is the electron self-energy. In the elastic approximation ($\omega \ll E$), the self-consistent equation for $\Gamma \equiv \Gamma_0$ can be solved in the way of Ref. [4]. In this case, the imaginary part of the electron Green's function $\mathrm{Im}G_N(E)$ has a semielliptic shape, with the level broadening $\Gamma = \sqrt{\Gamma_a^2 + \Gamma_r^2}$; here Γ_a and Γ_r are the Landau level broadening, induced by the interaction with vapor atoms and ripplons separately (Γ_r is twice as much as the result of the qualitative analysis of Ref. [17]).

The situation is much more complicated if $\omega \sim E$. Still, $\mathrm{Im}G_N(E)$ is assumed to be of a sharp semielliptic shape. In the ultra quantum limit ($N = 0$) the final equation for determining the level broadening Γ can be written as

$$\Gamma^2 = \Gamma_a^2 + \Gamma_*^2 \int_0^{(\Gamma/\hbar\omega_0)^{4/3}} U^2(x) e^{-x} \times \sqrt{1 - \left(\frac{\hbar\omega_0}{\Gamma}\right)^2 x^{3/2}}\, x^{-1} dx . \tag{2}$$

Here we introduced the notations,

$$U(x) = xw\left(\frac{x}{2\gamma^2\ell^2}\right) + \frac{eE_\perp \ell^2}{\Lambda_0}, \qquad \Lambda_0 = \frac{e^2(\epsilon - 1)}{4(\epsilon + 1)},$$
$$w(y) = -\frac{1}{1-y} + \frac{0.5}{(1-y)^{3/2}} \ln \frac{(1 + \sqrt{1-y})^2}{y}, \tag{3}$$
$$\omega_0^2 = \alpha(\sqrt{2}/\ell)^3/\rho, \qquad \Gamma_*^2 = \Lambda_0^2 k_B T/\pi\alpha\ell^4,$$

where ρ is the liquid helium mass density, ϵ is the dielectric constant, and γ is the parameter of the SE wave function $\langle 1 \mid z\rangle \propto z\exp(-\gamma z)$ which depends slightly on the holding electric field $E_\perp$.

In Eq. (2), the term proportional to $E_\perp^2$ has a logarithmic divergency for small q which should be cutoff at wave vectors $q \simeq \sqrt{n}$, where many-electron effects screen the electron-ripplon interaction. Still, in the LT limit that we are considering, only the small electron densities are important and this term is negligible at $B \geq 1$ T.

In the many-electron theory, the Coulomb-induced broadening Γ_C squared should be added to the right-hand side of Eq. (2). According to [14], in this treatment an electron feels the fluctuation field of other electrons as a random potential, since the density fluctuation has spectral intensities at very low frequencies. The fluctuation field, $E^* \simeq 0.84(4\pi k_B T n^{3/2}/\epsilon)^{1/2}$, entered the broadening $\Gamma_C = eE^*\ell$ calculated in Refs. [7,18].

To complete the quantum magnetotransport theory we should write the equation for the dynamic structure factor in the ultra quantum limit

$$S_0(q, \omega) = \frac{32\hbar}{3\pi\Gamma} \exp\left(-\frac{q^2\ell^2}{2}\right)\chi\left(\frac{\hbar\omega}{\Gamma}\right), \tag{4}$$

where

$$\chi(y) = \frac{3}{4}\int_{-1}^{1-y} \sqrt{1 - x^2}\,\sqrt{1 - (x + y)^2}\, dx .$$

The field dependence of σ_{xx}, which follows from Eq. (2) and equations for the conductivity, is shown in Fig. 2. The experimental data found from the damping of EMP, which are nearly field independent at $T = 0.3$ K, behave in accordance with the many-electron curve (solid). This curve describes the transition from the dashed curve 2, represented in the case of pure Coulomb broadening ($\Gamma = \Gamma_C$; the analog of the D-K theory), to the single-electron curve 1 ($\Gamma = \Gamma_r$), with the increase of the magnetic field. The dashed curve 2 is approximately 2.6 times higher than the result of D-K theory (dotted). This numerical factor is very important, since it is impossible to fit the D-K theory to the data just by replacing Γ_C by $\Gamma = \sqrt{\Gamma_{r,0}^2 + \Gamma_C^2}$ in the final conductivity equation (it would reduce $\sigma_{xx} \propto 1/\Gamma_C$ away from the data). Therefore we conclude that the way of treating the conductivity of highly correlated electron liquids proposed here is more adequate.

The temperature dependence of σ_{xx} which is crucial for the electron-ripplon scattering is shown in Fig. 3 for two

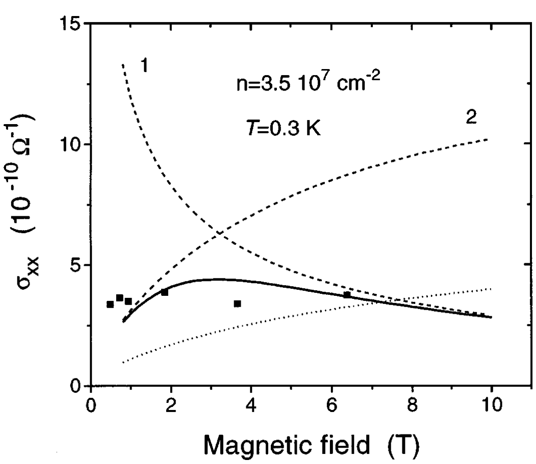

FIG. 2. The magnetoconductivity of SE as a function of the magnetic field for a fixed temperature: data (solid squares), many-electron theory presented (solid line), single-electron approximation (dashed line, 1), D-K many-electron theory (dotted line), and approximation $\Gamma = \Gamma_C$ (dashed line, 2).

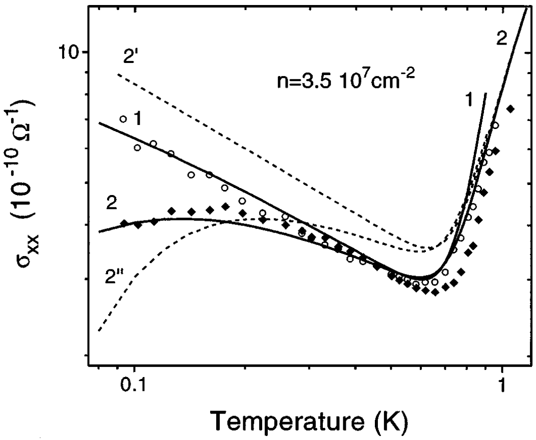

FIG. 3. Temperature dependence of the SE magnetoconductivity for two values of the magnetic field: $B = 1.84$ T [line 1 and data (open circles)]; $B = 6.4$ T [solid line 2—the many-electron theory, dashed line $2'$—the elastic many-electron theory, dashed line $2''$—the single-electron approximation, and data (solid diamonds)]. The many-electron theory presented (solid lines) has no fitting parameters.

values of B. In the LT regime, the many-electron curves (solid, 1, and 2) behave in different ways. The weak-field curve (1) is approximately linear in accordance with the elastic concept. The curve (solid, 2) plotted for the strong magnetic field ($B = 6.4$ T) is far away from the result of the elastic approximation (dashed curve, $2'$), showing different T dependence. The decrease of σ_{xx} is due to the narrowing of $\Gamma(T)$ beyond the quasielastic limit. The Coulomb correction to the level broadening, Γ_C, is still important, since it reduces the huge inelastic effect that is seen for the single-electron theory (dashed curve, $2''$). It should be noted that the contribution from the higher order two-ripplon scattering processes independent of level broadening decreases with lowering T; therefore it can be neglected here until $\sigma_{xx}(T)$ becomes much smaller than the minimum value at $T \simeq 0.65$ K.

The result shown in Fig. 3 proves that, at strong fields, the elastic many-electron theory cannot be used in the whole temperature range, where the electron-ripplon scattering dominates ($T < 0.7$ K). The T dependencies of the experimental data found from the damping of the EMP follow amazingly our many-electron curves. The Wigner solid transition, which slightly touched our data at $T = 0.13$, does not affect both sets of data ($B = 1.84$ T, 6.4 T), and therefore cannot be a cause of the deviations from the straight line started at $T = 0.7$ K for the strong-field data.

In conclusion, we have shown that the EMP-damping method can be used for determining the SE magnetoconductivity in the LT range, where the conventional method based on the capacitive coupling techniques fails. The theoretical concept of the inelastic quantum magnetotransport presented allows one to examine the validity of the previously used approaches, and to describe the intriguing temperature and field dependencies of the magnetoconductivity observed. Perfect agreement achieved between theory and experiment in the wide range of temperatures and magnetic fields provides important clues about the behavior of highly correlated 2D electron liquids in a quantizing magnetic field.

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