Frequency Entrainment in Optically Injected Semiconductor Lasers

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We theoretically predict and experimentally demonstrate the phenomenon of frequency entrainment in optically injected semiconductor lasers. The laser exhibits nonlinear subharmonic and ultraharmonic resonances for appropriate values of its frequency detuning from the externally injected signal. [S0031-9007(97)02786-5]

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The nonlinear response of semiconductor laser diodes subject to an external perturbation has long been the subject of intense investigation. They exhibit a wide variety of rich and complex dynamics which are interesting both from the physics and applications point of view. Component devices based on the induced laser diode nonlinearities are expected to play a key role in the design of future all-optical communication networks which transcend the speed and bandwidth limitations of conventional electronic systems. All-optical synchronization is indispensable in constructing such a transparent optical network [1]. A timing extraction scheme is required which recovers the timing information from an incoming optical data stream without the intermediate electronic stages. This approach involves the use of a laser diode that becomes synchronized to an injected optical signal containing the timing information.

In contrast to other lasers used in laboratories, gas [2] and solid-state [3] lasers, systematic experiments on the possible responses of an injected semiconductor laser system are rather difficult. This results from the fact that intensity fluctuations in semiconductor lasers occur at very fast time scales, typically several gigahertz. As a consequence, direct time-domain measurements are restricted to short time series which are not appropriate for systematic quantitative analysis. Most of our understanding of the induced instabilities is obtained from Fourier spectra [4]. In order to interpret them correctly, separate numerical studies of the laser model are needed which may identify specific bifurcations into periodic, and even more complicated, regimes. Still, a combined experimental and theoretical analysis of the response for large domains of the laser parameters is lacking.

In this Letter, we report the first experimental demonstration of frequency entrainment in a semiconductor laser subject to monochromatic optical injection. In addition, we analyze the experimental spectra by using approximate solutions of the laser equations without relying on numerical bifurcation studies. The slave laser becomes synchronized to the optically injected signal and exhibits nonlinear subharmonic or ultraharmonic resonances for values of the frequency detuning in the neighborhood of an integral multiple or submultiple, respectively, of the free-running relaxation oscillation frequency.

The single mode rate equations describing the operation of an optically injected semiconductor laser may be written in dimensionless form [5,6] as follows:

$$\dot{E} = (1 - ib)NE + \eta e^{-i\Omega s}, \tag{1}$$

$$T\dot{N} = P - N - (1 + 2N)|E|^2, \tag{2}$$

where E is the normalized complex electric field of the slave laser, N is the normalized excess carrier density above threshold, b ($\approx$5) is the linewidth enhancement factor, η is proportional to the amplitude of the externally injected field, $\Omega = \omega_m - \omega_s$ is the angular frequency detuning between the master and slave lasers, $T = \tau_s/\tau_p$ ($\approx 10^3$) is the ratio of the carrier lifetime, τ_s, to the photon lifetime, τ_p, $P \sim J/J_{th} - 1$ is the pumping above threshold, and the time s is normalized to τ_p.

The rate equations show excellent agreement with extensive experimental measurements, in various regimes of frequency detuning and injection strength [7]. Unfortunately, their direct numerical integration is a laborious task. Semiconductor laser parameter values vary considerably; therefore, a very systematic numerical investigation has to be performed for each particular case. Continuation methods may even fail to reveal coexisting attractors with different basins of attraction [8].

The objective of our investigation is to develop an alternative tool to anticipate the laser response without resorting to direct numerical simulations and, at the same time, reduce the high dimensionality of the parameter space in the rate equations. By taking into account the large values of the parameters T and b, which occur naturally in semiconductor lasers, the rate equations (1) and (2) are reduced to the following equation for the phase Φ of the injected-laser electric field [9]

$$\Phi''' + \xi\Phi'' + \Phi' = \Lambda\cos(\Phi + \Delta t), \tag{3}$$

where $\Delta = \Omega/\omega_r$ is the normalized frequency detuning, $\Lambda = \eta b/\omega_r$ is proportional to the amplitude of the injected field, and $\xi = \omega_r[(1 + 2P)/2P]$ is the damping rate of the laser relaxation oscillations. The quantity $\omega_r = \sqrt{2P/T}$ is the normalized free-running laser relaxation oscillation frequency, and the new time variable is $t = \omega_r s$.

The phase equation, Eq. (3), is a major simplification over the rate equations (1) and (2). It retains the differential order of the original system and the various parameters are grouped now into only three coefficients Δ, Λ, and ξ. It admits simple solutions which allow us to discuss the bifurcation possibilities and the experimentally relevant features of the optical spectrum. It also corrects the first order phase equation, Adler equation [10], that is frequently used in the literature on lasers subject to optical injection [11]. The phase equation has been investigated in the phase-locked regime for weak injection and zero frequency detuning [9]. That analysis captured several aspects of the numerical bifurcation diagram, namely, the fixed amplitude of the period-one solution and the period-doubling bifurcation. In our current analysis we perform a systematic investigation of the experimentally relevant domain of nonzero frequency detuning ($\Delta \neq 0$).

In the weak injection regime the nonlinear phase equation is analyzed using the method of multiple scales [12] in order to obtain an approximate solution to the laser actual response. The strength of the injection Λ is a small quantity and serves as the expansion parameter. The analysis focuses on the *phase drift* region where the slave laser is not phase locked to the injected signal. A distinct expression for the leading approximation to the phase of the electric field is obtained for different values of the frequency detuning. The results are summarized as follows:

$$\Phi = \delta_a t + (\Lambda Q_a)\sin C_a t + (\Lambda^2 W_a)\sin 2C_a t \tag{4}$$

$$= \delta_b t + (\Lambda Q_b)\sin C_b t + (\Lambda W_b)\sin(2C_b t + \phi_b) \tag{5}$$

$$= \delta_c t + (\Lambda Q_c)\sin(C_c t + B) + W_c \sin\left(\frac{C_c t}{2} + \phi_c\right). \tag{6}$$

The above equations illustrate the phase evolution in the three different regimes of the phase drift region. Equation (4) describes the *four-wave mixing* (FWM) response of the laser for arbitrary values of the frequency detuning Δ away from an integral multiple or submultiple of the free-running relaxation oscillation frequency ω_r. When the frequency detuning is in the neighborhood of ω_r/n or $n\omega_r$ (n being an integer) we observe the *nonlinear resonant* response of the system. In this regime the relaxation oscillations of the laser source are undamped and entrained by the integral multiple $n\Delta$ (*ultraharmonic* resonance) or the submultiple Δ/n (*subharmonic* resonance) of the frequency detuning. Equations (5) and (6) represent the ultraharmonic ($\Delta \approx 1/2$) and subharmonic ($\Delta \approx 2$) resonant responses, respectively [13]. The phase equation, therefore, provides a unique description of the laser response for each particular experimental condition (different Δ). In the above expressions $C_i = \Delta + \delta_i$. In Eq. (4), $Q_a^{-1} = C_a(1 - C_a^2)$ and $W_a^{-1} = 4C_a^2(1 - C_a^2)(1 - 4C_a^2)$. In Eq. (5), $Q_b = 8/3$ and $W_b^{-2} = 9[(\Delta - 1/2)^2/\Lambda^2 + (\xi/4\Lambda)^2]$. In Eq. (6), $Q_c = -J_0(W_c)/6$ and W_c is given by the implicit equation $(\Delta - 2)/\Lambda = \pm|(1 - 4W_c^{-2})J_2(W_c)|\{1 - [\xi W_c/2\Lambda J_2'(W_c)]^2\}^{1/2}$.

The expressions for the phase of the electric field can be used to compute analytically the frequency response $F(\omega)$ of the injected laser. For weak injection $F(\omega)$ is proportional to $|\int_{-\infty}^{\infty} e^{-i\omega t} e^{i\Phi} dt|^2$ which can be expanded in terms of Bessel functions. The computed frequency response may be contrasted to the experimentally captured spectra, in order to verify the validity of the theoretical predictions. Nevertheless, the formulation of Eqs. (4)–(6) allows for a straightforward comparison of the relative magnitudes of the various frequency components in each regime of the laser response.

The $\delta_i t$ term in each equation represents the injection-induced shift δ_i of the unperturbed-laser operating frequency which is of $O(\Lambda^2)$. The term $(\Lambda Q_i)\sin C_i t$ describes the laser response at the location of injection (*regenerative* signal) which is directly contributed by the externally injected field. The magnitude of the response of the regenerative signal is proportional to the injection strength Λ and inversely proportional to the value of the frequency detuning Δ [14]. At the FWM response of the laser, Eq. (4), the higher harmonic of the regenerative signal is a much weaker frequency component. It is described by $(\Lambda^2 W_a)\sin 2C_a t$ which is an $O(\Lambda^2)$ term. At the ultraharmonic resonant response, Eq. (5) ($\Delta \approx 1/2$), the higher harmonic of the regenerative signal increases considerably because of the resonance effect. It becomes a contribution of $O(\Lambda)$ denoted by the term $(\Lambda W_b)\sin(2C_b t + \phi_b)$. Compared to the ultraharmonic resonance, the subharmonic resonance, Eq. (6) ($\Delta \approx 2$), is predicted to be a much stronger phenomenon. The subharmonic component is represented by the zeroth order term $W_c \sin(\frac{1}{2}C_c t + \phi_c)$. The existence and significant difference in relative strengths of these nonlinear resonant responses have been verified experimentally.

To this end, we used two temperature and current stabilized AlGaAs index-guided semiconductor lasers [Hitachi HLP-1400] operating at 830 nm. The free-running relaxation oscillation frequency is $f_r = \omega_r/2\pi = 4.5$ GHz, corresponding to a pumping level of approximately 50% above threshold. Our setup is similar to the one described in [15]. The optical spectrum of the injected laser was measured with a Newport SR-240C scanning Fabry-Perot spectrometer, which has a free spectral range of 2 THz and a finesse of greater than 50 000 giving an optical frequency resolution of about 40 MHz.

Figure 1 depicts experimentally measured optical spectra of the injected laser. It demonstrates the phenomenon of ultraharmonic resonance as we keep the

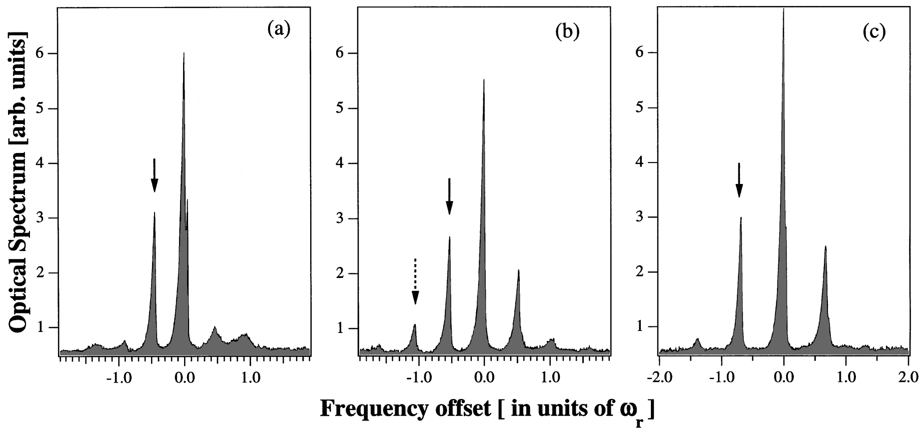

FIG. 1. Experimental optical spectra of the slave laser demonstrating the ultraharmonic resonance. The solid arrow indicates the location of injection (regenerative signal). The zero on the frequency axis coincides with the peak of the slightly shifted (due to the injection) slave laser operating frequency. (a) FWM response ($\Delta = 0.45$). (b) Ultraharmonic resonance ($\Delta = 0.52$). It is denoted by the dashed arrow. (c) The slave laser is back to the FWM response ($\Delta = 0.68$).

injection level constant and vary the detuning between the slave laser operating frequency and the injected signal. Figure 1(a) ($\Delta = 0.45$) illustrates the FWM response of the slave laser which is analytically described by Eq. (4). The solid arrow denotes the frequency response at the location of injection (regenerative signal), represented by $(\Lambda Q_a)\sin C_a t$. The weak sideband to the left indicates its higher harmonic corresponding to the $O(\Lambda^2)$ term $(\Lambda^2 W_a)\sin 2C_a t$. The coupling between the amplitude and phase fluctuations of the electric field, which is described by the linewidth enhancement factor, leads to the asymmetry of the sidebands on either side of the laser center line. Figure 1(b) ($\Delta = 0.52$) illustrates the ultraharmonic resonant response of the slave laser which is analytically formulated in Eq. (5). The frequency detuning is now in the neighborhood of $\omega_r/2$. The relaxation oscillation acts as a resonance and gives rise to a sharp frequency component at this location (denoted by the dashed arrow). This component is represented by the term $(\Lambda W_b)\sin(2C_b t + \phi_b)$ which is now of $O(\Lambda)$. In Figure 1(c) ($\Delta = 0.68$) we have increased further the frequency detuning and the laser response has regained its FWM character.

Similarly, Fig. 2 illustrates the subharmonic resonant response of the injected laser. The strength of the injection is kept constant, at the same level as in Fig. 1, and we vary the frequency detuning Δ. In Fig. 2(a) ($\Delta = 1.55$) the laser exhibits the characteristic FWM response with most of the power concentrated at the center line. The FWM response is again illustrated by Eq. (4). Because of the large value of the frequency detuning the laser response at the location of injection is weak. This verifies the prediction of the theoretical analysis regarding the inversely proportional relationship between the strength of the regenerative signal and the value of Δ [14] [compare with Fig. 1(a)]. In Fig. 2(b) the detuning has increased ($\Delta = 1.74$) and there is a clear indication of the onset of the period-doubling bifurcation. Figure 2(c) ($\Delta = 1.9$) shows the subharmonic resonant response of the system which is described by Eq. (6). The strong subharmonic frequency component (denoted by the dashed arrow) is represented by the zeroth order term $W_c \sin(\frac{1}{2}C_c t + \phi_c)$. The laser response has been altered dramatically. For even larger frequency detuning, as shown in Fig. 2(d) ($\Delta = 2.1$) the laser is ready to undergo a bifurcation out of the subharmonic region and regain its FWM character.

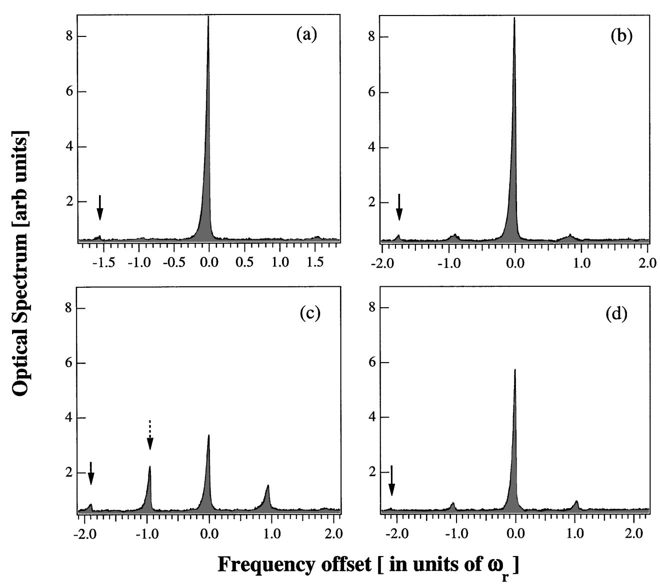

FIG. 2. Experimental optical spectra of the slave laser demonstrating the subharmonic resonance. The solid arrow indicates the location of injection (regenerative signal). (a) FWM response of the slave laser ($\Delta = 1.55$). (b) Onset of the period-doubling bifurcation ($\Delta = 1.74$). (c) Subharmonic resonance ($\Delta = 1.9$). It is denoted by the dashed arrow. (d) The slave laser is ready to undergo a bifurcation out of the subharmonic region and regain its FWM response ($\Delta = 2.1$).

Numerical integration of the laser rate equations (1) and (2) produces results which are in agreement with the theoretical predictions of the phase equation and the experimental observations. Figure 3 shows a typical bifurcation diagram of the extrema of the electric field amplitude. We keep the strength of injection and the values of the laser parameters constant as we vary the frequency detuning between the master and slave lasers. We clearly notice the primary resonance at $\Delta \approx 1$ and the strong subharmonic resonance at $\Delta \approx 2$. For $\Delta \approx 1/n$, with $n = 2, 3, \ldots$, we observe the ultraharmonic resonances which gradually diminish in magnitude as n increases.

In conclusion, we have demonstrated for the first time the existence of nonlinear subharmonic and ultraharmonic resonances in a semiconductor laser subject to monochromatic optical injection. Such a generic frequency conversion using semiconductor laser diodes raises significantly the prospects for future applications, including clock-distribution, high-speed time-division multiplexing/demultiplexing and optical frequency synthesis. Moreover, the theoretical analysis of the laser response complements our experimental investigation. It establishes the fundamental role of strong amplitude-phase coupling (large b) in inducing and further enhancing this nonlinear response. It also reveals that the information necessary to describe the system dynamics is embedded in the phase of the electric field. The success of the theoretical analysis in capturing the essential dynamical behavior of the system leads to corresponding analytical investigations of more complex optical systems (laser-diode feedback instabilities [6], evanescently coupled semiconductor lasers [16]) which can be described by analogous phase equations.

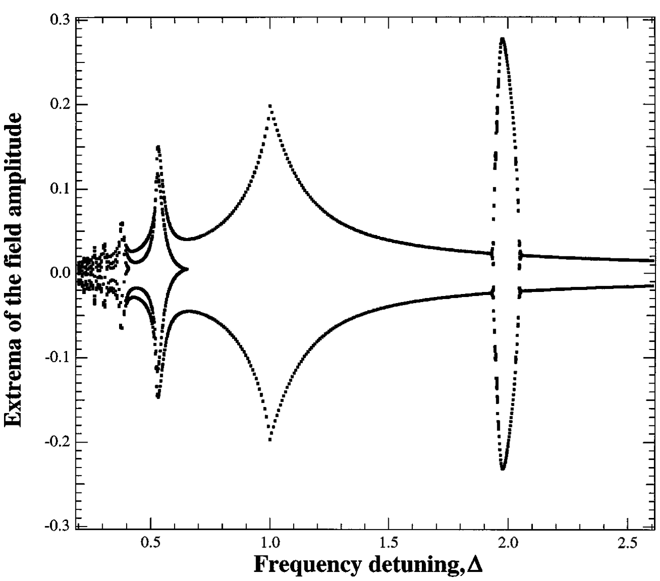

FIG. 3. Numerical bifurcation diagram of the extrema of the electric field amplitude, computed from the rate equations (1) and (2). We keep the laser parameters and injection strength constant ($b = 5$, $\eta = 10^{-3}$, $T = 10^3$, and $P = 0.6$) as we vary the detuning parameter $\Delta = \Omega/\omega_r$.

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