Systematic Description of Yrast Superdeformed Bands in Even-Even Nuclei of the Mass-190 Region

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Yrast superdeformed bands for even-even nuclei of the mass-190 region are described by the projected shell model. Excellent agreement with available data for all isotopes is obtained. Our calculation of electromagnetic properties and pairing correlations provides a microscopic understanding of the observed gradual increase of dynamical moments of inertia with angular momentum in this mass region and suggests that for superdeformation it is not very meaningful to distinguish between Coriolis antipairing and gradual high-j orbital alignment. [S0031-9007(97)02692-6]

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A slowly rotating nuclear system can often be characterized by a fixed deformation with pairing correlation among nucleons. As the system rotates more rapidly, these 2 degrees of freedom will be modified by the rotation. Three simple consequences may be identified: (1) the nuclear deformation can vary during the rotation, a phenomenon known as the *stretching effect* [1]; (2) the *Coriolis antipairing effect* (CAP) [2], which is caused by the weakening pairing correlations across many orbitals due to the Coriolis force; and (3) *rotation alignment* [3], which emphasizes an alignment along the rotation axis of a pair in an orbital particularly susceptible to the Coriolis effect. All these effects can lead to a variation in moment of inertia (MoI).

Generally, all of these effects may be expected in rotating nuclear systems, but in special cases one of them may dominate. For example, in rare-earth nuclei the observed sudden enhancement in the MoI associated with a backbend is typically dominated by rotation alignment of a nucleon pair from a high-j and low-Ω orbital [3], with CAP and stretching effects playing less important roles. However, for situations where the variation in the MoI is gradual, the measured γ-ray energies are often not sufficient to distinguish the effects that change the MoI. Additional measurements of transition quadrupole moments, g factors, or nucleon pair transfer reactions can provide information to disentangle these contributions, but these are more difficult than energy measurements.

The situation for superdeformed (SD) nuclei is less clear. In the SD nuclei of the mass-190 region, both kinematical ($\mathcal{J}^{(1)}$) and dynamical ($\mathcal{J}^{(2)}$) MoI for most SD bands exhibit a gradual increase as a function of increasing rotational frequency, with a more pronounced increase in $\mathcal{J}^{(2)}$. Thus, even though these are among the most deformed nuclei, they exhibit substantial deviation from rigid rotor behavior as the angular momentum increases. The usual understanding is that this behavior is caused by a gradual rotation alignment of pairs from high-j intruder orbitals [4]. There have been several approaches to the detailed calculation of SD bands and their properties [5–12]. Although these differ in particulars, a common feature is that the rotational degree of freedom is described by the cranking method. Thus no electromagnetic transition probabilities as a function of angular momentum have been calculated and there has been no quantitative prediction of the angular momentum for individual SD states.

Our calculations provide a full description of the SD yrast-band spectrum, electromagnetic properties, and pairing properties for the even-even Hg and Pb nuclei. We illustrate with a detailed description of the SD yrast band of ^{194}Hg. For this nucleus the γ-ray energies have been measured [4,13] and the transition quadrupole moments are known from lifetime measurements [14]. Moreover, linking transitions between the SD and normally deformed (ND) bands have been published [15,16], so that definite angular momenta can be assigned to the SD states. We shall demonstrate that the observed angular momenta are correctly reproduced by this calculation and that a gradually decreasing pairing caused by a combination of the CAP effect and rotation alignment of high-j pairs is responsible for the observed smooth increase of the MoI. Moreover, our calculations will indicate that in the SD mass-190 region (unlike for ND nuclei) it is no longer very meaningful to make a distinction between the CAP effect and high-j orbital alignment because the larger quadrupole-quadrupole correlations in SD systems imply that the high-j orbitals are more strongly mixed than for normal deformation (see Ref. [17]).

Our theoretical analysis for SD nuclei is based on the projected shell model (PSM) [18], which has been generalized from its description of ND systems [19]. The many-body wave function is a superposition of (angular momentum) projected multiquasiparticle states,

$$|\psi^I_M\rangle = \sum_\kappa f_\kappa \hat{P}^I_{MK_\kappa}|\varphi_\kappa\rangle , \quad (1)$$

where $|\varphi_\kappa\rangle$ denotes basis states consisting of the quasiparticle vacuum, two quasineutron and quasiproton, and four quasiparticle states. The dimension of the quasiparticle basis is about 100, and the deformation of the basis is fixed at $\epsilon_2 = 0.45$ for all nuclei calculated in this paper. In order to treat heavy SD nuclei we have extended the previously used single-particle configuration space [18,19] by a major shell each for neutrons and protons. For the Nilsson parameters κ and μ we take the N-dependent values from Ref. [20], subject to modifications introduced by Ref. [21].

We use the usual separable-force Hamiltonian described in [18,19] with spherical single-particle, residual quadrupole-quadrupole, monopole pairing, and quadrupole pairing terms. The strength of the quadrupole-quadrupole term is fixed in a consistent way when the deformation is determined [19]. Lack of SD data precludes determining the pairing interaction from experimental odd-even mass differences, so we have used the prescription introduced in Ref. [18]: We multiply the monopole pairing strengths G_M of Ref. [19] by 0.92 to accommodate the increase in the size of the basis for the present calculation. This amount of reduction is consistent with the principles described in Ref. [22]. For the quadrupole pairing interaction G_Q, a ratio $C = G_Q/G_M = 0.28$ is used; we shall discuss this choice further below. This single set of interactions is used for all nuclei discussed in this paper; it works equally well for odd-A and odd-odd nuclei of the same mass region [23]. Our wave functions are eigenstates of angular momentum obtained by diagonalizing a two-body, laboratory-frame Hamiltonian. Therefore effects beyond the simple mean field will be contained in the calculated expectation values, and our observables are calculated at discrete values of the angular momentum rather than rotational frequency (see Ref. [24]).

In Figs. 1 and 2, we compare calculated transition energies and dynamical MoI with the available measurements for yrast SD bands in even-even $^{188-196}$Hg and $^{190-198}$Pb isotopes. The agreement with data is very good, with even the sensitive dynamical MoI being reproduced well. The MoI rises gradually with angular momentum for all nuclei in this mass region; however, the rate of increase (slope of $\mathcal{J}^{(2)}$) varies from nucleus to nucleus. The differences in slope arise from the variation of effective interaction strengths and band crossing spin for different nuclei, which is a common occurrence in ND bands [25,26]. The experimentally observed segments of these bands are restricted by the technical problem of detection devices at higher spin and decay of SD states to ND states at lower spin. It is clear that the extension of data in known SD nuclei by even a few spin units, or observation of SD states in Pb or Hg isotopes 2 neutron numbers larger or smaller than presently known, could provide strong tests of these calculations.

The angular momenta for experimental states in Figs. 1 and 2 are those proposed by the corresponding experimentalists, but only for ^{194}Hg [15] and ^{194}Pb [16] are these measured quantities. For the ^{194}Hg example that we shall emphasize, the agreement with data is excellent, with the experimental and theoretical E_γ values virtually indistinguishable on the scale of the plot. For ^{194}Pb the agreement between experiment and theory for E_γ is almost as good, particularly in the lower angular momentum states. This provides strong theoretical support for recently measured spin values in these nuclei [15,16]. This indicates also that the present model could be a powerful tool to predict unknown spins for SD nuclei. For all the nuclei studied in Figs. 1 and 2, the direct or indirect experimental spin assignment is supported by our calculations. This differs from the SD nuclei in the mass-130 region, where for many SD bands we predict a positive shift for previous (indirectly assigned) spins by up to $6\hbar$ [18].

For ^{194}Hg, we predict a gradually decreasing $\mathcal{J}^{(2)}$ for the highest spins. The whole pattern forms a smooth bump with the maximum at $I = 42\hbar$. At this spin the SD ground-state band (g band) crosses, simultaneously, a 2-quasiproton band ($\pi i_{13/2}[\frac{3}{2}, \frac{5}{2}]$, $K = 1$) and a 2-quasineutron band ($\nu j_{15/2}[\frac{3}{2}, \frac{5}{2}]$, $K = 1$). After the crossing, the 2-quasiproton band is lower in energy and thus has larger weight in the yrast band for $I > 42\hbar$. Experimentally, the first observed downturn in $\mathcal{J}^{(2)}$ for the mass-190 region was reported for ^{194}Hg [13].

Changing the quadrupole pairing will shift the band crossing, thereby shifting the peak in the $\mathcal{J}^{(2)}$ curve. If, instead of the ratio $C = G_Q/G_M = 0.28$ employed in Figs. 1 and 2, we use $C = 0.20(0.36)$, the $\mathcal{J}^{(2)}$ peak for ^{194}Hg is shifted to $I = 38\hbar$ ($I = 46\hbar$), but the E_γ values are displaced by only a few keV. Similar results are found for the other isotopes displayed in Figs. 1 and 2. Thus the location of the maximum in $\mathcal{J}^{(2)}$ is sensitive to the quadrupole pairing strength, but our other results do not depend significantly on this parameter. This suggests that a systematic experimental determination of the turnover point in $\mathcal{J}^{(2)}$ could provide a sensitive measure of the quadrupole pairing in these SD states.

In Fig. 3 we show the theoretical values of the transition quadrupole moment Q_t, g factor, and pairing gaps (Δ_n and Δ_p) for the SD yrast band in ^{194}Hg. Rather constant Q_t [Fig. 3(a)] is found up to $I \approx 24\hbar$, with a value of 16.8 e b that is comparable to the measured average Q_t of 17.2 ± 2.0 e b [14]. Above that spin, the theoretical values are slowly quenched in a smooth manner. This is associated with gradual alignment processes, leading to a small reduction of collectivity. Therefore this calculation indicates that stretching is negligible for the range of measured angular momenta in the ^{194}Hg yrast band.

The calculated g factors are presented in Fig. 3(b). There are as yet no experimental data available for comparison. These quantities are sensitive to the alignment of individual nucleon pairs. Proton and neutron g factors are plotted separately, as is their sum. We observe a rather constant, small, and negative value for neutrons. The behavior of the total g factor is governed by that of the protons, which shows a gradual increase with angular momentum in the range $I = 24-44\hbar$. At $I = 44\hbar$, the total g factor reaches $Z/A \approx 0.41$ and saturates thereafter. The g factors suggest that the rotation alignment

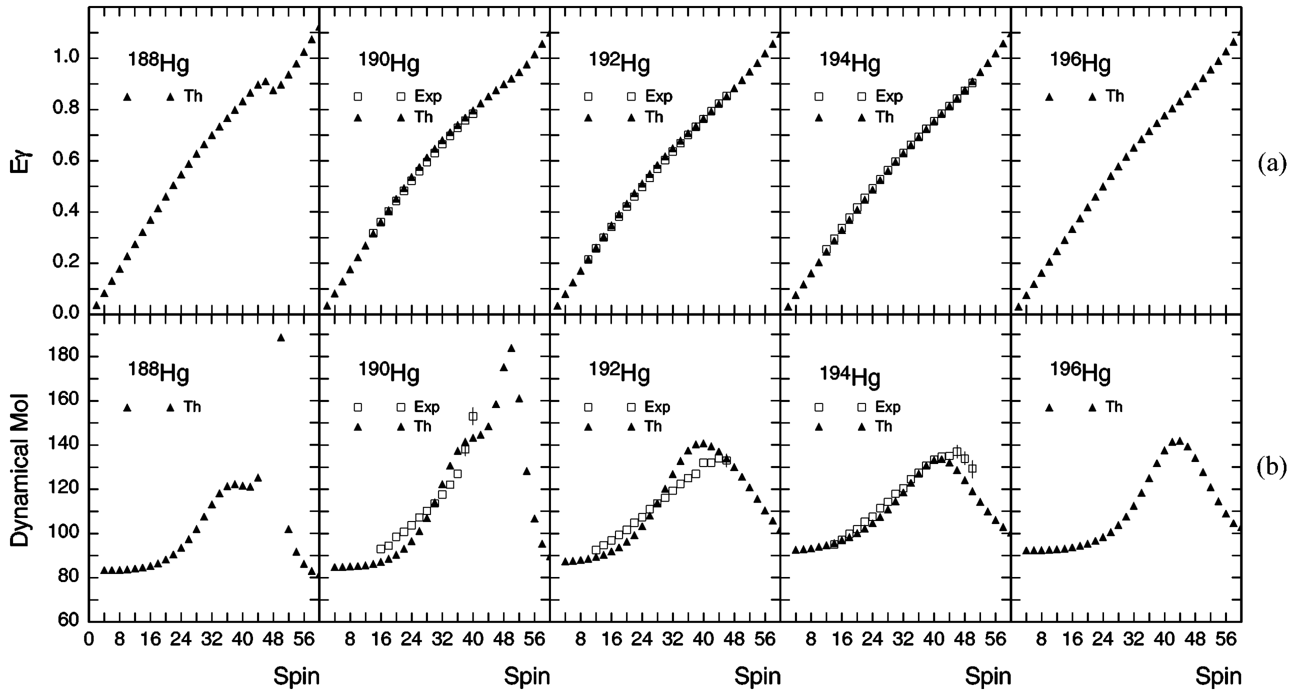

FIG. 1. Comparison of calculations with experimental data [15,30] for even-even $^{188-196}$Hg: (a) γ-ray energies defined by $E_\gamma(I) = E(I) - E(I-2)$ (MeV); (b) dynamical MoI defined by $\mathcal{J}^{(2)}(I) = 4/[E_\gamma(I) - E_\gamma(I-2)]$ ($\hbar^2$ MeV^{-1}).

of high-j orbitals enhances the MoI, but the alignment contribution seems insufficient to cause the pronounced increase in the MoI seen in Fig. 1 for ^{194}Hg. In particular, this cannot easily explain the increase in the MoI before $I = 24\hbar$, where the high-j pair alignment is small.

We show calculated pairing gaps in Fig. 3(c). Note that the quantities displayed are not BCS gaps; they are computed from the many-body wave functions, which incorporate dynamical effects in the pair field [24]. We observe steady quenching of proton and neutron pairing with increasing spin across the entire spin range. This smooth decrease of Δ corresponds to a collective CAP effect that enhances the MoI with increasing angular momentum. A somewhat steeper decrease in the gap is found for protons

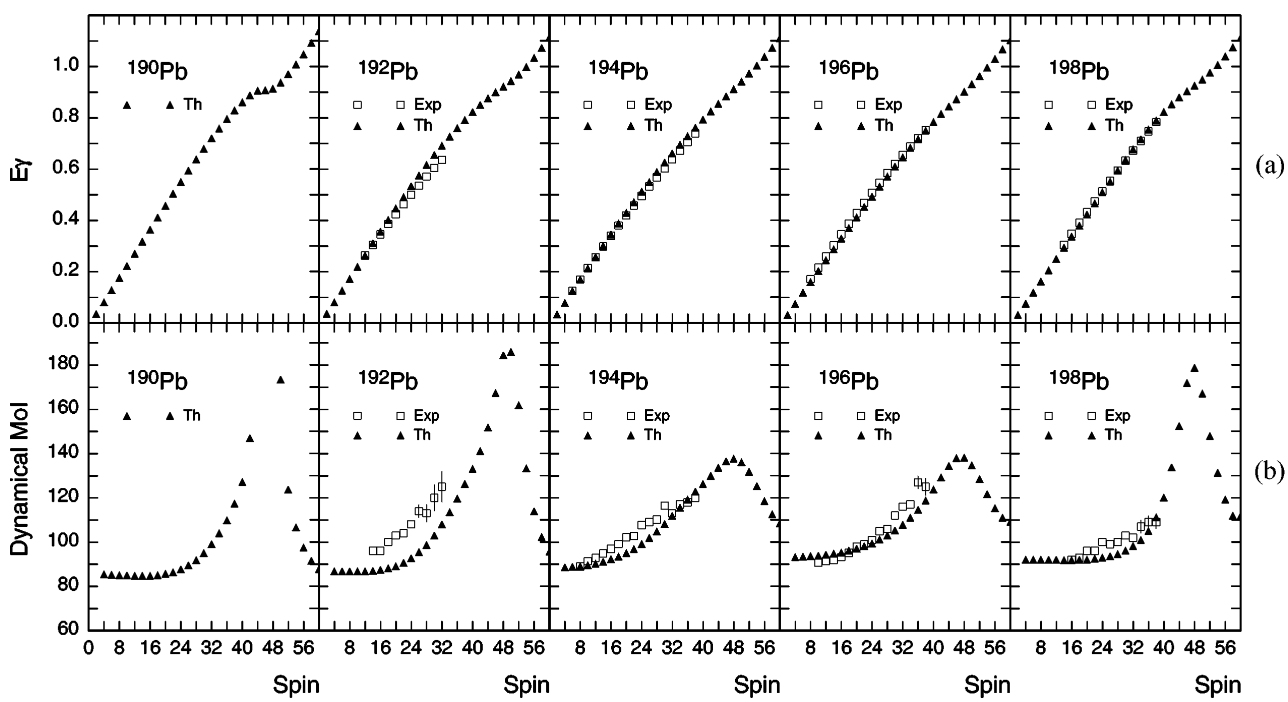

FIG. 2. As for Fig. 1, but for even-even $^{190-198}$Pb; data from Refs. [16,30].

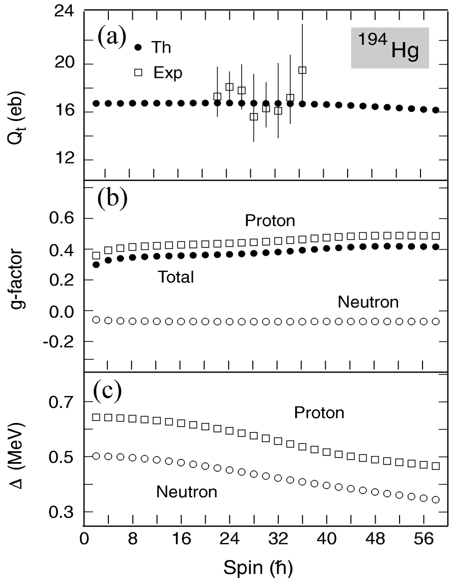

FIG. 3. For ^{194}Hg: (a) Comparison of the calculated transition quadrupole moments (in e b) with the data [14]. (b) Theoretical g factors; (c) Theoretical pairing gaps (in MeV).

in the range $I = 24-44\hbar$. This additional contribution comes from the increasing importance of high-j pair alignment, in accord with discussions above.

In previous discussions [4,12], the gradual rotation alignment of quasiparticles from high-j orbitals was stressed, but the important average contribution of *all states* through the CAP effect has not been emphasized. This observation is implicit in previous studies where it was noticed [4] that treatment of the collective motion in a cranking model with shell-correction calculations could have difficulties for a quantitative description of the MoI of a SD band. This is a well-known problem for the cranked shell model, which describes nicely the pair alignment, but not the MoI, particularly for cases where the CAP effect is important [27].

In conclusion, the projected shell model has been used to provide the first comprehensive theoretical description of the yrast SD band in even-even nuclei in the mass-190 region that simultaneously describes all observables, including the angular momentum. The increase of the MoI is caused by a smooth decrease of the pairing correlation within the nucleus due to the Coriolis antipairing effect, supplemented by a gradual alignment of high-j particles at higher angular momentum, with no evidence of deformation stretching. Our results suggest that for superdeformation there is no longer a clear distinction between high-j alignment and CAP effects, because the high-j orbitals are more strongly mixed in the case of superdeformation relative to normal deformation. These conclusions are supported by calculations of transition quadrupole moments, g factors, and pairing gaps. A recent experiment [28] shows that the quadrupole moments in the SD bands in ^{132}Ce and ^{131}Ce are nearly the same, even though these bands have different numbers of high-j quasiparticles. This observation strongly supports our conclusion in this paper and contradicts the predictions of cranked Woods-Saxon calculations [29]. Finally, these calculations provide a variety of predictions that could be tested by extension of extant SD data to somewhat higher spins and to other nuclei in this region, and by measurement of g factors.

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