Elliptical Flow: A Signature for Early Pressure in Ultrarelativistic Nucleus-Nucleus Collisions

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Elliptical energy flow in noncentral Au(11.7A GeV) on Au reactions is studied comparing results obtained in two different modes of the relativistic quantum molecular dynamics approach (mean field and cascade). The final azimuthal asymmetries in the central rapidity region are very sensitive to the pressure at maximum compression, because they involve a partial cancellation between early squeeze out and subsequent flow in the reaction plane. Analysis of elliptic flow may lead to much stronger constraints on the magnitude of pressure in ultradense nuclear matter than are available today. [S0031-9007(97)02794-4]

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A primary goal of heavy-ion physics is the creation and observation of the quark-gluon plasma (QGP). A first-order phase transition is generically associated with the presence of a “softest point” in the equation of state (EoS), since the pressure increases less than the energy density close to the critical temperature [1]. Incidentally, quantum chromodynamics (QCD) calculations on lattices show a softening of the EoS at temperatures in the transition region between pion gas and QGP. It is not yet settled whether QCD at finite temperature exhibits a true phase transition or just a rapid crossover between the two phases. Unfortunately, it is completely unknown from theory how the EoS changes if baryon number is added. Lattice QCD cannot address this question with current methods. Extremely baryon-rich matter is produced in today’s heavy ion experiments [2]. Can experimental results shed some light on the magnitude of the pressure at large baryon densities? Indeed, flow analysis can be utilized to study the transient pressure in nuclear collisions [3,4]. The expansion dynamics of equilibrating matter is governed by the EoS, because pressure gradients drive collective flows.

In this Letter, I focus on elliptical flow in the central rapidity region of nucleus-nucleus collisions with beam energies covered by current fixed-target programs at BNL and CERN (2−200A GeV). So far, the so-called radial and directed flows have been studied more extensively at these energies, theoretically [5,6] and experimentally [7,8]. I show that the existence of *two* transverse flow components in collisions with finite impact parameter provides additional valuable information on the transient pressure. In particular, the anisotropy of the transverse flow tensor turns out to be very sensitive to the pressure at maximum compression. While projectile and target nuclei are passing through each other spectator matter provides “confining walls” and magnifies the initial azimuthal asymmetry of the collision region.

At low beam energy, matter at central rapidity escapes preferentially orthogonal to the reaction plane. The spectator nucleons block the path of participant hadrons which try to escape from the collision zone. This is the experimentally observed squeeze-out effect [9]. At ultrahigh energies, particles produced in the central region do not interact with spectators, since the crossing time of projectile and target shrinks with the Lorentz factor gamma. If collective motion in transverse directions develops the almond-shaped geometry of the collision region clearly favors the directions (anti-) parallel to the impact parameter vector [10]. An interesting situation emerges for collision energies between the low and ultrahigh energies, basically the whole energy region covered by present fixed-target experiments. Taking collisions of equal mass nuclei moving nearly with the speed of light, the passage time of projectile and target spectators is approximately given by $2R_A/(\gamma c)$ (numerically 5.4 fm$/c$ at 12A GeV and 1.4 fm$/c$ at 160A GeV for the heaviest systems). Such a time scale neither covers the whole reaction time nor becomes irrelevant at these intermediate energies. As a consequence, the centrally produced matter is initially squeezed out orthogonal to the reaction plane. Afterwards, however, the geometry of the central region favors in-plane flow. The orientation of the final azimuthal asymmetry in particle, momentum, and energy flow is chiefly determined by the relative strength of the pressures in the initial compression and later expansion stage.

In general, flow anisotropies in the central region may turn out to be a more sensitive probe of the equation of state in ultradense matter than the other types of transverse flow. The developing flow anisotropies act against the initial asymmetry of the collision region which have created them in the first place. Thus they are rather sensitive to the early ultradense stages, much like the directed flow—the mutual deflection of projectile and target material in the reaction plane. However, directed flow receives a strong preequilibrium contribution as visible from the transverse momentum anticorrelation between nucleons close to original projectile and target rapidity. In contrast, transverse flow of matter in the central region is presumably less affected by nonequilibrium physics.

Furthermore, the highest energy and baryon densities are achieved in the central region for present experiments.

It should be mentioned that azimuthal asymmetries in noncentral *AA* collisions are currently being measured both at AGS and the higher CERN energies. E877 has analyzed the transverse energy and charged particle multiplicity distributions using the technique of Fourier decomposition [7,11]. A nonvanishing second or quadrupole moment which was found by E877 signals the elliptical deformation of the flow tensor. The preliminary E877 data indicate that the main flow directions of hadrons at central (pseudo-)rapidities are indeed parallel to the impact parameter. NA49 has also reported preliminary data on elliptical transverse energy flow patterns for Pb(158*A* GeV) on Pb reactions [12].

In the following, I employ a transport model, relativistic quantum molecular dynamics (RQMD) [13], to calculate the azimuthal asymmetries in the energy deposition for Au(11.7*A* GeV) on Au collisions in noncentral collisions. RQMD is based on string and resonance excitations in the primary collisions of nucleons from target and projectile. Overlapping color strings may fuse into so-called ropes. Subsequently, the fragmentation products from rope, string, and resonance decays interact with each other and the original nucleons, mostly via binary collisions. These interactions drive the system towards equilibration [14] and are responsible that collective flow develops, even in the preequilibrium stage. The model contains some option which allows one to vary the pressure in the high-density state. In RQMD baryons may acquire effective masses in the medium. They are generated by introducing Lorentz-invariant quasipotentials into the mass-shell constraints which simulate the effect of "mean fields" [15]. There are no potential-type interactions in the so-called cascade mode of RQMD. In this mode, the equilibrium pressure is simply an ideal gas of hadrons and resonances. Its equation of state is very similar to the one calculated by the Bern group in Ref. [16], because the spectrum of included resonance states is nearly the same.

It should be noted that propagating strings modify the equation of state as well. This correction is small, however, in equilibrium at relevant temperatures around 150 MeV. Although not realized in RQMD the collision term in the equations of motion may contribute to the equilibrium pressure, in general. For instance, repulsive trajectories are selected for colliding baryons with some probability in a new version of the ARC model [17]. In Refs. [18,19] it was concluded that a cascade lacks some pressure in comparison to experimental data for *AA* collisions at 10−15*A* GeV. This result was seemingly contradicted by the findings in [17], based on ARC calculations. However, the latter calculations contain some nonideal gas pressure contributions from the repulsive trajectory prescription.

RQMD results for ultrarelativistic nucleus-nucleus collisions have been compared to measurements by most major experimental collaborations [2], showing usually reasonable or good agreement. Various experimental data—e.g., directed and total transverse momenta [11,20]—which have been taken in the AGS energy region around 12*A* GeV seem to hint that the generated pressure in RQMD is too "soft" if it is used in its cascade mode. Therefore we compare here the result obtained in the cascade mode with a calculation in which the quasipotentials generate additional pressure due to repulsion at baryon densities larger than ground state density. Potential parameters have been selected for RQMD (version 2.3) which bring the generated transverse momenta in agreement with available data similarly as it was done in [18]. Since a first-order phase transition from a resonance gas into a QGP softens the equation of state, it would act in the opposite direction to repulsive mean fields. Its inclusion in RQMD would require even more repulsion in the hadronic stage.

In the following, the question will be addressed how sensitive the azimuthal asymmetry of energy flow in the central rapidity region is to the pressure in the compression and expansion stages. For this purpose, I have analyzed the evolution of the pressure in a particular reaction, Au(11.7*A* GeV) on Au at an impact parameter of 6 fm. Applying the virial theorem, the pressure in each of the three space directions is determined from

$$P^i \cdot V = \left\langle \sum_M \left(\mathbf{p}^i(M)\mathbf{v}^i(M) + \sum_N \mathbf{F}^i_{MN}\mathbf{r}^i(M) \right) \right\rangle, \quad i = 1,2,3\,. \tag{1}$$

$\mathbf{p}$, $\mathbf{v}$ denote the hadron's momentum and velocity, respectively, and $\mathbf{F}_{MN}$ the force which baryon N exerts on M. The summations over N, respectively, M, include only hadrons inside a cylindrical volume $V = 2\pi(xR_A)^3/\gamma$ centered at the origin (with $x = 0.3$). Nucleons which have not collided yet are not included in the evaluation of the pressure. The right-hand side of Eq. (1) has been evaluated for approximately 400 events in the mean-field mode of RQMD. An event average has been taken (indicated by the $\langle\rangle$ symbols). Analogously, local energy and baryon densities (e and ρ_B) have been calculated. Since no nonstatistical difference between the two transverse pressure components was found, their average was taken and is dubbed "pressure" in the following. The time evolution of pressure, local energy, and baryon density are displayed in Fig. 1. The upper part shows the evolution in the e-p plane, the lower part in the e-ρ_B plane. It becomes apparent from Fig. 1 that the time of maximum compression coincides with the time at which the spectators are about to leave the collision zone. The energy and baryon density at maximum compression are approximately 1.3 GeV/fm^3 and $3.5\rho_0$, respectively. It is noteworthy that these values are close to the region for which a phase transition into a QGP is usually expected. The pressure in the expansion stage is somewhat larger than

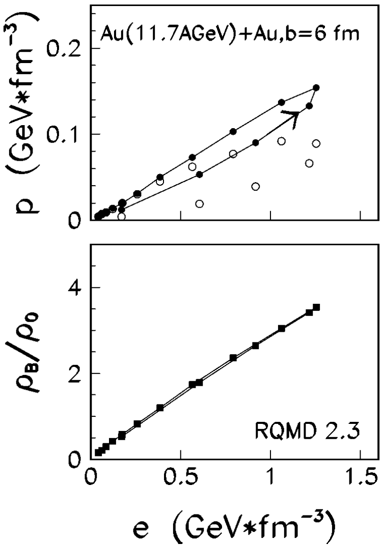

FIG. 1. Time evolution of transverse pressure p, local maximum energy, and baryon density (e, respectively, ρ_B/ρ_0) in the collision center for the reaction Au(11.7A GeV) on Au at impact parameter 6 fm. The results were obtained using the RQMD model (version 2.3) in the mean-field mode. The upper part shows the evolution in the e-p plane, the lower part in the e-ρ_B plane. The symbols represent the values of these quantities, with time in the center of mass increasing in steps of 1 fm/c each, starting 1 fm/c after the two nuclei have touched each other. Time direction is indicated by an arrow. The contribution from the kinetic part of the pressure alone is also displayed (open circles).

in the compression stage. This reflects the presence of a strong preequilibrium component which tends to soften the transverse pressure. In Fig. 1 the contribution from the kinetic part of the pressure [the first term on the rhs of Eq. (1)] is also shown (open symbols). Roughly, the kinetic and the potential part contribute equally at the time of maximum compression.

The azimuthal asymmetry in the energy flow can be quantified by defining the following variable:

$$E_{\mathrm{dir}} = \sum_{M} E(M)\mathrm{sgn}(\phi)\,. \tag{2}$$

The summation over M includes hadrons only which are close to center-of-mass rapidity. ϕ is defined as the angle of a hadron's momentum with respect to the impact parameter vector. sgn (ϕ) is defined to be $+1$ in the cones with an opening angle of 45° around $\phi = 0$ and 180°, -1 elsewhere. Figure 2 displays the time evolution of E_{dir}. In the time span up to maximum compression, E_{dir} acquires negative values (squeeze out), because the pressure mostly from the repulsive potentials pushes the hadrons against the confining spectator material in the reaction plane and into the vacuum orthogonal to it. After the spectators are gone ($t > 5$ fm) E_{dir} gets positive contributions with increasing time. Finally, the in-plane flow dominates for this reaction. Therefore the major energy flow axis is parallel to **b**. For comparison,

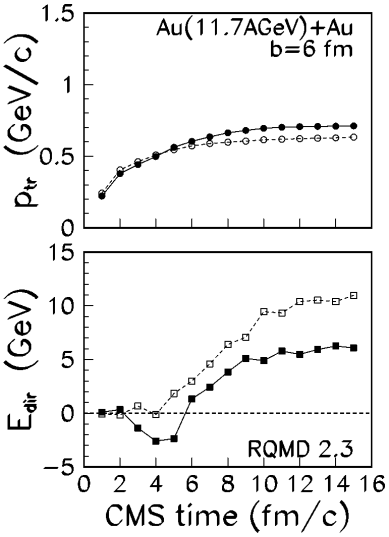

FIG. 2. Time evolution of the average transverse momentum of nucleons p_{tr} (top) and E_{dir} (bottom). The variable E_{dir} is defined in Eq. (2)). Closed (open) symbols refer to the calculation in RQMD mean-field (cascade) mode. A rapidity cut ± 0.7 around center-of-mass rapidity was imposed.

Fig. 2 shows the evolution of the same quantity, but calculated in the RQMD cascade mode. Because of the preequilibrium effect, the effective transverse equation of state is ultrasoft. There is no visible squeeze out present at the early times. The pressure at later times is smaller than in the mean-field mode. However, the final azimuthal asymmetry expressed by its E_{dir} value is approximately 60% larger, because the initial squeeze out is absent. Note that the rapidity cut which goes into the definition of E_{dir} is essential that this observable is dominated by the quadrupole component of the transverse energy flow. Without cut, the dipole component (directed in-plane flow) would lead to larger E_{dir} values in mean-field as compared to the cascade mode.

In addition, Fig. 2 shows the evolution of the average nucleon transverse momentum (in the same rapidity window as for E_{dir}). The mean fields push the nucleons to larger p_{tr} values which is favored in comparison to measurements. Other transport calculations for AGS energies (done with the ARC and the ART codes [21]) confirm that some additional repulsion—beyond the kinetic pressure from an ideal resonance gas—improves agreement with data. However, experimental data for transverse spectra do not provide a clue whether the observed larger stiffness in comparison to the pure cascade result results from additional pressure in the early or in the late stage. Here, analysis of the azimuthal asymmetries provides information of uttermost importance. Late pressure strengthens the in-plane flow while pressure in the compression stage in which the central region is confined by spectators weakens it. The repulsive potentials which are built into the RQMD model act mostly early and therefore weaken the final asymmetries. The RQMD prediction for the finally observable energy flow with respect to the reaction plane (cf. Fig. 3) is readily testable using the present experimental setup of the E877 group. We take the *qualitative* agreement with preliminary E877 data which show the major flow (anti-) parallel to the impact parameter as an encouraging sign that the model can be useful to extract the early produced pressure from experimental data.

Summarizing, I have studied azimuthal asymmetries of the energy flow in the central collision zone in relation to the transient pressures. The power of elliptical flow analysis becomes apparent if it is combined with measurement of the average flow in transverse directions, the radial flow. Early and late pressures contribute with opposite signs to the elliptic deformation but with equal signs to the average transverse flow. This beautiful feature makes it possible to gain separate information on the early and late pressures. The sensitivity of elliptical flow to the early pressure opens up possibilities for a rich research program. For instance, the beam energy at which final squeeze out disappears and is replaced by domination of in-plane flow will strongly depend on the early pressure. Asymmetric collisions, variations of centrality triggers and cuts on momenta or particle species could provide more detailed information about its dependence on baryon density. Elliptical flow analysis may become extremely useful in a joint effort of experimentalists (e.g., E877, NA49, and E895 [22]) and theorists to determine the pressure in matter with baryon densities of several times ground state density.

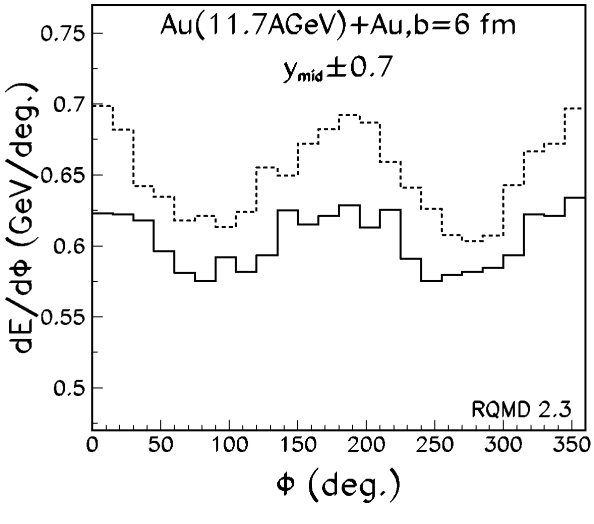

FIG. 3. Differential energy flow distribution $dE/d\phi$ as a function of the angle with respect to the impact parameter vector ϕ. The results were obtained for the same system as in Figs. 1 and 2 and the same acceptance cuts as in Fig. 2. Straight (dashed) line histogram refers to the calculation in the RQMD mean-field (cascade) mode.

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