Impossibility of the Cylindrically Symmetric Einstein-Straus Model

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The classical Einstein-Straus problem treats the influence of the *expansion* of the universe on the *static* vacuum surrounding a spherically symmetric object. In this Letter we study the next simplest step by dropping the assumption of spherical symmetry and considering the case of cylindrical symmetry. Our main result is that the cylindrically symmetric analog of the Einstein-Straus model is impossible, even without any restriction on the matter content of the static cavity. This fact forbids the embedding of some static objects (say strings or similar objects) into the standard cosmological models. [S0031-9007(97)02807-X]

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This Letter deals with the question of whether or not the space surrounding astrophysical objects is influenced by the expansion of the Universe. We usually tend to consider the gravitational fields of the above-mentioned objects as *static*, and yet we also believe that the Universe they live in is expanding. Of course, this is an old problem, and has been studied several times since the appearance of general relativity. Perhaps the first attempt to give an answer to this problem was that of McVittie [1], later developed and completed by Järnefelt [2] (see also for a brief historical survey [3]), where they tried to discover the possible influence of the cosmic expansion on planetary orbits. Nevertheless, it is commonly accepted that the first formally correct description of the above problem was done by Einstein and Straus [4] back in 1945.

The classical Einstein-Straus paper [4] gave a model for a spherically symmetric compact object surrounded by vacuum inside a spatially homogeneous and isotropic universe. The main aim was the determination of "the influence of the expansion of the space on the (vacuum) gravitational fields surrounding the individual stars" [4], and the method was to ascertain whether or not it is possible to match the spherically symmetric vacuum (and hence *static*) Schwarzschild solution to an expanding (and hence *nonstatic*) exterior Robertson-Walker (RW) cosmological space-time across a hypersurface preserving the spherical symmetry. The well-known result was that it is possible to match any *dust* RW geometry with the Schwarzschild vacuum metric across any co-moving 3-sphere as long as the total mass contained inside the 3-sphere in the RW part is exactly the mass of the Schwarzschild "hole." Thus, the result is that there is no influence of the cosmic expansion on the static Schwarzschild vacuole.

This has been the standard answer for many years, even though there have appeared other new attempts to construct more sophisticated models in which the expansion may influence other types of cavities (see, for instance, [5–7]; this is also related to the problem of formation of voids in the Universe. We refer the reader to the excellent review of these matters given in [3]). Nevertheless, and apart from first order deviations [6], as far as we are aware *all* the known models have *spherical symmetry*.

Therefore, it arises naturally the question of how much Einstein-Straus's general conclusion depends on the assumption of spherical symmetry. In this paper we give a perhaps surprising answer to this question, since we attack the next simplest problem (cylindrical symmetry) and thereby we prove rather easily the impossibility of having a static cavity matched to a standard RW cosmological universe whenever the cavity has a border containing a cylindrical-like piece. This problem makes sense, of course, because the RW geometry itself has global cylindrical symmetry. Notice also that since we are dealing with junction conditions, which are *local*, our result applies to many situations such as cylinders (for strings), coin-shaped objects, or more complicated ones (bottle-shaped things, and many others), *as long as* they can be considered *locally* cylindrically symmetric.

For the sake of generality we do not make any assumption on the energy-momentum tensor of the static cavity, and thus our results apply to either case of considering only the interior of the static objects, or together with their possible static exteriors (vacuum or with electromagnetic fields). The general result we obtain is that *a nonstatic Robertson–Walker metric cannot be matched to any cylindrically symmetric static metric across a nonspacelike hypersurface preserving the symmetry.* Notice that, by the usual duality between the interior and the exterior metric (remember that, for example, the Einstein-Straus model is mathematically equivalent to the Oppenheimer-Snyder [8] collapsing dust with vacuum exterior), we are solving at the same time the problem of finding a RW interior to any static cylindrically symmetric space-time.

It might seem strange trying to match a static metric with a nonstatic one, but that is precisely the point in the Einstein-Straus work: a static cavity can be contained within the expanding Universe (of course, the vacuole is getting bigger as the hypersurface surrounding it is expanding with the Universe). For our case the problem is also meaningful since we prove here explicitly that the matching of two *arbitrary* cylindrically symmetric space-times, one static and the other nonstatic, is feasible in principle, and *only* when we restrict the nonstatic one to be a RW model the impossibility arises.

The junction conditions for two general cylindrically symmetric space-times across a timelike or spacelike hypersurface preserving the symmetry can be found in [9]. Nevertheless, we are interested in nonspacelike hypersurfaces in general, even changing its character from point to point, because a possible matching across a partly null hypersurface would be a satisfactory situation since we could have an object surrounded by a static region whose rim is expanding at the speed of light. Therefore, we need the junction conditions for general hypersurfaces, which have been recently given in [10] (see also [11,12] for the specific particular case of null hypersurfaces). (Actually, by using the junction conditions for *general* hypersurfaces we will also solve the problem for partly spacelike hypersurfaces, which may describe phase transitions in the Universe.)

Let us consider the most general whole cylindrically symmetric line element, given in coordinates $\{t, r, \varphi, z\}$ by [13–15]

$$ds^{2+} = -\hat{A}^2 dt^2 + \hat{B}^2 dr^2 + \hat{C}^2 d\varphi^2 + \hat{D}^2 dz^2, \quad (1)$$

where $\hat{A}$, $\hat{B}$, $\hat{C}$, and $\hat{D}$ depend on t and r. (We use the term "whole" in the sense of [13], p. B232 (see also [14]): cylindrically symmetric systems invariant also under reflection in any plane containing the symmetry axis or perpendicular to it. In fact, we can also obtain our result in the general cylindrically symmetric case, that is, with a crossed term $d\varphi dz$ in (1), but we have preferred to keep the simplicity as our first goal is the case of a RW metric (which has whole cylindrical symmetry).) This will describe the nonstatic region, while the static one is given by the analogous line element in coordinates $\{T, \rho, \tilde{\varphi}, \tilde{z}\}$

$$ds^{2-} = -A^2 dT^2 + d\rho^2 + C^2 d\tilde{\varphi}^2 + D^2 d\tilde{z}^2, \quad (2)$$

where now A, C, and D depend *only* on ρ (and the function in front of $d\rho^2$ has been put equal to 1 without loss of generality). The parametric forms of a general hypersurface σ preserving the cylindrical symmetry are given by

$$\begin{aligned} &\sigma^+ : \{t(\lambda), r(\lambda), \varphi = \phi, z = \zeta\}, \\ &\sigma^- : \{T(\lambda), \rho(\lambda), \tilde{\varphi} = \phi, \tilde{z} = \zeta\}, \end{aligned} \quad (3)$$

for the $(+)$ and $(-)$ space-times, respectively, where $\{\xi^a\} \equiv \{\lambda, \phi, \zeta\}$ $(a, b = 1, 2, 3)$ are intrinsic coordinates in the hypersurface and $t(\lambda), r(\lambda), T(\lambda), \rho(\lambda)$ are arbitrary functions. The three independent vector fields $\{\partial/\partial\xi^a\}$ tangent to σ can be pushed forward to the space-times giving

$$\vec{e}_1{}^+ = \dot{t}\,\frac{\partial}{\partial t} + \dot{r}\,\frac{\partial}{\partial r}\bigg|_{\sigma^+}, \qquad \vec{e}_2{}^+ = \frac{\partial}{\partial \varphi}\bigg|_{\sigma^+},$$
$$\vec{e}_3{}^+ = \frac{\partial}{\partial z}\bigg|_{\sigma^+}, \quad (4)$$

$$\vec{e}_1{}^- = \dot{T}\,\frac{\partial}{\partial T} + \dot{\rho}\,\frac{\partial}{\partial \rho}\bigg|_{\sigma^-}, \qquad \vec{e}_2{}^- = \frac{\partial}{\partial \tilde{\varphi}}\bigg|_{\sigma^-},$$
$$\vec{e}_3{}^- = \frac{\partial}{\partial \tilde{z}}\bigg|_{\sigma^-}. \quad (5)$$

These space-time vector fields are defined *only* on the hypersurface.

The first fundamental forms inherited by $\sigma^\pm$ from the $(\pm)$ space-times are, respectively,

$$\begin{aligned} ds^{2+}|_{\sigma^+} = {} & (-\hat{A}^2\dot{t}^2 + \hat{B}^2\dot{r}^2)d\lambda^2 \\ & + \hat{C}^2 d\phi^2 + \hat{D}^2 d\zeta^2|_{\sigma^+}, \end{aligned} \quad (6)$$

$$\begin{aligned} ds^{2-}|_{\sigma^-} = {} & (-A^2\dot{T}^2 + \dot{\rho}^2)d\lambda^2 \\ & + C^2 d\phi^2 + D^2 d\zeta^2|_{\sigma^-}. \end{aligned} \quad (7)$$

Now, the first part of the junction conditions is always the equality $ds^{2+}|_{\sigma^+} = ds^{2-}|_{\sigma^-}$ [10–12,16,17], which in this case read simply as

$$-\hat{A}^2\dot{t}^2 + \hat{B}^2\dot{r}^2 \overset{\sigma}{=} -A^2\dot{T}^2 + \dot{\rho}^2, \quad (8)$$

$$\hat{C} \overset{\sigma}{=} C, \qquad \hat{D} \overset{\sigma}{=} D, \quad (9)$$

where $\overset{\sigma}{=}$ means that both sides of the equality must be evaluated on $\sigma^\pm$.

In order to impose the remaining junction conditions we need the normal forms to the hypersurface, defined by the vectors (4) and (5) through the condition $n_\mu e^\mu_a = 0$, so that they take the form

$$\begin{aligned} \boldsymbol{n}^+ &= \hat{A}\hat{B}(-\dot{r}\,dt + \dot{t}\,dr)|_{\sigma^+}, \\ \boldsymbol{n}^- &= \tilde{\epsilon}A(-\dot{\rho}\,dT + \dot{T}\,d\rho)|_{\sigma^-}, \end{aligned} \quad (10)$$

where we have chosen them with the same modulus at both sides of the hypersurface by using (8), and the sign $\tilde{\epsilon}$ defines their two possible relative orientations [18,19]. (We cannot normalize the normals as we are treating with general hypersurfaces.) In the usual situations with a timelike matching hypersurface, the remaining junction conditions demand the equality of the second fundamental forms $K^\pm_{ab} = -n^\pm_\nu e^{\pm\mu}_a \nabla^\pm_\mu e^{\pm\nu}_b$ inherited by $\sigma^\pm$ from the $(\pm)$ space-times [10,11,16]. However, for the case of general hypersurfaces these junction conditions must be appropriately generalized. It is known [10] that the proper junction conditions for the general case are simply $H^+_{ab} \overset{\sigma}{=} H^-_{ab}$ where the symmetric tensors $H^\pm_{ab}$ generalizing the second fundamental forms are defined by

$$H^\pm_{ab} = -\ell^\pm_\nu e^{\pm\mu}_a \nabla^\pm_\mu e^{\pm\nu}_b, \quad (11)$$

$\vec{\ell}^{\,\pm}$ being the so-called rigging vectors, that is, any vector field defined only on $\sigma^\pm$ and *not* tangent to $\sigma^\pm$ (obviously, for the case of timelike hypersurfaces the normal vector is itself a rigging and hence H_{ab} coincides with the second fundamental form by choosing $\vec{\ell} = \vec{n}$). The above junction conditions $H^+_{ab} \overset{\sigma}{=} H^-_{ab}$ do not depend on the specific choice of the rigging [10].

The riggings are defined by $n^{\pm}_{\mu}\ell^{\pm\mu} \neq 0$, and in addition they must be chosen such that

$$\ell^{+}_{\mu}\ell^{+\mu} \stackrel{\sigma}{=} \ell^{-}_{\mu}\ell^{-\mu}, \qquad \ell^{+}_{\mu}e_{a}^{+\mu} \stackrel{\sigma}{=} \ell^{-}_{\mu}e_{a}^{-\mu}, \tag{12}$$

so as to assure their mutual identification. Therefore, a suitable choice for $\vec{\ell}^{\,\pm}$ is

$$\begin{aligned} \vec{\ell}^{\,+} &= -\frac{\dot{r}}{\hat{A}^2}\frac{\partial}{\partial t} + \frac{\dot{t}}{\hat{B}^2}\frac{\partial}{\partial r}\bigg|_{\sigma^+}, \\ \vec{\ell}^{\,-} &= G\left(-\alpha^2\frac{\dot{\rho}}{A^2}\frac{\partial}{\partial T} + \dot{T}\frac{\partial}{\partial \rho}\right)\bigg|_{\sigma^-}, \end{aligned} \tag{13}$$

where G and α are the solutions of the simple system of equations

$$\begin{aligned} -\frac{\dot{r}^2}{\hat{A}^2} + \frac{\dot{t}^2}{\hat{B}^2} &\stackrel{\sigma}{=} G^2\left(-\alpha^4\frac{\dot{\rho}^2}{A^2} + \dot{T}^2\right), \\ 2\dot{r}\dot{t} &\stackrel{\sigma}{=} G(\alpha^2 + 1)\dot{T}\dot{\rho}, \end{aligned} \tag{14}$$

in order to comply with (12). Then, the nontrivial junction conditions $H^{+}_{ab} \stackrel{\sigma}{=} H^{-}_{ab}$ come from $H_{\lambda\lambda}, H_{\phi\phi}$, and $H_{\zeta\zeta}$ and their explicit form reads

$$\begin{aligned} &\dot{r}\ddot{t} + \dot{t}\ddot{r} + \dot{r}\left(\frac{\hat{A}_{,t}}{\hat{A}}\dot{t}^2 + 2\frac{\hat{A}_{,r}}{\hat{A}}\dot{r}\dot{t} + \frac{\hat{B}\hat{B}_{,t}}{\hat{A}^2}\dot{r}^2\right) \\ &\quad + \dot{t}\left(\frac{\hat{B}_{,r}}{\hat{B}}\dot{r}^2 + 2\frac{\hat{B}_{,t}}{\hat{B}}\dot{r}\dot{t} + \frac{\hat{A}\hat{A}_{,r}}{\hat{B}^2}\dot{t}^2\right) \\ &\stackrel{\sigma}{=} G\left(\alpha^2\dot{\rho}\ddot{T} + \dot{T}\ddot{\rho} + 2\alpha^2\dot{\rho}^2\dot{T}\frac{A_{,\rho}}{A} + \dot{T}^3AA_{,\rho}\right), \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{r}\frac{\hat{C}_{,t}}{\hat{A}^2} - \dot{t}\frac{\hat{C}_{,r}}{\hat{B}^2} &\stackrel{\sigma}{=} -G\dot{T}C_{,\rho}, \\ \dot{r}\frac{\hat{D}_{,t}}{\hat{A}^2} - \dot{t}\frac{\hat{D}_{,r}}{\hat{B}^2} &\stackrel{\sigma}{=} -G\dot{T}D_{,\rho}, \end{aligned} \tag{16}$$

where in (16) we have used (9). Equations (8), (9), and (14)–(16) constitute the full set of *matching conditions.*

Let us see what information we can extract from the matching conditions. First of all, the derivatives along the hypersurface of (9) give

$$\dot{t}\hat{C}_{,t} + \dot{r}\hat{C}_{,r} \stackrel{\sigma}{=} \dot{\rho}C_{,\rho}, \qquad \dot{t}\hat{D}_{,t} + \dot{r}\hat{D}_{,r} \stackrel{\sigma}{=} \dot{\rho}D_{,\rho}. \tag{17}$$

Combining (17) and (16) we get the following statements (otherwise σ would not exist):

(i) $\hat{D}_{,t} \stackrel{\sigma}{=} \hat{C}_{,r} \stackrel{\sigma}{=} \hat{D}_{,r} \stackrel{\sigma}{=} \hat{C}_{,t} \stackrel{\sigma}{=} 0 \Longleftrightarrow D_{,\rho} \stackrel{\sigma}{=} C_{,\rho} \stackrel{\sigma}{=} 0$.

(ii) $\hat{D}_{,t} \stackrel{\sigma}{=} \hat{D}_{,r} \stackrel{\sigma}{=} 0 \Longrightarrow D_{,\rho} \stackrel{\sigma}{=} 0$.

(iii) $\hat{C}_{,t} \stackrel{\sigma}{=} \hat{C}_{,r} \stackrel{\sigma}{=} 0 \Longrightarrow C_{,\rho} \stackrel{\sigma}{=} 0$.

(iv) $\dot{r} = 0 \Longleftrightarrow \dot{\rho} = 0$, and then necessarily $\hat{C}_{,t} \stackrel{\sigma}{=} \hat{D}_{,t} \stackrel{\sigma}{=} 0$.

(v) $\dot{t} = 0 \Longleftrightarrow \dot{T} = 0$, and then necessarily $\hat{C}_{,t} \stackrel{\sigma}{=} \hat{D}_{,t} \stackrel{\sigma}{=} 0$.

Thus, we can consider two different cases:

Case (a): This case is defined by $D_{,\rho} \stackrel{\sigma}{=} C_{,\rho} \stackrel{\sigma}{=} \hat{D}_{,t} \stackrel{\sigma}{=} \hat{D}_{,r} \stackrel{\sigma}{=} \hat{D}_{,r} \stackrel{\sigma}{=} \hat{C}_{,t} \stackrel{\sigma}{=} 0$, and the whole system of matching conditions is completed with (8), (9), (14), and (15). This case must be treated separately because (16) and (17) are satisfied identically and do not provide any further information.

Case (b): This is the generic case defined by $C^2_{,\rho} + D^2_{,\rho} \stackrel{\sigma}{\neq} 0$. In this case, taking into account the above statements (i), (ii), and (iii), Eqs. (17) and (16) lead to

$$D_{,\rho}\hat{C}_{,t} \stackrel{\sigma}{=} C_{,\rho}\hat{D}_{,t}, \qquad D_{,\rho}\hat{C}_{,r} \stackrel{\sigma}{=} C_{,\rho}\hat{D}_{,r}, \tag{18}$$

and these two equations imply in turn

$$\hat{D}_{,t}\hat{C}_{,r} - \hat{D}_{,r}\hat{C}_{,t} \stackrel{\sigma}{=} 0, \tag{19}$$

which is an important relation since it involves *only* quantities of the exterior (+) space-time. We shall call it the *exterior condition.*

Without loss of generality, we can always assume that $C_{,\rho} \stackrel{\sigma}{\neq} 0$ for case (b) due to the symmetry of the equations under the simultaneous interchange of the D and C functions. Then, after substituting the explicit values of G and α coming from (14), the complete set of matching conditions for the case (b) can be finally written after some long but straightforward calculations as

$$AC_{,\rho}\dot{T} \stackrel{\sigma}{=} \tilde{\epsilon}\left(\hat{A}\frac{\hat{C}_{,r}}{\hat{B}}\dot{t} + \hat{B}\frac{\hat{C}_{,t}}{\hat{A}}\dot{r}\right), \tag{20}$$

$$C^2_{,\rho} \stackrel{\sigma}{=} \frac{\hat{C}^2_{,r}}{\hat{B}^2} - \frac{\hat{C}^2_{,t}}{\hat{A}^2}, \tag{21}$$

$$\begin{aligned} \dot{T}C^2_{,\rho}A_{,\rho} &\stackrel{\sigma}{=} \left(\frac{\hat{C}^2_{,r}}{\hat{B}^2} - \frac{\hat{C}^2_{,t}}{\hat{A}^2}\right)\left(\frac{\hat{A}_{,r}}{\hat{B}}\dot{t} + \frac{\hat{B}_{,t}}{\hat{A}}\dot{r}\right) - \frac{\hat{C}_{,t}}{\hat{A}}\frac{\hat{C}_{,r}}{\hat{B}} \\ &\quad \times\left(\frac{\hat{B}_{,t}}{\hat{B}}\dot{t} + \frac{\hat{B}_{,r}}{\hat{B}}\dot{r}\right) + \frac{\hat{C}_{,t}}{\hat{A}}\frac{\hat{C}_{,r}}{\hat{B}}\left(\frac{\hat{A}_{,t}}{\hat{A}}\dot{t} + \frac{\hat{A}_{,r}}{\hat{A}}\dot{r}\right) \\ &\quad - \frac{\hat{C}_{,r}}{\hat{B}}\left(\frac{\hat{C}_{,tt}}{\hat{A}}\dot{t} + \frac{\hat{C}_{,tr}}{\hat{A}}\dot{r}\right) + \frac{\hat{C}_{,t}}{\hat{A}}\left(\frac{\hat{C}_{,tr}}{\hat{B}}\dot{t} + \frac{\hat{C}_{,rr}}{\hat{B}}\dot{r}\right), \end{aligned} \tag{22}$$

together with (9) and (19) [and one of the (18) if $\dot{\rho} = 0$].

Consider then the pertinent physical problem of determining the interior (−) space-time given the exterior one. Thus, we assume that we know $\hat{A}, \hat{B}, \hat{C}, \hat{D}$ *explicitly.* The first thing to do is check whether or not the exterior condition (19) is satisfied. If (19) holds identically, which may happen, for example, when $\hat{C} = \hat{D}$, or may lead to cases (i)–(iii), then no information comes from it.

In general, however, (19) will not be satisfied identically, and therefore it gives an explicit relation between $t(\lambda)$ and $r(\lambda)$ or, in other words, it provides the hypersurface σ^+ as seen from the exterior. Then, the (−) space-time will get completely determined, except in the above especial case (iv), as follows: from (21) we obtain $C_{,\rho}$ on the hypersurface. Then, from the derivative (17) of (9) we find $\dot{\rho}$, and thereby $\rho(\lambda)$ up to an additive constant. The combination of this $\rho(\lambda)$ and (9), taking into account that C and D depend only on ρ, determines completely the functions $C(\rho)$ and $D(\rho)$ [and (18) is automatically satisfied]. Then, from (22), (20), and $\rho(\lambda)$ we get $A_{,\rho}/A$, and upon integration the function $A(\rho)$ up to an irrelevant multiplicative constant. Finally, the last relation (20) provides $T(\lambda)$, which together with $\rho(\lambda)$ gives the hypersurface σ^- as seen from the interior explicitly. (We have tacitly assumed that $\dot{T} \neq 0$. If $\dot{T} = 0$ the reasoning is identical, but A is not determined.) In summary, we have proved that a general nonstatic metric (1) can be matched to a static metric (2) *provided there exists a hypersurface in which the exterior condition (19) holds.* Furthermore, the static space-time is completely determined in general.

With all this information at hand, let us finally attack the problem for a RW exterior. The RW metric in explicitly cylindrically symmetric form is [3]

$$ds^{2+} = -dt^2 + a^2(t)[dr^2 + \Sigma^2(r,\epsilon)\,d\varphi^2 + \Sigma^2_{,r}(r,\epsilon)\,dz^2], \qquad (23)$$

where $a(t)$ is the scale factor and $\Sigma(r,\epsilon)$ satisfies $\Sigma^2_{,r} = 1 - \epsilon\Sigma^2$ with $\epsilon^3 = \epsilon$, or equivalently

$$\Sigma(r,\epsilon) = \begin{cases} \sinh r, & \epsilon = -1, \\ r, & \epsilon = 0, \\ \sin r, & \epsilon = 1, \end{cases} \qquad (24)$$

where ϵ is the curvature index so that $\epsilon = 1, 0, -1$ for closed, flat, or open RW models, respectively. Thus, we have now $\hat{A} = 1$, $\hat{B} = a$, $\hat{C} = a\Sigma$, and $\hat{D} = a\Sigma_{,r}$, and the necessary exterior condition (19) becomes

$$\begin{aligned} 0 &\overset{\sigma}{=} (a\Sigma_{,r})_{,t}(a\Sigma)_{,r} - (a\Sigma_{,r})_{,r}(a\Sigma)_{,t} \\ &= aa_{,t}(\Sigma^2_{,r} + \epsilon\Sigma^2) = aa_{,t}\,, \end{aligned} \qquad (25)$$

that is

$$a_{,t} \overset{\sigma}{=} 0\,. \qquad (26)$$

This means that either $\dot{t} \overset{\sigma}{=} 0$ or a is constant in a neighborhood of σ. In the first case the matching hypersurface is spacelike, and in the second case the RW metric is, in fact, static. Therefore, we have arrived at the main result: *a nonstatic RW space-time cannot be matched to a cylindrically static metric across a nonspacelike hypersurface.* This implies that the cylindrical analog of the Einstein-Straus problem has no solution.

For the sake of completeness, let us consider the possible matching defined by (26) for a nonstatic RW metric. From (26) we know that the matching hypersurface must be the spacelike hypersurface $\sigma^+ : t = t_0$ (correspondingly $\sigma^- : T = T_0$) where the expansion of the RW model vanishes, *if it exists.* The complete set of the remaining matching conditions, after a short manipulation, is then

$$a\Sigma \overset{\sigma}{=} C, \qquad a^2 \overset{\sigma}{=} \epsilon C^2 + D^2, \qquad C^2_{,\rho} \overset{\sigma}{=} \Sigma^2_{,r}\,. \qquad (27)$$

In this case, the static partner (we do not use the term "interior" because the matching hypersurface is an instant of time) is determined and the metric functions are

$$C(\rho) = a(t_0)\Sigma\left(\frac{\rho - n}{a(t_0)}, \epsilon\right),$$
$$D(\rho) = a(t_0)\sqrt{1 - \epsilon\Sigma^2\left(\frac{\rho - n}{a(t_0)}, \epsilon\right)}, \qquad (28)$$

where n is a constant and $A(\rho)$ is arbitrary. In particular, the matching of a RW space-time with a static *vacuum* metric [15] is possible only for a flat ($\epsilon = 0$) RW with Minkowski space-time. An explicit smooth example can be found in Ref. [20].

As a closing remark, we can notice that the negative result in this Letter supports the known instability of the Einstein-Straus model [3]. Furthermore, our results seem to imply that the problem of the influence of the expansion on the space surrounding individual objects has no satisfactory answer in general relativity yet.

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