New Universality at the Magnetic Field Driven Insulator to Integer Quantum Hall Effect Transitions

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We present a magnetotransport study of a disordered two-dimensional hole system in a strained Ge quantum well. As the magnetic field is increased, a clear transition from a low magnetic field insulator to the $\nu = 1$ quantum Hall state at the lowest density range (controlled by a gate), and to the $\nu = 3$ state at higher densities, is observed. We find that these transitions are characterized by a new universality: At the critical point, the diagonal and Hall resistivities are equal, within experimental uncertainty. These results are in conflict with the "floating" scenario suggested by Khmel'nitzkii [JETP Lett. **38**, 552 (1983)] and Laughlin [Phys. Rev. Lett. **52**, 2304 (1984)]. [S0031-9007(97)02740-3]

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Shortly following the discovery of the integer quantum Hall effect (IQHE), it became clear that its observation requires the existence of extended states below the Fermi energy. Since it is believed that, in the absence of magnetic field (B), all two-dimensional electron systems (2DES's) are insulating [1], Khmel'nitzkii [2] and Laughlin [3] suggested that the energies of these extended states, E_n, "float" up from the center of the Landau level (LL) and follow

$$E_n = (n + 1/2)\hbar\omega_c\{1 + 1/(\omega_c\tau)^2\}, \qquad (1)$$

where $\omega_c = eB/m^*$ is the cyclotron frequency, m^* is the effective mass, and τ is the classical elastic collision time. Indeed, $E_n \to \infty$ as $B \to 0$, rendering all electronic states in 2DES localized at $B = 0$. This conjecture has been the center of a recent debate [4–6].

Building on the notion of floating, and extending it to the fractional quantum Hall regime by means of a bosonic Chern-Simons mapping, Kivelson-Lee-Zhang (KLZ) [7] proposed the existence of a global description of 2DES at high B. This description can be summarized by a set of correspondence rules relating the physical properties of the system at different LL filling factors (ν's), and a selection rule that forbids transitions to the insulating phase from all but the principal QHE states, such as $\nu = 1, 1/3, 1/5$, etc. By successively applying these rules, KLZ constructed a theoretical phase diagram for 2DES in the disorder-B plane (top-left inset of Fig. 1).

While it appears that most experiments are in support of the correspondence principle of KLZ, their selection rule, which stems from the floating picture, seems to be at odds with many recent results [8]. One can crudely categorize the observed violations of the selection rule into two classes. The first class is that of very high mobility samples, where the forbidden transitions are attributed to the formation of a Wigner solidlike order in the system [9], which is not taken into account in the KLZ theory. The transitions to the insulating phase in this class have been recently discussed in Ref. [10].

In this paper, we center our attention on the second class, which is that of very low mobility samples which exhibit insulating behavior at $B = 0$ and undergo a transition to the IQHE phase at higher B. In such samples, transitions between low-B insulating behavior and either the $\nu = 1$ [11] or the $\nu = 2$ IQHE [12–14] states were observed. This apparent inconsistency had prompted

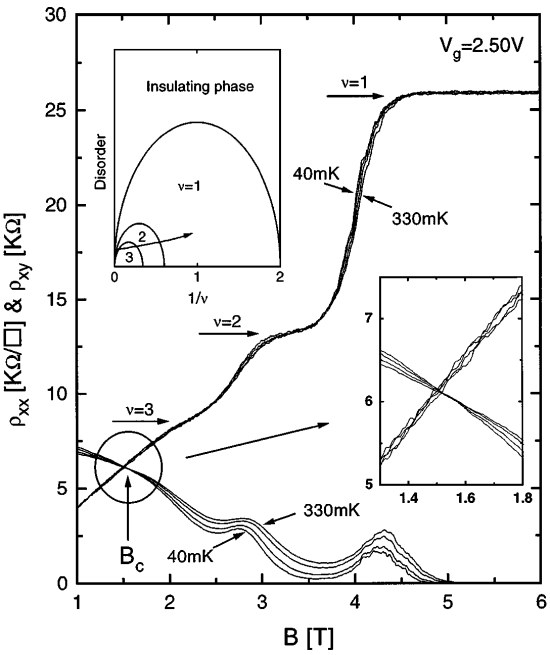

FIG. 1. ρ_{xx} and ρ_{xy} versus magnetic field (B) for $V_g = 2.5$ V at $T = 40$, 130, 230, and 330 mK. B_c (1.56 T) denotes the critical magnetic field at which the transition from an insulator to QHE occurs. The top-left inset shows the IQHE portion of the theoretical phase diagram suggested by KLZ. The arrow indicates a possible trajectory for a sample. The bottom-right inset shows a magnified view around the transition.

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theoretical attempts to incorporate the spin degree of freedom in the KLZ framework [15]. It is clear that the discrepancies among experiments on samples of different materials and parameters necessitate further studies.

A 2D hole system (2DHS) in a strained Ge layer, where the spin splitting of the lowest LL is calculated to be about one-third of the cyclotron gap [16], makes a natural choice for this purpose. We performed a systematic study on this system where the hole density (p) could be varied over a wide range of $0.7 \times 10^{11} < p < 5.4 \times 10^{11}$ cm^{-2} by a metallic front gate. We focus on the low density range where the system exhibits insulating behavior at low B, followed by a transition to a QHE phase at higher B. Two main findings will be discussed. The first is the observation of clear, well-defined, transitions from the insulator to either the $\nu = 1$ IQHE state at the lowest densities or, for the first time, the $\nu = 3$ IQHE state at higher densities. The second finding is that, on the insulator to IQHE phase boundary, the transition points are characterized by the equality of the diagonal and Hall transport coefficients, $\rho_{xxc} = \rho_{xyc}$, within experimental error. Both findings are in apparent conflict with the KLZ theory and the floating scenario.

The Ge layer, which contains the 2DHS, is under a compressive strain that significantly modifies the valence band structure. The strain removes the degeneracy of the valence band at zone center, leaving a single, doubly degenerate band as the lowest-energy state. The strain-split energy difference is estimated to be $\sim$110 meV at zone center, using a $\boldsymbol{k} \cdot \boldsymbol{p}$ Luttinger model that includes the strain but neglects confinement effects [17]. The effective mass of the 2DHG in the strained Ge channel at low magnetic field was measured to be $0.1m_e$ from cyclotron resonance [18], where m_e is the bare electron mass. The Fermi energy estimated from the 2D hole density using this mass value is approximately 13 meV for the highest hole density in our study, $p = 5.4 \times 10^{11}$ cm^{-2}. Therefore, band nonparabolicity and valence band mixing effects are not significant in this study.

The sample we studied was grown by the molecular-beam epitaxy (MBE) technique. It consists of a graded buffer $Si_{1-y}Ge_y$ layer grown on a Si substrate, followed by a uniform buffer $Si_{0.4}Ge_{0.6}$ layer, and a 150 Å thick Ge layer sandwiched in between $Si_{0.4}Ge_{0.6}$ layers, where Boron modulation doping is placed [19]. In this structure, the $Si_{0.4}Ge_{0.6}$ layers are fully relaxed, while the Ge layer is under 1.6% compressive strain. In order to continuously vary the effective disorder *in situ*, a gated structure is fabricated to change the density that modifies the screening of the disorder potential. A simple metal-semiconductor field-effect transistor (MESFET) structure was not successful due to the lack of a good Schottky barrier forming metal. To circumvent this difficulty, a metal-oxide-semiconductor field-effect transistor (MOSFET) structure was made by growing an insulating layer between a metal and the cap layer. A low temperature insulator, plasma-enhanced chemical-vapor-deposition (PECVD) silicon oxynitride, was used to keep the sample structure intact. A standard Hall bar sample was prepared with a Ti/Al gate and Al/Au alloyed ohmic contacts. The channel is 50 μm wide and the voltage probes are 200 μm apart.

The measurements were done in a dilution refrigerator at temperatures (T's) ranging from 30 to 350 mK in a magnetic field up to 13 T. A phase sensitive four-terminal ac lock-in technique was used with an excitation current of 0.1 nA to minimize heating of the sample. dc measurements were occasionally performed to check for consistency. From the low field Hall effect and the Shubnikov–de Haas (SdH) measurements, it was found that the sample had the density of 5.4×10^{11} cm^{-2} and mobility of 13 000 cm^2/V sec at zero gate voltage. By changing the gate voltage (V_g), we could lower the density down to 0.7×10^{11} cm^{-2}, at $V_g = 3.1$ V. The gate leakage current was negligible within the capability of our experimental setup ($<10^{-11}$ A) throughout our gate voltage range.

In Fig. 1, we show the diagonal, ρ_{xx}, and Hall, ρ_{xy}, resistivities at $V_g = 2.50$ V and $T = 40$, 130, 230, and 330 mK. Both resistivities are measured by the sheet and Hall resistances, respectively. At high B ($B > 4.5$ T), a fully developed $\nu = 1$ IQHE is observed, with vanishing ρ_{xx}, and ρ_{xy} quantized to h/e^2. Well-developed $\nu = 2$ and $\nu = 3$ IQHE states are also present at lower B values, characterized by strong deviations from the linear dependence of ρ_{xy} on B, and the accompanying minima in the ρ_{xx} traces. At $B = 1.56$ T, there is a clear transition magnetic field, B_c, where the ρ_{xx} value is temperature independent. For $B < B_c$, the system behaves like an insulator in the sense that ρ_{xx} increases as temperature decreases. At $B = 0$, the values of ρ_{xx} are 8.44, 8.28, 8.12, and 8.00 K Ω for the T's shown here. On the other hand, above B_c, ρ_{xx} decreases as the temperature decreases and ρ_{xy} starts developing a plateau, indicating the quantum Hall state. We thus identify the T independent point B_c as the insulator to the $\nu = 3$ IQHE transition point.

The observation of the low-B insulator to the $\nu = 3$ quantum Hall state transition is a clear violation of the KLZ selection rule [8,9], which is one of the key ingredients in constructing the global phase diagram [7]. Consider the phase diagram in the top-left inset of Fig. 1. Since all higher IQHE states are completely nested inside the $\nu = 1$ to insulator phase boundary, the allowed transitions involving the insulator can only take place along this phase separation line, i.e., only the insulator to $\nu = 1$ (0–1) transition is allowed. Therefore, any direct transition between an insulating phase and an IQHE state with $\nu > 1$ is a possible violation of the KLZ phase diagram, and may be regarded as inconsistent with the floating picture. We wish to point out that while earlier observations of a direct insulator to $\nu = 2$ (0–2) transition, which appears to be in conflict with the theoretical phase diagram, were reconciled by assuming that the lowest LL is spin degenerate [12], the insulator

to $\nu = 3$ transition cannot be explained using similar reasoning, since the $\nu = 3$ IQHE takes place at the second LL, separated from the first by a cyclotron (rather than spin) gap. It is possible that, at these low-B transitions, significant mixing of the states at different LL's takes place, and the simple floating picture should be modified accordingly.

To show the evolution of the sample as V_g or, equivalently, the disorder increases, we plot in Fig. 2 the two sets of B field traces of ρ_{xx} taken at $V_g = 2.72$ V [Fig. 2(a)] and 2.85 V [Fig. 2(b)], for several T's ranging from 30 to 350 mK. The insets show the traces of ρ_{xy} at $T = 120$ mK and the magnified views of the crossing points. At $V_g = 2.72$ V, $\nu = 2$ and $\nu = 3$ IQHE states are on the verge of disappearing, while the $\nu = 1$ IQHE state still remains strong, as is evident from the wide minimum where ρ_{xx} is vanishingly small. At $V_g = 2.85$ V, all the IQHE states have disappeared, except the $\nu = 1$ IQHE state. The low-B insulator to IQHE transition B_c's are 1.90 T for $V_g = 2.72$ V, and 2.19 T for $V_g = 2.85$ V. At $B \approx 9$ T, the system undergoes another transition from $\nu = 1$ to insulator. Shahar *et al.* [20] performed an extensive study of this transition and showed that, at the transition point, ρ_{xx} appears to be universal and close to h/e^2. In our sample, the contacts deteriorate at higher B and this transition is not so well resolved.

In Fig. 3, we plot the values of ρ_{xx} and ρ_{xy} at the low-B transition point, for all the transitions we observed. As V_g changes from 2.1 to 3.0 V, the values of ρ_{xxc} and ρ_{xyc} monotonically increase, (faster than a power law or an exponential form), and appear to saturate at the value of the quantum of resistance, h/e^2. Interestingly, for all transitions, ρ_{xxc} is very close in value to ρ_{xyc}. The same relation is also apparent in previously reported experiments on GaAs samples [11,13,14,21], even though the ρ_{xyc} (and ρ_{xxc}) range is limited in those cases. Incidentally, we note that this relation also holds for the $\nu = 1$ to insulator transition [22], but not for the $\nu = 1/3$ to insulator transition, where $\rho_{xyc} \approx 3$ but $\rho_{xxc} \approx 1$, both in units of h/e^2 [20,23].

Next, we consider the possible consequences of the $\rho_{xxc} = \rho_{xyc}$ observation. Since we are unaware of any realistic model pertaining to the insulator-QHE transition, we will limit our discussion to the framework of the Drude model, keeping in mind that it is not, *a priori*, applicable to either the critical point of the transition or the insulating and QHE phases. In the Drude model, $\rho_{xxc} = \rho_{xyc}$ implies that $\omega_c\tau = 1$. First, we note that $\omega_c\tau = 1$ also implies $l_B\sqrt{\nu} = l_{\rm el}$, where application of the KLZ theory becomes dangerous for small ν's [7]. Here l_B and $l_{\rm el}$ are the magnetic length and the elastic mean free path, respectively. While, on general grounds, one indeed expects a crossover from the low-B behavior to the QHE regime, where $l_B \sim l_{\rm el}$, our observation of a clear, well-defined, critical transition point, where $l_B\sqrt{\nu} = l_{\rm el}$, warrants further theoretical investigations. Second, according to Eq. (1), the energy of the extended states is at a minimum when $\omega_c\tau = 1$. It is hard to see how the transition between the insulator and the IQHE phase, which, in the floating picture, is to occur when the Fermi energy crosses E_n, would coincide with a minimum of E_n.

Finally, in Fig. 4, we construct an experimental phase diagram by using our values of B_c together with the

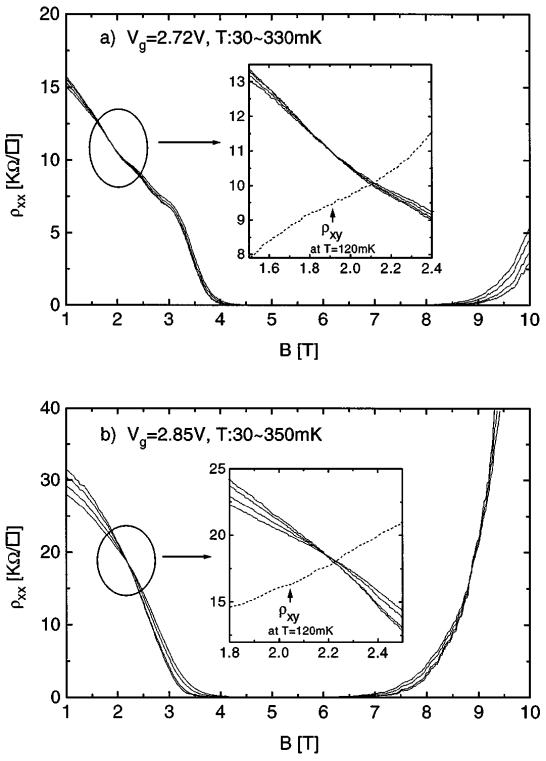

FIG. 2. ρ_{xx} vs B traces for two V_g values. The insets show traces of ρ_{xy} at 120 mK and magnified views of the crossing points. (a) $V_g = 2.72$ V and $T = 30$, 120, 240, and 320 mK. (b) $V_g = 2.85$ V and $T = 30$, 120, 250, and 350 mK.

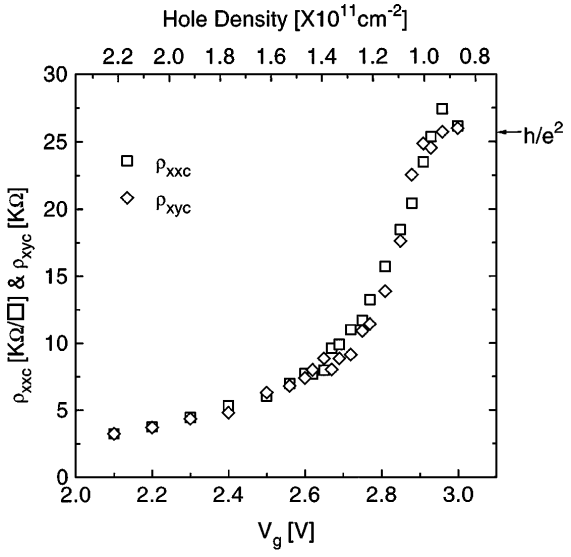

FIG. 3. The critical resistivities, ρ_{xxc} and ρ_{xyc}, versus the applied gate voltage. The top axis shows the corresponding hole density. The arrow on the right axis indicates the quantum of resistance, h/e^2.

position of ρ_{xx} peaks between the quantum Hall plateaus at different gate voltages. We also included the transition points from $\nu = 1$ QHE to the high-B insulator, deduced from the value of B where $\rho_{xx} = h/e^2$, which is where the transition to insulator is expected to occur [20]. This phase diagram clearly shows the transition from a low-B insulator to the $\nu = 3$ QHE state in $V_g \leq 2.75$ V. At $V_g > 2.85$ V, an insulator to $\nu = 1$ QHE transition exists. Because of the finite T in our experiments, we are unable to resolve whether, for our sample, a direct transition to the $\nu = 2$ IQHE can occur at $2.75 < V_g < 2.85$ V. Notice also that the transition boundary between the low-B insulator and the $\nu = 1$ IQHE appears to be only weakly dependent on magnetic field, and occurs at $B \sim 2$ T. On the other hand, the boundary between the $\nu = 1$ IQHE state and the high-B insulator follows the $\nu = 1/2$ line deduced from the corresponding hole density.

The inconsistency of our results with the floating scenario can be most conveniently expressed by comparing our experimental phase diagram with the theoretical one (see the inset of Fig. 1). Because of the characteristic of the floating scenario, transition patterns such as insulator-1-2-3-2-1-insulator or insulator-1-2-1-insulator are expected. Such patterns have not been observed in our experiment or, as far as we know, in previous experiments.

In summary, we conducted a magnetotransport study of a highly disordered 2DHG in a strained Ge channel MOSFET structure. We observe a transition from a low-B insulator to $\nu = 3$ IQHE as well as a transition from an insulator to $\nu = 1$ IQHE as the disorder is increased. For all the transitions we observed, we find that the values of the diagonal resistivity at the transition are equal to those of the Hall resistivity within experimental error. Using our results, we construct an experimental phase diagram in the disorder versus magnetic field plane. Our results are in conflict with the floating scenario as an explanation for the low-B transitions between the insulator and the IQHE phase.

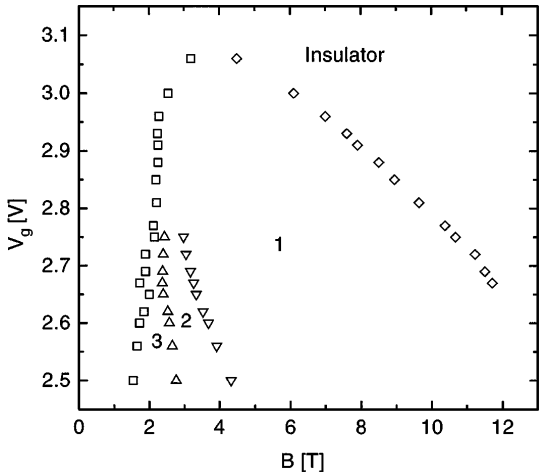

FIG. 4. The experimental phase diagram for 2DHG in our strained Ge quantum well. Different symbols denote different transitions; (□) low-B insulator to QHE transition, (△) $\nu = 3$ to $\nu = 2$ QHE transition, (▽) $\nu = 2$ to $\nu = 1$ QHE transition, and (◇) $\nu = 1$ to high-B insulator transition deduced from B values where $\rho_{xx} = h/e^2$.

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