Decay of High Order Optical Vortices in Anisotropic Nonlinear Optical Media

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We present an experimental and theoretical study of the decay of high order optical vortices in media with an anisotropic nonlocal nonlinearity. Vortices with charge n decay into an aligned array of n vortices of unit charge. [S0031-9007(97)02727-0]

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Vortical excitations with quantized circulation appear in superfluids, superconductors, and nonlinear optics in the presence of a repulsive or self-defocusing nonlinearity. The nonlinear Schrödinger equation with cubic nonlinearity has been used extensively to describe vortex solitons in two transverse dimensions, as was done first in the context of vortex lines in superfluids by Ginzburg and Pitaevskii [1]. Much recent attention has been attracted by vortices and vortex solitons in nonlinear optics [2,3]. Idealized Kerr-type optical media with a cubic, isotropic, and local nonlinearity have been shown experimentally to support stable $(2 + 1)D$ vortex solitons [4].

While vortices with unit topological charge have been studied extensively much less is known about higher order vortices. The cubic nonlinear Schrödinger equation $[\partial/\partial x - (i/2)\nabla^2]B(\vec{r}) = -i|B|^2 B(\vec{r})$ has vortex solutions with integer charge m of the form $B(\vec{r}) \sim f(r)e^{im\theta}$, where r is the radius, θ is the azimuthal angle, and $f(r) \sim r^{|m|}$ as $r \to 0$ and $f(r) \sim 1 - m^2/2r^2$ for $r \to \infty$. As was shown in Ref. [1] a high order vortex of charge m is energetically unfavorable compared to m vortices of unit charge. It is therefore often assumed that high order vortices will always decay. Nonetheless, the mode of decay of a high order vortex has not been established, and recent calculations suggest that in the context of the cubic nonlinear Schrödinger equation high order vortices are metastable [5]. The nonlinear metastability of high order vortices in Kerr media may be viewed as evidence for the weak interaction of unit-charge vortices in such media. It has been shown [6] that for large intervortex separation the dynamics of vortices satisfying the cubic nonlinear Schrödinger equation correspond to point vortex dynamics in an ideal two-dimensional fluid. A pair of point vortices with equal vorticity in an ideal fluid rotate about their mutual center, while maintaining a constant separation [7]. The corresponding rotation of quantized vortices with the same charge and a relatively large separation has been observed in both linear [8] and nonlinear [9] optics.

We show here that the weak interaction of optical vortices with the same topological charge, and the close analogy with point vortices in a fluid, are not general results. We present the first experimental demonstration of the decay of a high order vortex in a nonlinear medium with nonlocal anisotropic response. Upon breakup of the input high-charge vortex the resulting charge-one vortices repel each other and form an array aligned perpendicular to the anisotropy axis. The same scenario holds for several closely spaced charge-one vortices launched into the medium. They repel each other and their characteristic separation at the final stages of the instability may exceed their initial separation by orders of magnitude. Our results suggest that the high order vortex decay is also possible in local isotropic media and we outline the conditions for which it could be observed.

As a background for subsequent results we discuss two possible mechanisms that split a high-charge vortex into a set of charge-one vortices. The results of this splitting may subsequently be enhanced by the nonlinearity leading to the observation of vortex decay. The first mechanism is due to the fact that high order vortices are topologically unstable and separate into a set of charge-one vortices in the presence of a small amount of noise, even in the framework of linear optics. This circumstance was pointed out in Ref. [10] and can be illustrated as follows. The structure of the electromagnetic field near the center of a radially symmetric vortex of charge m in the presence of noise is given by $B(\vec{r}) + \epsilon(\vec{r})$, where $B = r^{|m|}\exp(im\theta)$, and $\epsilon = |\epsilon(\vec{r})|\exp[i\theta_\epsilon(\vec{r})]$ is a random complex function. Zeros of the total field are determined by the equation $B + \epsilon = 0$, the solution of which gives m roots $\vec{r} = \vec{r}_j$ $(j = 0 : m - 1)$

$$\vec{r}_j = |\epsilon(0)|^{1/|m|}\exp[i\pi(2j + 1)/m + i\theta_\epsilon(0)/m]. \quad (1)$$

The results of the above topological decay may not be very dramatic or even noticeable. The characteristic separation between the formed vortices is determined by the level of the noise. Subsequent linear diffraction does not change the situation considerably. If the level of noise is small, the set of charge-one vortices remains close together, compared to the characteristic width of a vortex core, and for many purposes may still be treated as a high-charge single vortex. Some nonlinearities such as a local Kerr response seemingly do not affect the situation appreciably [5]. The nonlocal anisotropic response obtained in photorefractive media with an externally applied electric field [11,12] changes the situation dramatically, resulting in a nonlinear instability of the noise-created charge-one vortices. The vortices repel each other and their final separation may exceed their initial separation (due to noise-induced splitting) by orders of magnitude.

The second mechanism is due to propagation effects. An elliptically shaped high-charge vortex embedded in a Gaussian beam may split into charge-one vortices as the result of linear diffraction when it propagates in the medium. In the case of anisotropic media the nonlinearity itself serves as the generator of the ellipticity regardless of the initial boundary conditions. In the case of isotropic media the ellipticity should be introduced explicitly by, e.g., a specific choice of anisotropic boundary conditions.

We believe that in our experiments both of the above mechanisms contribute to the vortex decay, though the numerics show that vortex decay in these media is possible even in the absence of any internal noise. We also predict that decay of a high order vortex should be observable in isotropic media if the input boundary conditions are anisotropic. The linear diffraction then splits the vortex into charge-one vortices. In the absence of the nonlinearity the separation of these resulting vortices is typically much less than the size of their cores. Even though a local Kerr nonlinearity does not change the intervortex separation appreciably as compared to the linear case, it significantly reduces the size of the vortex cores thereby making the vortices physically distinct.

Vortex decay was observed using a photorefractive crystal as the nonlinear medium. The photorefractive nonlinearity is proportional to the product of the materials Pockels coefficient with the static electric field generated by the optical beam. The electric field is found by solving a particular form of Poisson's equation for the electrostatic potential ϕ, with a source charge distribution due to light induced charge transport. The photorefractive nonlinearity is thus nonlocal. Given the electrostatic potential the perturbation to the refractive index is $\delta n_{ij} \sim r_{ijk} \partial \phi / \partial x_k$, where r is the electro-optic tensor. The anisotropic nature of r results in a highly anisotropic nonlinear response.

The experimental arrangement was similar to that used in Ref. [12]. A 30 mW He-Ne laser beam ($\lambda = 0.63\ \mu$m) was passed through a variable beam splitter and a system of lenses controlling the size of the beam waist. The beam was directed into a photorefractive crystal of SBN:60 doped with 0.002% by weight Ce. The beam propagated perpendicular to the crystal $\hat{c}$ axis ($= z$ axis), and was polarized along it. The crystal measured 19 mm along the direction of propagation, and was 5 mm wide along the $\hat{c}$ axis. The experimentally measured value of the relevant component of the electro-optic tensor was found to be equal to $r_{33} = 150$ pm/V. A variable dc voltage was applied along the $\hat{c}$ axis to control the value of the nonlinearity coefficient. The crystal was illuminated by a source of incoherent white light to increase the level of the effective saturation intensity. Images of the beam at the output face of the crystal were recorded with a CCD camera. Beams with embedded vortex structures were produced with the help of computer-generated holograms corresponding to the interference pattern created by the target structure of the field and a plane reference beam. Diffraction of the laser beam by the hologram read out the target structure and embedded it in the beam.

Figure 1 presents experimental observation of the instability of a charge-two vortex embedded in a Gaussian beam. Figure 1(a) shows the output beam with no applied voltage (zero nonlinearity). The output intensity distribution in Fig. 1(a) corresponds to an annular ring with internal and external diameters of about 106 and 260 μm, respectively. Figure 1(b) shows the interferogram of the output beam and a spherical reference wave. A double spiral demonstrates the existence of a charge-two vortex on the beam. Increasing the applied external voltage (the nonlinearity) in Figs. 1(c) and 1(d) to 300 and 840 V, respectively, results in splitting of the input vortex into two charge-one vortices, their increasing separation and alignment along the y axis, perpendicular to the applied field, and the direction of anisotropy. The same behavior but for vortices with charges of three and five is shown in Fig. 2. A slight tilting of the line of vortices away from the y axis is due to the combined effects of the anisotropy inducing alignment along y, and the phase structure of the vortices causing a clockwise rotation. Changing the sign of the

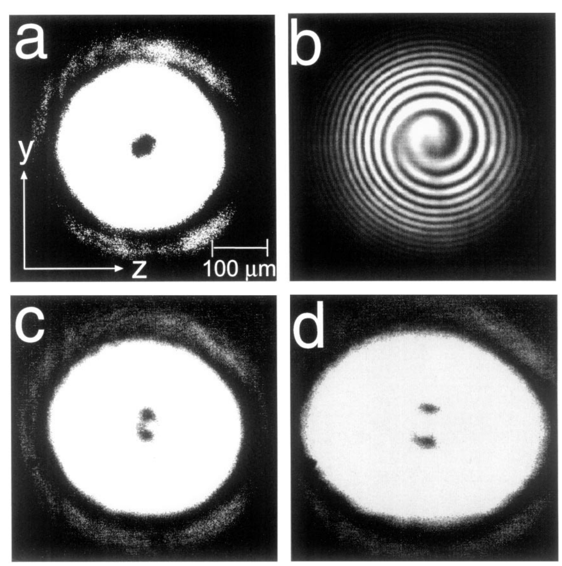

FIG. 1. Observed evolution of a charge-two vortex for applied voltages of 0 (a), 300 (c), and 840 V (d). Two unwinding spirals on the interferogram (b) demonstrate the existence of a double charge on the beam.

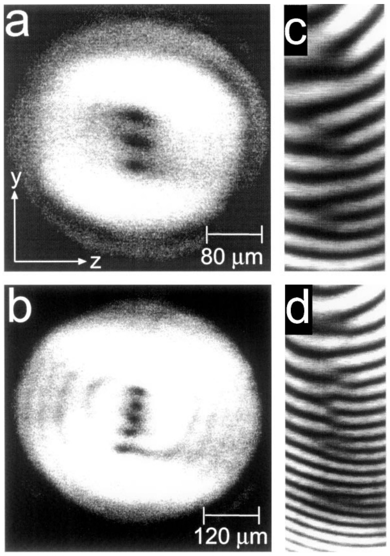

FIG. 2. Decay of charge-three (a) and charge-five (b) vortices obtained with applied voltages of 550 and 820 V, respectively. The corresponding interferograms are shown in (c) and (d).

input high order vortex leads to a slight tilting away from the y axis in the opposite direction. The asymptotic structure of a row of vortices of the same charge is an open question.

For theoretical interpretation of the results we use the set of equations describing propagation of an optical beam $B(\vec{r})$ in a photorefractive self-focusing or self-defocusing medium as developed in [11,12]

$$\left[\frac{\partial}{\partial x} - \frac{i}{2}\nabla^2\right]B(\vec{r}) = -i\,\frac{\partial \varphi}{\partial z}\,B(\vec{r}), \qquad (2a)$$

$$\nabla^2\varphi + \nabla \ln(1 + |B|^2)\cdot\nabla\varphi = \frac{\partial}{\partial z}\ln(1 + |B|^2). \qquad (2b)$$

Here $\nabla = \hat{y}(\partial/\partial y) + \hat{z}(\partial/\partial z)$ and φ is the dimensionless electrostatic potential induced by the beam with the boundary conditions $\nabla\varphi(\vec{r} \to \infty) \to 0$. The dimensionless coordinates (x, y, z) are related to the physical coordinates (x', y', z') by the expressions $x = \alpha x'$ and $(y, z) = \sqrt{k\alpha}\,(y', z')$, where $\alpha = (1/2)kn^2 r_{\mathrm{eff}} E_{\mathrm{ext}}$. Here k is the wave number of light in the medium, n is the index of refraction, r_{eff} is the effective element of the electro-optic tensor, and E_{ext} is the amplitude of the external field directed along the z axis far from the beam. The normalized intensity $|B(\vec{r})|^2$ is measured in units of saturation intensity I_d, so that the physical beam intensity is given by $|B(\vec{r})|^2 I_d$. The minus sign on the right hand side of Eq. (2a) corresponds to a self-defocusing nonlinearity. Equations (2a) and (2b) are valid for relatively wide beams and neglect the part of the nonlinearity responsible for asymmetric stimulated scattering since it is not important in the range of parameters discussed here (for details see [11]).

Numerical solutions of Eqs. (2a) and (2b) describing nonlinear evolution of a charge-two optical vortex for different values of the nonlinearity are shown in Fig. 3. The input intensity distribution of the field was taken to be

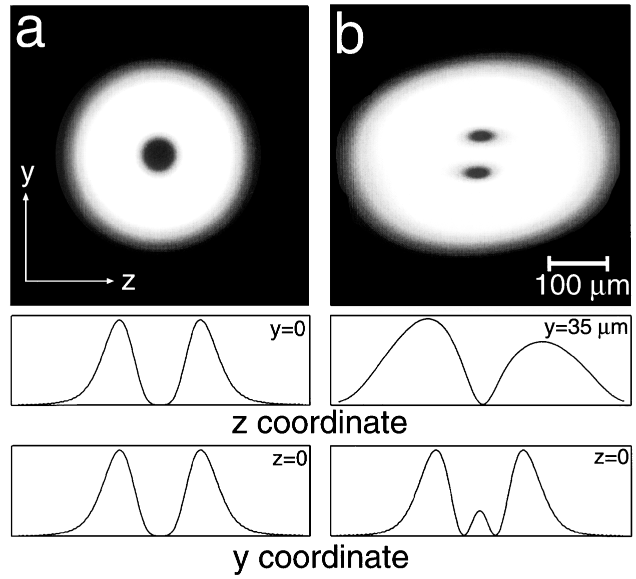

FIG. 3. Numerical results showing evolution of a charge-two vortex in photorefractive media for applied voltages of 0 (a) and 840 V (b). The curves below show the corresponding intensity profiles.

$$B(x = 0, \vec{r}) = \sqrt{I_{\mathrm{in}}}\, r \exp(-r^2/d_G^2 + 2i\theta), \qquad (3)$$

where $r = \sqrt{y^2 + z^2}$ and d_G is the diameter of the Gaussian beam. For $d_G = 22.5\ \mu$m the corresponding input intensity distribution is that of an annular ring with internal and external diameters of about 24 and 55 μm, respectively. The parameter I_{in} was chosen such that the maximum input intensity was about 0.3. The run was carried out without noise. A small amount of random noise added to the input optical field did not change the results in any qualitative way. The output intensity distribution in the absence of the nonlinearity is given by Fig. 3(a) and corresponds to an annular ring having approximately 115 and 260 μm internal and external diameters, respectively. Figure 3(b) corresponds to the applied voltage of 840 V and demonstrates instability and breakup of this vortex into two charge-one vortices that repel each other and move apart along the y axis. The magnitude of the effect is proportional to the value of the nonlinearity. The graphs under frames 3(a) and 3(b) show cross sections of the corresponding intensity distributions along the y and z axes.

Alignment of the output charge-one vortices along the y axis is due to the material anisotropy which prevents the nonlinear rotation of a corotating pair seen in isotropic Kerr media [9]. We have conducted calculations with a high order vortex plus different realizations of small superimposed noise, and also with several charge-one vortices placed close together in different configurations with respect to the anisotropy axis. In all cases the resulting charge-one vortices end up aligned along the y axis.

As an example Fig. 4 shows the spatial dynamics of a charge-three vortex embedded in a Gaussian beam. The input distribution of the field was chosen to be

$$B = \tanh^3(r/d_v)\exp(-r^2/d_G^2 - 3i\theta). \qquad (4)$$

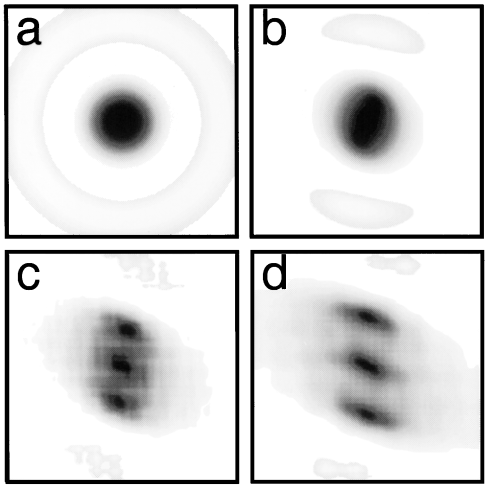

FIG. 4. Spatial intensity distribution of an input charge-three vortex at propagation distances of 0 (a), 7 (b), 14 (c), and 20 mm (d). The size of the frames is 150 μm.

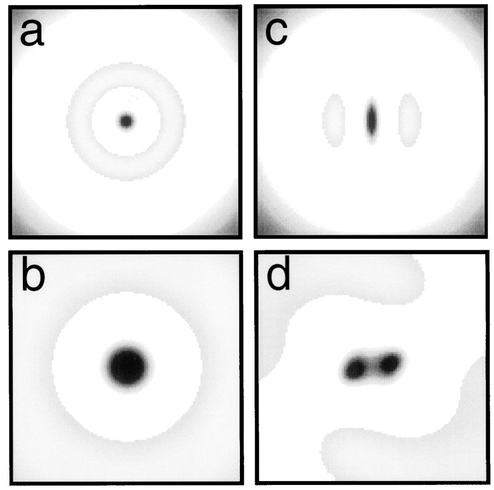

FIG. 5. Spatial intensity distribution of a round (a),(b) and elliptical (c),(d) charge-two vortex in an isotropic Kerr medium. Frames (a),(c) are the input; (b),(d)—the output.

Here $r = \sqrt{y^2 + z^2}$, and d_v and d_G are the diameters of the vortex and the Gaussian beam, respectively. The input intensity distribution of the field corresponded to an annular ring with internal and external diameters equal to 70 and 250 μm, respectively. Frames 4(a)–4(d) correspond to the intensity distribution inside the medium at distances $x = 0$, 7, 14, and 20 mm, respectively. The size of each frame is equal to 150 μm. As the beam propagates, the input vortex splits up into three charge-one vortices that orient themselves along the y axis perpendicular to the direction of the applied electric field [Figs. 4(b) and 4(c)]. Each of the resulting vortices is predominantly stretched along the direction of the anisotropy (the z axis). This stretching is in correspondence with our previous results on the dynamics of unit-charge vortices [13].

Results of numerical analysis of spatial evolution of a radially symmetric and elliptical charge-two vortex embedded in a Gaussian beam in a medium with isotropic Kerr nonlinearity is shown in Fig. 5. The analysis has been carried out for the cubic Schrödinger equation with the input boundary conditions taken as

$$B(x = 0) = \tanh\left(\sqrt{\frac{y^2}{d_{v,y}^2} + \frac{z^2}{d_{v,z}^2}}\right) \exp\left(-\frac{r^2}{d_G^2} + 2i\theta\right). \tag{5}$$

Frames 5(a) and 5(b) are the input ($x = 0$) and the output ($x = 20$) intensity distribution of a radially symmetric vortex $d_{v,z} = d_{v,y} = 3.5$ embedded in a Gaussian beam with $d_G = 15$. Frames 5(c) and 5(d) show the same but for an initially elliptic vortex with $d_{v,z} = 2.3$ and $d_{v,y} = 6.8$. Figure 5 demonstrates the metastability of a round vortex and the possibility of observing a high-charge vortex decay in isotropic media for anisotropic boundary conditions.

In summary we presented an experimental and theoretical study of the decay of high order optical vortices in media with an anisotropic nonlocal nonlinearity. Vortices with charge n decay into n vortices of unit charge and form a linear array aligned perpendicular to the direction of the anisotropy. Our results suggest that the decay of a high-charge vortex may be observed in local isotropic media for anisotropic boundary conditions.

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