Long-Lived States of Positronium in Crossed Electric and Magnetic Fields

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We show that crossed electric and magnetic fields provide a unique way for stabilizing simple matter-antimatter systems. Theoretical calculations on positronium in crossed fields predict the existence of long-lived states in which the average positron-electron separation is several thousand angstroms. These delocalized states are due to the existence of an outer well in the potential for certain values of the pseudomomentum and field strength. The near zero probability for positron-electron overlap suppresses direct annihilation processes. Transition moments between the ground state in the outer well and the Coulomb states are also extremely small, resulting in lifetimes up to the order of one year. [S0031-9007(96)02149-7]

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The lowest order decay rate for positronium is

$$\Gamma = \sigma v \rho \,, \tag{1}$$

where σ is the plane-wave cross section for free pair annihilation, v is the relative velocity of the electron and positron, and ρ is the square of the wave function evaluated at contact. Higher order corrections involve powers and logarithms of the fine structure constant α. For a complete review of the theory of positronium including decay rates, energy levels, and the Zeeman effect, see Refs. [1,2]. In this paper we predict the existence of states of positronium in crossed electric and magnetic fields in which the average electron-positron distance is several thousand angstroms, and the probability density within hundreds of angstroms of the contact point is nearly zero. Dipole transitions to the Coulomb states with nonzero probability at the origin are shown to be vanishingly small. We therefore propose that crossed fields can be used to stabilize positronium or other particle-antiparticle pairs.

The problem of two-body systems in crossed electric and magnetic fields has been a topic of intense investigation during the past decade [3–10]. Unlike the field-free problem, the center-of-mass motion of the system cannot be separated from the internal motion. When center-of-mass effects are treated exactly, the crossed fields problem and the problem of transverse motion in a purely magnetic field can be treated in a unified way, and the relative motions of the systems are equivalent. A conserved quantity for these systems is the pseudomomentum $\mathbf{K} = \mathbf{P} + \frac{e}{2}\mathbf{B} \times \mathbf{r}$, which represents a generalization of the field-free total momentum $\mathbf{P}$ to the case of external fields [11].

For neutral systems, one can carry out a pseudoseparation of the center of mass and internal motion [11,12]. The effective Hamiltonian for the internal motion is connected to the center-of-mass motion via the eigenvalue $\mathbf{K}$ of the pseudomomentum. Dippel *et al.* [13] showed the existence of a gauge-independent potential picture for neutral two-particle systems in external fields and derived an effective potential for the internal motion of the system. They further showed the existence of delocalized states of hydrogen for certain values of pseudomomentum and field strength.

For positronium, we consider the case of two equal masses $m_1 = m_2 = m_e$. Without loss of generality, we choose $\mathbf{B} = B\hat{\mathbf{z}}$, $\mathbf{E} = 0$, and $\mathbf{K} = K\hat{\mathbf{y}}$. Components of $\mathbf{K}$ parallel to $\mathbf{B}$ shift the energy only by a constant amount. The external electric field and the motional electric field due to the collective motion of the system in the magnetic field yield equivalent electric Stark terms in the Hamiltonian. Results for nonvanishing electric fields can be obtained from the zero field results by replacing $\mathbf{K}$ with $\mathbf{K}' = \mathbf{K} - 2m_e\mathbf{v}_D$; $\mathbf{v}_D = \mathbf{E} \times \mathbf{B}/B^2$ is the classical drift velocity in crossed fields. We use atomic units throughout ($v_o = c\alpha$): $B = mv_o/ea_o = 1$ a.u. corresponds to 2.350×10^5 T and $E = mv_o^2/ea_o = 1$ a.u. to 5.142×10^9 V/cm.

The effective Hamiltonian for the internal motion (neglecting spin) $H = T + V$ is given in Cartesian coordinates by

$$H = p^2 + \frac{B^2}{4}(x^2 + y^2) + \frac{BKx}{2} - \frac{1}{\sqrt{x^2 + y^2 + z^2}} + \frac{K^2}{4} \,. \tag{2}$$

Note that the gerade/ungerade symmetry in x is broken by the interaction with the motional electric field $BK/2$,

whereas y and z parity are still conserved quantities. If $K^3 > \frac{27}{2}B$, then the potential V has a local maximum at the saddle point x_s and an outer well with a local minimum at x_0 on the negative x axis. For positronium, such wells can easily be achieved by laboratory field strengths. Another important property of the potential V is its confining behavior for large distances perpendicular to the magnetic field. As a consequence ionization can occur only in a direction parallel to the magnetic field.

Additional corrections to the energy due to spin, relativistic, and QED effects are small and can be calculated using perturbation theory. We note that the field shifts the total energy (and the ionization energy) by a constant amount proportional to the electron and positron gyromagnetic ratio [1]. None of these corrections are expected to influence the prediction of the long-lived delocalized states.

For illustrative purposes, we choose $K = 0.4$ and $B = 5 \times 10^{-5}$, although the long-lived states of positronium are predicted for a range of values of K and B. Our choice of parameters is easily accessible by laboratory standards and motivated by the relatively small number of bound states in the Coulomb well which makes the computation tractable. In Fig. 1, we show an intersection of the potential V along the x direction. The potential has a local minimum in the outer well $V_0 = V(x_0 = -7987.46a_o) = -1.2510 \times 10^{-4}$ a.u. The local maximum at the saddle point $V_s = V(x_s = -322.81a_o) = 3.3739 \times 10^{-2}$ a.u. is well above the ionization energy $E_i = B = 5 \times 10^{-5}$ a.u.

While the ionization energy is independent of K, the saddle point and outer well can be greatly varied by changing the value of the pseudomomentum. As K decreases (with fixed B), x_0 and x_s approach each other and both V_s and V_0 decrease in energy; they coincide when $K^3 = \frac{27}{2}B$. As K increases, x_0 and x_s move away from each other, and both the well minimum and the saddle point increase in energy. The well minimum is always less than zero.

At $K = 0.4$, the ionization energy is much closer to the bottom of the well than to the saddle point. The barrier width at the ionization energy is over $7000a_o$. This suggests that the bound state spectrum of the exact Hamiltonian should be nearly identical to the spectra from the two isolated potential wells: a set of Coulomb states localized at the origin and a set of outer well states centered about the minimum.

At low fields, the low-lying states of the spectrum in the Coulomb well can be approximated using perturbation theory (PT). At $K = 0.4$, only states with $n \leq 2$ lie below the ionization threshold. The interaction with the field couples the $2s$ and $2p_x$ in first order; degenerate PT gives the splitting between these two states. Nondegenerate PT gives the first order shift to the $1s$, $2p_y$, and $2p_z$ levels. Second order corrections due to $BKx/2$ are of the same magnitude (10^{-8} a.u.) as the first order corrections due to $B^2(x^2 + y^2)/4$.

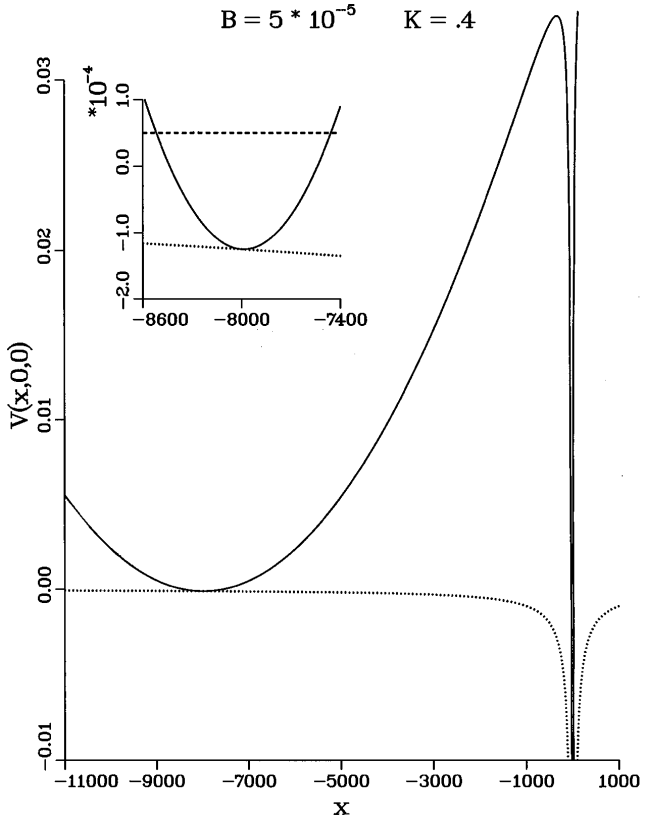

FIG. 1. Potential $V(x, y = 0, z = 0)$ of the internal motion of positronium at $B = 5 \times 10^{-5}$ and $K = 0.4$ (solid line). Also shown is the field free Coulomb potential for zero pseudomomentum (dotted line). The inset is an amplification of the outer well in the region of the minimum showing the ionization energy (dashed line).

An approximate spectrum for the outer well can be obtained by expanding the Coulomb potential around the minimum of the outer well [13]. Keeping terms to order x_0^{-3}, the potential has the form of an anisotropic harmonic oscillator (HO) with the well defined spectrum

$$E_{n_x n_y n_z} = \left(n_x + \frac{1}{2}\right)\omega_x + \left(n_y + \frac{1}{2}\right)\omega_y + \left(n_z + \frac{1}{2}\right)\omega_z + C\,, \tag{3}$$

where

$$\omega_x = \sqrt{\left(B^2 + \frac{4}{x_0^3}\right)},$$

$$\omega_y = \sqrt{\left(B^2 - \frac{2}{x_0^3}\right)},$$

$$\omega_z = \sqrt{\frac{-2}{x_0^3}},$$

$$C = \frac{2}{x_0} - \frac{B^2 x_0^2}{4} + \frac{K^2}{4}\,. \tag{4}$$

For $K = 0.4$ and $B = 5 \times 10^{-5}$, ω_z is less than 4% of ω_x and ω_y; all the low-lying excited states involve excitations in the z direction only.

The validity of this isolated potential approach depends on the amount of tunneling through the barrier, or in other words, the amount of mixing between the Coulomb states and the outer well states. If the barrier is sufficiently high and wide, the eigenstates of the the two isolated wells will not mix. The probability density will reach nearly zero near the middle of the barrier. If positronium is in an outer well state, the electron (or positron) cannot tunnel through the barrier and annihilate.

On the other hand, if probability density "leaks" through the barrier, tunneling processes will contribute to annihilation. The two wells can no longer be treated as isolated; for energies above the outer well minimum, the wave functions will mix leading to probability in both wells, and the energy levels will no longer be identical to those of the isolated systems. The annihilation rate will depend on the probability density at the origin.

In order to estimate the tunneling probability from the outer well ground state into the Coulomb well, we use the semiclassical approximation for the one-dimensional tunneling probability [14]

$$P = \exp\left[-2\int_{x_1}^{x_2} |p(x)|\,dx\right], \tag{5}$$

where x_1 and x_2 are the two classical turning points and $|p(x)| = \sqrt{V(x,0,0) - (\omega_x/2 + C)}$. Lifetimes for tunneling are expected to be of the order of some characteristic time $\tau \approx 2\pi/\omega_x$ divided by the probability P. For $B = 5 \times 10^{-5}$ and $0.0878 \geq K \geq 0.0908$, the harmonic ground state lies above V_s; there are no outer well states, only Coulomb states and saddle states which have probability in both wells. (Saddle states can occur any time the saddle point lies below the ionization energy; for certain choices of B and K, all three types of states—Coulomb, outer well, and saddle—can be found in the spectrum.) The tunneling probability drops dramatically with increasing K, and the outer well state of positronium can be considered stable for $K > 0.1$. The corresponding tunneling lifetime is greater than one year.

To confirm the validity of the isolated potential energy spectrum and the semiclassical one-dimensional result for the tunneling probability, it is necessary to obtain a direct solution of Eq. (2) that is converged over all space. In order to obtain an accurate representation of the wave function in the Coulomb well, the outer well, *and* the barrier region, we use the three-dimensional adaptive finite element (FE) method [15]. With local interpolation functions and automatic grid refinement, one can obtain a wave function that gives highly accurate results for the energy spectrum, dipole transition rates, and tunneling probability.

In Table I we compare the FE energy spectrum using the exact potential with the two approximations for the isolated wells. For the Coulomb states, the FE energies agree with the PT results to within the accuracy of both calculations ($\approx 10^{-8}$ a.u.). For the outer well, the harmonic approximation gives four digit accuracy for the ground state. The discrepancy here is attributed to the higher order corrections from the Coulomb potential that were neglected in the HO approximation. Also shown in Table I are the results for the FE, PT, and HO dipole moments.

In order to determine the tunneling probability we integrated the probability density for the outer well ground state in both the barrier region and in the Coulomb well. The probability density in the barrier region between the two classical turning points at $x = 7787.18a_o$ and $x = -25.094a_o$ is 8×10^{-2}. Consistent with the semiclassical prediction, the total probability density on the other side of the barrier from $x = -25.096$ to infinity is 4×10^{-20}. Hence there is no mixing between the outer well state and the Coulomb states and the tunneling probability is for all practical purposes negligible.

In addition to tunneling processes, an outer well state can also decay to a Coulomb well state via a dipole transition. However, the probability for such a process is not unrelated to the tunneling rate. If the tunneling rate is very small, neither the Coulomb states nor the outer well states will penetrate the barrier region significantly. One would then expect the *overlap* of the two states to be very small. While transition moments between the Coulomb states agree with the first order PT results, the transition moments between an outer well state and a Coulomb state

TABLE. I. The energy spectrum and dipole moments in atomic units obtained from a finite element (FE) calculation at $B = 5 \times 10^{-5}$ and $K = 0.4$. Results for the Coulomb states are compared with perturbation theory (PT); first and second order energy corrections were calculated to an accuracy of 10^{-9}. Dipole moments were obtained with first order PT wave functions. Results for the outer well states are compared with the harmonic oscillator (HO) approximation ($n_x = 0, n_y = 0$).

		Energy	$\langle x \rangle$
PT	$1s$	−0.209999996	−0.00028
FE	$n = 1$	−0.209999994	−0.00036
PT	$2s + 2p_x$	−0.022559995	−6.012
FE	$n = 2$	−0.022560000	−6.013
PT	$2p_x$	−0.022500022	−0.0114
FE	$n = 3$	−0.022500027	−0.0125
PT	$2p_y$	−0.022499997	−0.0114
FE	$n = 4$	−0.022500002	−0.0125
PT	$2s - 2p_x$	−0.022439995	5.988
FE	$n = 5$	−0.022440000	5.987
HO	$n_z = 0$	−0.000074127	−7987.46
FE	$n = 6$	−0.000074135	−7987.60
HO	$n_z = 1$	−0.000072146	−7987.46
FE	$n = 7$	−0.000072188	−7987.89
HO	$n_z = 2$	−0.000070165	−7987.46
FE	$n = 8$	−0.000070275	−7988.18

are about 12 orders of magnitude smaller. Although the criterion for the dipole approximation is not rigorously met for a transition from an outer well state to the $1s$ state, the near zero overlap between the wave functions guarantees that higher order multipole moments will also be negligible.

Our highly accurate numerical results are consistent with the isolated potential picture and indicate that for certain values of the pseudomomentum and field strength, positronium in an outer well state is prevented from undergoing annihilation for a very long time. Excitation to a saddle state (if it exists) or the continuum followed by rapid deexcitation to a Coulomb state is the most direct process leading to annihilation.

How then would one create such a state in a laboratory field? We suggest two possible mechanisms. The first involves the reverse process we just described leading to annihilation: UV laser excitation to a saddle state or the continuum followed by IR stimulated emission to the outer well state. This scenario seems more likely to be successful in a situation where *both* outer well and saddle states exist; a full study of the dependence of the spectra on K and B will be presented in a future work. The second possibility is to create the outer well state through an inelastic collision of positron with hydrogen; in order to increase the probability of such a reaction, a low-temperature plasma could be used. Despite the technical challenges of both proposed experiments, once a delocalized state is created it will remain in that state indefinitely if shielded from ionizing radiation. The large dipole moment (see Table I) will make detection easy. We hope that the prospect of creating a stable bound state of a particle-antiparticle pair will motivate experimental efforts.

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