Stability of Elastic Glass Phases in Random Field *XY* Magnets and Vortex Lattices in Type-II Superconductors

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A description of a dislocation-free elastic glass phase in terms of domain walls is developed and used as the basis of a renormalization group analysis of the energetics of dislocation loops added to the system. It is found that even after optimizing over possible paths of large dislocation loops, their energy is still very likely to be positive when the dislocation core energy is large. This implies the existence of an equilibrium elastic glass phase in three-dimensional random field *XY* magnets, and a dislocation-free, bond orientationally ordered "Bragg glass" phase of vortices in dirty type-II superconductors. [S0031-9007(97)02715-4]

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It has been believed for a long time that systems with quenched randomness that couples to a continuous symmetry order parameter cannot exhibit long range order in less than four dimensions [1–3]. Recently for a class of random systems including *XY* magnets in a random magnetic field, the absence of long range order has been proven rigorously [2]. Yet an intriguing open question remains: for weak randomness in such systems, are they simply disordered at low temperatures or can phases exist which exhibit some kind of topological or other type of order that distinguishes them from high temperature disordered phases? This issue has resurfaced in the context of vortices in high temperature superconductors; various authors have either implicitly assumed [4], raised the question of [5], or conjectured [6], the existence of an *elastic vortex glass* phase which is locally latticelike and is free of large dislocation loops. Such a phase would probably have power law Bragg-like singularities in its structure factor and true bond-orientational long range order, [6] thus providing a counterexample to the general conjecture mentioned above [3].

In the simpler context of three-dimensional random field *XY* magnets, Gingras and Huse [7] have conjectured, and given some numerical evidence in support of, the existence of a phase transition to a defect-free phase for weak randomness at low temperatures. Yet at this point, no convincing analytical arguments to support or deny the existence of an elastic glass phase have been put forth [6,8], although the delicate balance between elastic, randomness, and dislocation energies has been pointed out by Giamarchi and Le Doussal [6].

In this Letter, we explicitly study the stability of a putative elastic glass phase in a three-dimensional random field *XY* model to dislocation loops—to avoid confusion, we will refer to the relevant topological defects in all these systems as "dislocations." To do this we must first reconsider the behavior of the ground state and excitations of an elastic glass model with dislocations excluded by fiat. The framework that will be developed naturally allows one to analyze the energetics of a dislocation loop that is added to the system. If the core energy of the dislocation line is sufficiently large, it is found that, even after optimizing over the possible paths of a dislocation loop, large loops cost energy with high probability. This situation can be achieved for a weak random field *XY* model and we thus conclude that an elastic glass phase *should exist* in this system in three dimensions.

By analogy, our results are applied to the elastic vortex glass.

Our basic starting point will be the elastic glass model with Hamiltonian

$$\mathcal{H} = \frac{1}{2}\sum_{(xy)}[\varphi(x) - \varphi(y)]^2 - h\sum_x \cos[\varphi(x) - \gamma(x)], \tag{1}$$

with $\gamma(x)$ independent quenched random variables on each site uniformly distributed on $[-\pi, \pi]$ and $\varphi(x)\epsilon(-\infty, \infty)$. This model has been extensively studied by renormalization group (RG) and other techniques [6,9]. At all temperatures the behavior is controlled by a zero temperature fixed point whose properties yield disorder averaged (denoted by an overbar) mean square phase variations

$$\overline{\langle[\varphi(x) - \varphi(y)]^2\rangle} \approx 2A \ln|x - y| \tag{2}$$

at large distances, with A a universal coefficient computable in a $4 - \epsilon$ expansion [6]. It is believed that the mean correlation function will decay as $\overline{\langle e^{i\varphi(x)}e^{-i\varphi(y)}\rangle} \sim \frac{1}{(x-y)^\eta}$. These results have primarily been obtained from coarse graining or Fourier space representation of the phase variables. Unfortunately, this framework does not appear to be naturally amenable to consideration of dislocations, for these intrinsically involve physics on many length scales.

A complementary and more complete picture of the elastic glass phase can be constructed in terms of domain walls which turn out to be the natural objects at long length scales. This is most easily seen by studying the

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limit $h \to \infty$ so that $\varphi(x) = \gamma(x) + 2\pi n(x)$ with $\{n(x)\}$ integers. The ground state can then be represented [up to a uniform shift in the $\{n(x)\}$] by the oriented surfaces (with a "+" side) through which $n(x)$ changes by $+1$. The scale invariance suggests that the nested set of closed surfaces enclosing a point x will typically occur on logarithmically spaced scales $1, B, B^2, \ldots$. Since the orientations of these surfaces on widely separated scales should be independent, the sum of the $\sim 2\log_B |x - y|$ random terms in $\Delta\varphi$ from crossing the surfaces that enclose x and then those that enclose y will yield $\varphi(x) - \varphi(y) \sim \pm\sqrt{\ln|x - y|}$. In contrast, the coarse grained φ averaged over regions of size of order $\frac{1}{2}|x - y|$ will vary from x to y by only $O(1)$.

Rather than working with the surfaces across which $n(x)$ changes, it is useful, as for Ising spin glasses [10], to consider configurations *relative* to the ground state $\{n_G(x)\}$ with some chosen fixed boundary conditions; any state can then be represented by the set of *oriented domain walls* across which $n(x) - n_G(x)$ increases by one. A crucial question is the typical energy of the minimal domain wall excitation, ε_L, that surrounds a chosen volume L^3.

The observation that the coarse grained φ has variation of $O(1)$ strongly suggests from the scaling of the elastic energy in Eq. (1) that $\varepsilon_L \sim L^\theta$ with $\theta = 1$ (generally $\theta = d - 2$). This can be seen more explicitly by fixing $\varphi(x_1 = 0, x_2, x_3) = 0$ in a system of size $L \times L \times L$ and letting $\varphi(x_1 = L, x_2, x_3)$ change from zero, which defines the reference state $\{n_G(x)\}$, to 2π, with periodic boundary conditions in $x_2 \to x_2 + L$ and $x_3 \to x_3 + L$. This forces a single domain wall spanning the system and changes the energy by E_L. We can now make use of the powerful statistical symmetry of the model Hamiltonian Eq. (1): if φ is replaced by $\varphi = \varphi_D + \psi$ with φ_D single valued module 2π and $\nabla^2\varphi_D = 0$ (with lattice derivative operators) then the *statistical* properties of

$$\tilde{\mathcal{H}}(\psi) = \mathcal{H} - \frac{1}{2}\sum |\nabla\varphi_D|^2 \qquad (3)$$

with constant boundary conditions on ψ are identical to those of $\mathcal{H}(\varphi)$ with constant boundary conditions on φ. Choosing $\varphi_D = \frac{2\pi x_1}{L}$, this implies that the mean energy of the forced domain wall is $\overline{E}_L = \frac{1}{2}(2\pi)^2 L$. The necessary balance of the random part of the energy with the mean elastic part ($\sim L$), implies that this is consistent only if the variations of the spanning wall energy, δE_L, are also of order L; i.e., $\theta = 1$. The result that E_L is much less than the area of the walls implies that the walls will be fractal.

If, in the L^3 system, the boundary condition at $x_1 = L$ is changed to $\varphi = 2\pi s$, then s spanning domain walls will be forced. Because these are closer together for larger s, they can less easily optimize their positions and their energy will be larger. Following arguments for confined directed polymers, [11] we consider each section of domain wall of scale L/s. Roughly, these are transversely confined by the others on scale L/s. Thus we guess that the mean energy of a section will be of order L/s yielding, with s^3 sections, a total increase in the mean energy $\overline{E}_L \sim Ls^2$ in agreement with the result from the statistical symmetry.

The main lesson from the above is that sections of optimal domain walls on scales L that are *restricted on the same scale* L, typically have energy that is distributed with positive mean and variations both of order L. If we try to put several distinct domain walls of scale L into a volume of order L^3, the energy of each of them will have to increase.

We now consider inserting a single dislocation loop of radius R into a fixed position of the system by making φ multivalued with $\nabla \times \nabla\varphi = 2\pi$ on plaquettes through which the dislocation loop passes and $\nabla \times \nabla\varphi = 0$ elsewhere. The statistical symmetry implies, via Eq. (3), with φ_D the dislocation solution in the pure system, that the extra ground-state energy, D_R, due to the dislocation has mean

$$\overline{D}_R = 2\pi R[\epsilon_0 + \pi \ln R], \qquad (4)$$

where ϵ_0 is the core energy per unit length of the dislocation. How can we understand this in terms of domain walls? The dislocation loop forces in a single domain wall that spans the loop. The sections of the wall of scale $L = 1$ adjacent to the loop are restricted on this scale by their attachment to the loop and thus each have mean energy of order 1 and well-separated sections have roughly independent variations of this same order. The sections of scale $L = 2$ are attached to these which restricts them on scale $L = 2$ and gives each a mean energy of order 2; and so on, on scales 4, 8, 16—as illustrated in Fig. 1—up to $L \sim R$. Thus each factor of 2 in scale will contribute a factor of R to $\overline{D}_R$ yielding the $R \ln R$ of Eq. (4). But the variations $\delta D_R \equiv D_R - \overline{D}_R$ will be much smaller: from each scale L there will be a random contribution $\pm(R/L)^{\frac{1}{2}}L$, from the sum of order R/L roughly independent variations of sections of scale L. The typical variations in D_R are thus $\delta D_R \sim R$ which are dominated by the *largest* scale sections of the wall.

For a large *fixed* dislocation loop, the energy is thus very likely to be large and positive. But we must consider the optimization of the dislocation energy over all possible paths of the dislocation loop in a region of volume of order R^3. We do this by an approximate RG analysis of the effects of sections of the wall at each length scale on the minimum energy path of a segment of the dislocation loop.

We focus on transverse deformations of a coarse-grained dislocation segment, Δ, by distances of order W. The position of the coarse-grained dislocation core cannot be specified on scales less than W: the actual dislocation path will be the minimum energy dislocation which is restricted to be within a specified tube—the coarse-grained core—of diameter W. The effects of smaller scale deformations of

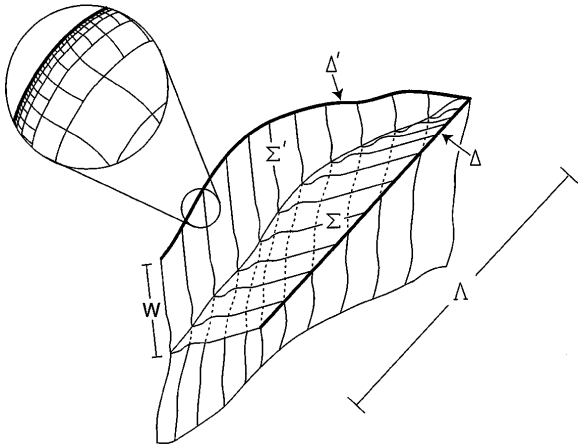

FIG. 1. Schematic of a straight dislocation segment, Δ, of length Λ, and a distorted segment Δ' with the corresponding minimum energy walls Σ and Σ' attached. At distances more than the scale of the distortion, W, from Δ and Δ', Σ and Σ' coincide as shown. The magnified region illustrates the argument discussed in the text: the various different scale sections of the wall shown have roughly independent energies. In actuality, the walls will be fractal.

the dislocation and the concomitant changes in the sections of the wall, Σ, attached to it on scales smaller than W are included in an effective mean dislocation energy per unit length, $\tilde{\epsilon}_W$, with local variations around this value. If the dislocation were straight on smaller scales, $\tilde{\epsilon}_W$ would be simply $\epsilon_0 + \pi \ln W$ but it will be reduced from this by optimization over the smaller scale deformations. Our task is to iteratively understand how the deformations on a scale W change $\tilde{\epsilon}$ on larger scales.

In the continuum approximation (valid on large scales), the typical excess energy cost of a transverse distortion of a segment of length Λ of the dislocation segment by an amount $W \ll \Lambda$ will be $\frac{1}{2}\tilde{\epsilon}_W W^2/\Lambda$ from the extra length of the dislocation. Such a distorted dislocation segment, Δ', will have a different spanning wall, Σ', attached to it. But if Σ' were typically different from Σ out to distances of order Λ from Δ, it would imply the existence of many $(\sim\Lambda/W)$ distinct minimal walls in a volume Λ^3, as can be seen by considering a sequence of roughly parallel dislocation segments each separated by W. From the energetics of many walls forced by boundary conditions analyzed earlier, we see that this would imply an energy per wall much larger than $O(\Lambda)$, contradicting the requirement that these all be minimal walls. What should be expected, instead, is that the minimal spanning walls, Σ and Σ', attached to Δ and Δ', will typically only differ in a strip of width W near the dislocation, with Σ and Σ' coinciding further away, as shown in Fig. 1.

Each wall section of scale W attached to Δ or Δ' will be roughly independent and the mean energy $(\sim W)$ of each of these sections of the two wall configurations will be the same but with random differences between them also of order W. Thus the *total difference* in the energies of Σ and Σ' will be the sum of $\frac{\Lambda}{W}$ random terms and hence of order $\pm(\frac{\Lambda}{W})^{\frac{1}{2}}W$. Balancing this difference against the excess effective core energy cost of the distortion, from above, yields the typical length Λ_W over which transverse deformations of size W will occur:

$$\Lambda_W \sim W\tilde{\epsilon}_W^{\frac{2}{3}}, \tag{5}$$

which is indeed $\gg W$ if $\tilde{\epsilon}_W$ is large.

At the next length scale, $2W$, the effective core energy $\tilde{\epsilon}$ will change since it should include the effects of the dislocation deformations and wall sections within a core tube of diameter $2W$. The inclusion of the wall segments of scale W attached to the dislocation increases the mean energy of a length Λ_W segment by $\sim W(\Lambda_W/W)$ but the optimization analyzed above over the scale W deformation of the dislocation decreases it by $\sim(\Lambda_W/W)^{\frac{1}{2}}W$—a smaller amount. Thus we find that

$$\tilde{\epsilon}_{2W} \approx \tilde{\epsilon}_W + \pi \ln 2 - \alpha/\tilde{\epsilon}_W^{\frac{1}{3}}, \tag{6}$$

with α some coefficient.

This is our key result: although the arguments leading to Eq. (6) will break down for small $\tilde{\epsilon}_W$, the *form* should be correct for large $\tilde{\epsilon}_W$. The RG flow of Eq. (6) implies that the delicate balance [6] between the $R \ln R$ terms in D_R and its $\pm R$ variations, as well as that implied by the almost linear growth in Eq. (5) of dislocation, distortions W_Λ, with Λ, is resolved for large $\tilde{\epsilon}$ by the dominance of the deterministic terms in the dislocation energy over even the optimal random ones.

For sufficiently large ϵ_0, the renormalized energy of the lowest energy dislocation loop of radius R in a volume $\sim R^3$ can now be obtained by renormalizing until a scale W_R at which $\Lambda_W \sim R$. On longer scales, the optimal dislocation loop of radius R will look essentially circular but can still reduce its energy by rotating or moving within the region of volume $\sim R^3$. From scales W in the range $R > W > W_R$, the renormalization of $\tilde{\epsilon}$ will have a similar form to Eq. (6) but with the last term replaced by $-\alpha'(W/R)^{\frac{1}{2}}$. The mean energy of the optimal dislocation loop of radius R in a volume of order R^3 will thus be

$$\overline{D}_R \approx 2\pi R\{\pi \ln R + \epsilon_0 - O[\epsilon_0^{\frac{2}{3}}, (\ln R)^{\frac{2}{3}}]\} \tag{7}$$

for large ϵ_0.

Because the energies of small scale distortions of distant parts of the dislocation loop will be uncorrelated, the largest scales $W \sim R$ should dominate the variations in D_R, yielding

$$\delta D_R \sim R \ll \overline{D}_R. \tag{8}$$

Large dislocation loops with negative energy will thus be very improbable.

To complete the analysis we need to consider the effects of dislocation loops on each other. Some small loops will of course appear for any ϵ_0. In the presence

of an applied $\nabla\varphi$ the effect of these will be to decrease the effective long wavelength elastic constant of the elastic glass phase and concomitantly the mean energy of large dislocation loops, by allowing spanning walls with small holes in them. But if ϵ_0 is greater than some critical value, ϵ_{0c}, these effects will yield only finite renormalizations and the elastic glass phase will be stable. In contrast, for $\epsilon_0 < \epsilon_{0c}$, $\tilde{\epsilon}_W$ will decrease with length scale eventually becoming negative and leading to the proliferation of dislocations of size greater than a correlation length ξ, even at zero temperature. The decrease of $\tilde{\epsilon}$ with length scale for small $\tilde{\epsilon}$ can be seen from Eq. (6); although not quantitatively correct for small $\tilde{\epsilon}$, the arguments leading to Eq. (6) include enough of the relevant physics to yield a critical ϵ_0.

So far, we have focused on an unphysical limit of infinite h with ϵ_0 tuned by hand. But the results will apply much more generally. For weak randomness ($h \ll 1$) in a three-dimensional XY magnet, the crossover length scale, ξ_p, above which the random fields become important, diverges as $\xi_p \sim h^{-2}$. On scales L larger than ξ_p, the randomness typically prevents the system from being able to find more than one optimal configuration in a region: another configuration that differed over most of a region of size L^3 would be expected to differ in energy by of order $h^4 L^3$ which, since this is much larger than the basic energy scale L for excitations and stiffness, is highly unlikely. Thus we expect configurations with different boundary conditions (or those differing by large dislocation loops) to differ substantially by other than multiples of 2π only on lower dimensional subsets: these will be domain walls with thickness $\sim\xi_p$.

Our analysis can be carried through for small h if the system is first coarse grained to a scale $W \sim \xi_p$. This yields an effective core energy $\tilde{\epsilon}_{\xi_p} \approx \pi \ln \xi_p$ from distances within ξ_p of the dislocation since the energy on these scales is mostly the deterministic elastic energy. Larger scale deformations of a dislocation are similar to those studied above. If h is sufficiently small, $\tilde{\epsilon}_{\xi_p}$ will be large and the energy gained from optimizing the dislocation path will be small; the elastic glass phase will thus be *stable* for weak random fields. Because D_R/R grows with length scale, thermal fluctuations will be irrelevant and thus the phase will persist at low temperatures [10].

Aside from the complications of dislocations (and domain walls) with different Burgers' vectors and of coupling of the real and order parameter spaces, the arguments given above for XY random field magnets should apply also for three-dimensional vortex lattices in weakly disordered type-II superconductors. Hexatic long range order and power law singularities at Bragg peak positions should be observable in the resulting elastic glass phase, [6] especially if the exponent η is small, and the linear resistivity should be zero [4]. An intermediate quasi-long range ordered hexatic phase with no large disclination loops can also exist [3]. Analogous phases can exist for solids in porous media and in other systems.

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