Collective Switching and Inversion without Fluctuation of Two-Level Atoms in Confined Photonic Systems

Sajeev John and Tran Quang
Department of Physics, University of Toronto, 60 St. George Street, Toronto, Ontario Canada M5S 1A7
(Received 30 August 1996)

We demonstrate population inversion and sub-Poissonian excitation statistics of N two-level atoms in the context of collective resonance fluorescence. This occurs within photonic band gap and other confined photonic systems that exhibit sharp features in the optical density of states. When the deviation in the photon density of states between the Mollow spectral components is considerable, the atoms switch collectively from ground to excited states at a critical value of the applied laser field. This suggests a new mechanism of sub-Poissonian pumping of lasers, fast optical switching, and optical transistor action. [S0031-9007(97)02714-2]

PACS numbers: 42.65.Pc, 42.50.Fx, 42.50.Lc

Quantum optics in confined photonic systems [1] such as microcavities, optical fibers, optical wires, and photonic band gap (PBG) materials exhibits novel features arising from our ability to tailor the photon density of states (DOS) in a prescribed manner. These features include inhibition of spontaneous emission [2–4], photon localization in a PBG [5,6], quantum collapses and revivals of atomic population inversion [7,8], quantum Rabi splitting of atomic levels, and photon-atom bound states [9–11]. The distinguishing common feature of the confined photonic systems is that the photonic mode density exhibits rapid variation with frequency at certain edge or cutoff frequencies. In the optical fibers, the mode density vanishes abruptly below a waveguide cutoff frequency ω_0. For $\omega \geq \omega_0$, the mode density of the fiber diverges as $(\omega - \omega_0)^{-1/2}$ [3]. A similar situation appears in the optical wires [4]. In photonic band gap materials, the DOS exhibits band edge and other van Hove singularities. At the band edge frequency ω_{edge}, this can take the form of a step discontinuity (two-dimensional PBG) or a singularity of the form $|\omega - \omega_{\mathrm{edge}}|^{1/2}$ in a true three-dimensional PBG.

In this paper, we study collective atomic population inversion and statistics of atoms driven by a laser field in confined photonic systems. When the deviation of the photonic mode density between the two Mollow sidebands is large, strong atomic population inversion occurs. We show that when the number of atoms, N, is large, collective switching from the ground state into excited state occurs at a sharp threshold value of the applied field intensity. We show that, under certain conditions, the statistics of the atoms in the excited state can be strongly sub-Poissonian. This suggests a new mechanism of sub-Poissonian pumping for lasers, fast optical switching, and large differential optical gain relevant to an all-optical transistor.

Consider a system of N identical two-level atoms driven by a strong external laser field and coupled to the radiation field reservoir of the confined photonic material. The atoms have excited state $|2\rangle$, ground state $|1\rangle$, resonant transition frequency ω_{21}, and may interact with lattice vibrations of the host photonic material. The Hamiltonian of the system in the interaction picture takes the form, $H = H_0 + H_1 + H_{\mathrm{dephase}}$, where

$$H_0 = \frac{1}{2}\hbar\Delta J_3 + \hbar\varepsilon(J_{12} + J_{21}) + \sum_\lambda \hbar\delta_\lambda a_\lambda^\dagger a_\lambda, \quad (1)$$

$$H_1 = i\hbar\sum_\lambda g_\lambda(a_\lambda^\dagger J_{12} - J_{21}a_\lambda). \quad (2)$$

Here $J_{ij} = \sum_{k=1}^N |i\rangle_k\langle j|_k$ $(i,j = 1,2)$ are the collective atomic operators; $J_3 = J_{22} - J_{11}$ describes the atomic population inversion; a_λ and $a_\lambda^\dagger$ are the radiation field annihilation and creation operators; $\Delta = \omega_a - \omega_L$; and $\delta_\lambda = \omega_\lambda - \omega_L$. ω_a, ω_L, and ω_λ are the atomic resonant frequency, the applied field frequency, and the frequency of a mode λ, respectively. ε is the resonant Rabi frequency of the applied field and g_λ is the atom-radiation field coupling constant. The Hamiltonian H_{dephase} describes additional dephasing interactions which may arise from atomic collisions and the scattering of phonons from the impurity atoms if the atoms are embedded in the solid part of a dielectric material. We assume for simplicity that the *phonon* DOS is broad and displays no sharp features. In this case, the dephasing part of the master equation for the atomic density operator can then be written as [12]

$$\left(\frac{\partial\rho}{\partial t}\right)_{\mathrm{dephase}} = (\gamma_p/2)(2J_3\rho J_3 - J_3^2\rho - \rho J_3^2), \quad (3)$$

where γ_p is a phenomenological dephasing decay rate.

It is convenient to express the atomic operator J_{ij} in the Schwinger (boson) representation [13,14]. We write $J_{ij} = a_i^\dagger a_j$ $(i,j = 1,2)$, where $a_i^\dagger$ and a_i satisfy bosonic commutation relations and describe creation and annihilation of atoms in the state $|i\rangle$ with the additional constraint that $a_1^\dagger a_1 + a_2^\dagger a_2 = N$. The atom-applied field part of the Hamiltonian H_0 can be diagonalized using the following canonical transformation [13]: $a_1 = \cos\phi q_1 + \sin\phi q_2$, $a_2 = -\sin\phi q_1 + \cos\phi q_2$. This leads to the dressed state Hamiltonian,

$$H_0 = \hbar\Omega R_3 + \sum_\lambda \hbar\delta_\lambda a_\lambda^\dagger a_\lambda, \quad (4)$$

where $\sin^2\phi = \frac{1}{2}[1 - \mathrm{sgn}(\Delta)/(4\varepsilon^2/\Delta^2 + 1)^{1/2}]$, $\Omega = (\varepsilon^2 + \Delta^2/4)^{1/2}$, $R_{ij} = q_i^\dagger q_j$ $(i,j = 1,2)$ are the dressed atomic operators, and $R_3 = R_{22} - R_{11}$. Clearly, $q_i^\dagger$ and q_i satisfy the bosonic commutation relations and represent creation and annihilation of atoms in the dressed state $|\tilde{i}\rangle$.

In dressed state basis $|\tilde{i}\rangle$, J_{21} and J_3 in the interaction Hamiltonian H_1 and $(\partial\rho/\partial t)_{\mathrm{dephase}}$ must be replaced by $J_{21} = \sin\phi\cos\phi R_3 + \cos^2\phi R_{21} - \sin^2\phi R_{12}$ and $J_3 = \cos(2\phi)R_3 - \sin(2\phi)(R_{21} + R_{12})$. We define the time-dependent interaction picture Hamiltonian $\tilde{H}_1(t) = U^\dagger(t)H_1U(t)$ where $U(t) = \exp(-iH_0t/\hbar)$. The dressed-state collective atomic operators in this interaction picture exhibit the time dependence $\tilde{R}_{21}(t) = \tilde{R}_{21}(0)\exp(2i\Omega t)$, $\tilde{R}_{12}(t) = \tilde{R}_{12}(0)\exp(-2i\Omega t)$, and $\tilde{R}_3(t) = \tilde{R}_3(0)$. Hereafter we drop the tilde on the interaction picture operators. Clearly R_3, R_{12}, and R_{21} can be considered as the source operators for the central component, left and right sidebands of the Mollow triplet at frequencies ω_L, $\omega_L - 2\Omega$, and $\omega_L + 2\Omega$, respectively. In this interaction picture, the interaction Hamiltonian H_1 takes the form

$$H_1 = i\hbar\sum_\lambda g_\lambda\big(\sin\phi\cos\phi\, a_\lambda^\dagger R_3 e^{i\delta_\lambda t} + \cos^2\phi\, a_\lambda^\dagger R_{12}e^{i(\delta_\lambda - 2\Omega)t} - \sin^2\phi\, a_\lambda^\dagger R_{21}e^{i(\delta_\lambda + 2\Omega)t}\big) + \mathrm{H.c.} \tag{5}$$

The dephasing part of the master equation remains in the same form (3) except with

$$J_3 = \cos(2\phi)R_3 - \sin(2\phi)(R_{21}e^{2i\Omega t} + R_{12}e^{-2i\Omega t}). \tag{6}$$

The collective spectral and statistical properties of these spectral components in free space (where the photonic DOS is smooth and featureless) can be found in [14,15]. In this paper we consider the case when the DOS at the atomic transition frequency exhibits a step discontinuity or some other singularity so that the resulting Mollow spectral components experience strongly different mode densities. We also assume for simplicity that the photonic mode density, while singular at one frequency, is constant over the immediate spectral regions surrounding the dressed-state resonant frequencies ω_L, $\omega_L - 2\Omega$, and $\omega_L + 2\Omega$. In this case, the radiative part [12] of the master equation for the reduced atomic density operator, ρ, takes the form

$$\left(\frac{\partial\rho}{\partial t}\right)_{\mathrm{rad}} = -\frac{1}{\hbar^2}\int_0^t dt'\,\mathrm{Tr}_R\{[H_1(t),[H_1(t'),\chi(t')]]\}. \tag{7}$$

Here χ is the density operator of the full atomic system plus electromagnetic reservoir, $\rho = \mathrm{Tr}_R\{\chi\}$, and Tr_R denotes a trace over the reservoir variables. In the Born approximation [12,14–16], we replace $\chi(t')$ in Eq. (7) by $\rho(t')R_0$, where R_0 is an initial reservoir density operator. This corresponds to the second order perturbation theory in the interaction between atoms and reservoir. It assumes that changes in reservoir as a result of atom-reservoir interaction are negligible. Our second major simplification is the Markovian approximation which replaces $\rho(t')$ by $\rho(t)$. That is to say, we ignore the memory effects such as those arising form photon localization [6]. The dressed-state master equation for the density operator ρ in the *Born-Markoff approximation* for the case of a strong external field takes the form

$$2\frac{\partial\rho}{\partial t} = A_0[R_3\rho R_3 - R_3^2\rho] + A_-[R_{21}\rho R_{12} - R_{12}R_{21}\rho] + A_+[R_{12}\rho R_{21} - R_{21}R_{12}\rho] + \mathrm{H.c.} \tag{8}$$

Here $A_0 = \gamma_0\sin^2\phi\cos^2\phi + \gamma_p\cos^2(2\phi)$, $A_- = \gamma_-\sin^4\phi + \gamma_p\sin^2(2\phi)$, and $A_+ = \gamma_+\cos^4\phi + \gamma_p\sin^2(2\phi)$. The spontaneous emission decay rates $\gamma_0 = 2\pi\sum_\lambda g_\lambda^2\delta(\omega_\lambda - \omega_L)$, $\gamma_- = 2\pi\sum_\lambda g_\lambda^2\delta(\omega_\lambda - \omega_L + 2\Omega)$, $\gamma_+ = 2\pi\sum_\lambda g_\lambda^2\delta(\omega_\lambda - \omega_L - 2\Omega)$ are proportional to the density of modes at the dressed-state transition frequencies. In deriving (8), we have also used the secular approximation [14–16] for strong applied laser field with $\Omega \gg N\gamma_0$, $\Omega \gg N\gamma_-$, $\Omega \gg N\gamma_+$. That is, the fast oscillating terms with frequencies 2Ω and 4Ω in the master equation were ignored. Physically, it means the three Mollow spectral components are well separated and the overlap between them is negligible. We note, finally, that the master equation (8) reduces to the free space case 14,15] when $\gamma_0 \cong \gamma_- \cong \gamma_+$.

It is convenient to introduce the dressed state ket vector $|n\rangle \equiv |N - n, n\rangle$ which denotes a symmetrized N-atom state in which $N - n$ atoms are in the lower dressed state $|\tilde{1}\rangle$ and n atoms are excited to the upper dressed state $|\tilde{2}\rangle$. Using the harmonic oscillator property of the Schwinger bosons, it follows that $R_{12}|n\rangle = \sqrt{n(N-n+1)}\,|n-1\rangle$, $R_{21}|n\rangle = \sqrt{(N-n)(n+1)}\,|n+1\rangle$, and $R_3|n\rangle = (2n - N)|n\rangle$. Using these rules and the master equation (8), it is straightforward to verify that diagonal matrix elements of the density operator $P_n \equiv \langle n|\rho|n\rangle$ satisfy the equation

$$\frac{\partial P_n}{\partial t} = n(N - n + 1)[A_-P_{n-1} - A_+P_n] - (n \to n+1). \tag{9}$$

In the steady state limit, $\partial\rho/\partial t = 0$, the off-diagonal elements of the density matrix vanish, and the diagonal elements can be found by the detailed balance from Eq. (9) as $P_n = P_0\xi^n$, where $\xi = A_-/A_+$ and $P_0 = (\xi - 1)/(\xi^{N+1} - 1)$. Using atomic distribution function P_n, we derive $\langle n\rangle$ and $\langle n^2\rangle$ in the form

$$\langle n\rangle = P_0[N\xi^{N+2} - (N+1)\xi^{N+1} + \xi]/(\xi - 1)^2, \tag{10}$$

$$\langle n^2\rangle = P_0[N^2\xi^{N+3} - (2N^2 + 2N - 1)\xi^{N+2} + (N+1)^2\xi^{N+1} - \xi^2 - \xi]/(\xi - 1)^3, \tag{11}$$

where $\langle\cdots\rangle$ stands for a steady state expectation value. The atomic population per atom on the upper dressed state and bare state can be found for the case of $N \gg 1$ as

$$\langle R_{22}\rangle/N \cong \begin{cases} 1, & \text{if} \quad \xi > 1\,, \\ 1/2, & \text{if} \quad \xi = 1\,, \\ 0, & \text{if} \quad \xi < 1\,, \end{cases} \tag{12}$$

$$\langle J_{22}\rangle/N \cong \begin{cases} \cos^2\phi, & \text{if} \quad \xi > 1\,, \\ 1/2, & \text{if} \quad \xi = 1\,, \\ \sin^2\phi, & \text{if} \quad \xi < 1\,. \end{cases} \tag{13}$$

Clearly, the atomic population displays a sharp collective jump in which the active region of the photonic material switches from an absorptive medium to a gain medium as a function of the control laser field (change in the angle ϕ). A probe laser beam will experience a substantial differential gain when the control laser amplitude is in the vicinity of $\xi = 1$. In this sense, the system acts as a quantum optical transistor. For a *single* atom case, Eq. (12) is replaced by $\langle R_{22}\rangle/N = \xi/(\xi + 1)$. That is, there is no jump in a single atom case. It is apparent from the above analysis that phonon mediated dephasing processes have a tendency to destroy the collective switching. However, the influence of dephasing can be reduced or eliminated by increasing the number of atoms N or by detuning the control laser field frequency so that $\sin^2(2\phi)$ is small. In Fig. 1 we plot $\langle J_{22}\rangle/N$ as functions of resonance Rabi frequency ε for the case of $\gamma_-/\gamma_+ = 10^{-3}$ and $\gamma_p/\gamma_+ = 0.5$. A large jump in the DOS, of this nature, may arise in a 3D PBG material [17,18]. In other confined photonic systems such as wires and fibers, the jump in the DOS may be much weaker. In Fig. 2, we show the collective jump for $0.3 < \gamma_-/\gamma_+ < 0.5$ for $N = 5000$. It is clear that a sizable switching behavior can be achieved even for small DOS variations and substantial phonon dephasing ($\gamma_p/\gamma_+ = 0.5$) when a large number of atoms responds collectively. This is possible when the left sideband lies in the gap of a PBG or in the cutoff region of the optical fibers and wires, while the right sideband lies outside of the gap or of the cutoff region. It is apparent from Figs. 1 and 2 that in the case of $N \gg 1$ ($N = 5 \times 10^3$ for Fig. 2 and the solid curve in Fig. 1) the atomic system switches very sharply from the ground state to the excited states at the critical value of ε. We note finally that the collective time scale for this switching is proportional to N^{-1}. As a result, this effect may be relevant for very fast optical switching devices [19].

To characterize atomic fluctuations in the excited dressed and bare states, we introduce the Mandel q parameters $Q_d = (\langle R_{22}^2\rangle - \langle R_{22}\rangle^2)/\langle R_{22}\rangle$ and $Q_b = (\langle J_{22}^2\rangle - \langle J_{22}\rangle^2)/\langle J_{22}\rangle$. The detailed analytical expression for Q_d and Q_b can be easily found using Eqs. (10) and (11). In particular, when $N \gg 1$, Q_d and Q_b are found as

$$Q_d \cong \begin{cases} 1/N, & \text{if} \quad \xi > 1\,, \\ N/12, & \text{if} \quad \xi = 1\,, \\ 1/(1 - \xi), & \text{if} \quad \xi < 1\,, \end{cases} \tag{14}$$

$$Q_b \cong \begin{cases} \sin^2\phi(\xi + 1)/(\xi - 1), & \text{if} \quad \xi > 1\,, \\ (N + 2)/6, & \text{if} \quad \xi = 1\,, \\ \cos^2\phi(\xi + 1)/(1 - \xi), & \text{if} \quad \xi < 1\,. \end{cases} \tag{15}$$

It is apparent from Eqs. (14) that for $N \gg 1$ and $\xi > 1$, the q-Mandel parameter $Q_d \cong 0$. That is, the dressed-state atomic population inversion with strong sub-Poissonian atomic statistics occurs. Clearly, the dipole dephasing due to phonons has only a limited influence on the sub-Poissonian distribution of atoms on the upper dressed state. In contrast, the atom statistics on the excited bare state $|2\rangle$ depends quite strongly on the dephasing decay rate γ_p. For example, in the case of $\gamma_-/\gamma_+ \ll 1$, Eq. (15) reduces to $Q_b \cong \cos^2\phi + 8(\gamma_p/\gamma_+)\sin^2\phi$. That is, Q_b tends to zero only if $\cos^2\phi \ll 1$ and $\gamma_p/\gamma_+ \ll 1$. It is useful to note here that in free space no population inversion is available in this system [14,15] and that the distribution of atoms on the excited state $|2\rangle$ is super-Poissonian rather than sub-Poissonian.

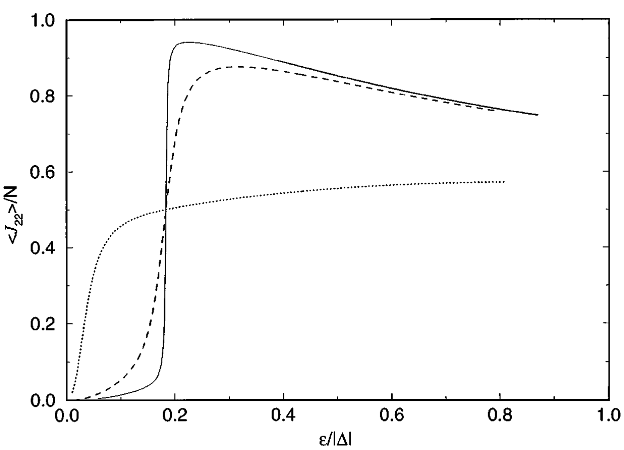

FIG. 1. Atomic population per atom on the bare excited states $\langle J_{22}\rangle/N$ as a function of $\varepsilon/|\Delta|$ for $\gamma_-/\gamma_+ = 10^{-3}$, $\gamma_p/\gamma_+ = 0.5$, $\Delta = -1$, and for $N = 10$ (dotted curve), 500 (dashed curve), and 5000 (solid curve).

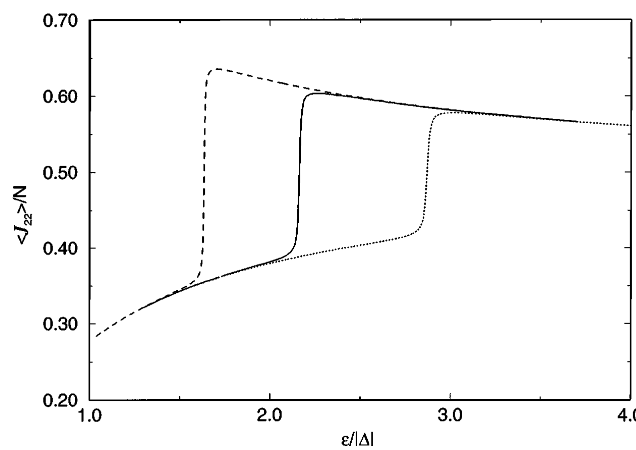

FIG. 2. Atomic population per atom on the bare excited states $\langle J_{22}\rangle/N$ as a function of $\varepsilon/|\Delta|$ for $\gamma_p/\gamma_+ = 0.5$, $\Delta = -1$, $N = 5000$, and for $\gamma_-/\gamma_+ = 0.3$ (dashed curve), 0.4 (solid curve), and 0.5 (dotted curve).

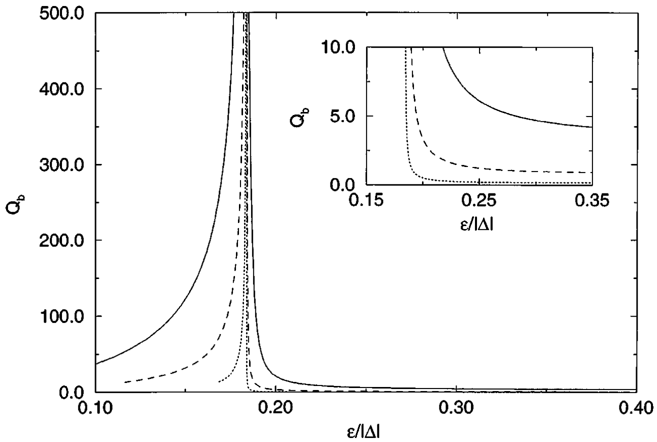

FIG. 3. q-Mandel's parameter Q_b as a function of $\varepsilon/|\Delta|\gamma_-/\gamma_+ = 10^{-3}$, $\Delta = -1$, $N = 5000$, and for $\gamma_p/\gamma_+ = 0.5$ (solid curve), 0.1 (dashed curve), and 0.01 (dotted curve). Inset shows an expanded view of the same curves in the regime of sub-Poissonian statistics of excited atoms.

In Fig. 3 plot Q_b as functions of resonance Rabi frequency ε for the case of $N = 5 \times 10^3$, $\gamma_-/\gamma_+ = 10^{-3}$, and for different values of γ_p/γ_+. At the switching threshold there is a large (proportional to the number of atoms N) increase in fluctuations, characteristic of a phase transition. It is apparent from Fig. 3 that the Mandel parameter Q_b for atoms on the bare excited state can be small for the case when the dephasing decay rate caused by atomic collisions or phonons is small compared to the radiative decay rate outside the gap ($Q_b \cong 0.19$ for $\gamma_p/\gamma_+ = 0.01$). It suggests that the above considered system may be relevant for a new mechanism of sub-Poissonian pumping for lasers [20,21] and dressed-state lasers [22]. Lasers exhibiting sub-Poissonian photon statistics may have applications in noiseless optical data transfer and detection of gravitational waves.

The analysis we have presented in this paper provides a qualitative picture of collective switching, sub-Poissonian statistics, and optical amplification in the regime of strong external laser fields. By using a strong laser field, it is possible to drive the Mollow spectral components away from the photon DOS singularity so that over the width of the individual sidebands, the DOS is smooth. For weaker fields, the singularity in the DOS can lead to important non-Markovian effects. A more detailed calculation for a specific van Hove singularity may lead to the lower threshold and much faster switching. In particular, the collective time scale factor near a three-dimensional photonic band edge has been shown to be proportional to N^2 [6]. That is, the switching speed may be proportional to N^2 when photon localization and other non-Markovian effects are included. These problems will be discussed in detail elsewhere.

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada and the Ontario Laser and Lightwave Research Centre, and a grant from William F. McLean.

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