Equation of State of Asymmetric Nuclear Matter and Collisions of Neutron-Rich Nuclei

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(Received 5 September 1996)

The ratio of pre-equilibrium neutrons to protons from collisions of neutron-rich nuclei is studied as a function of their kinetic energies. This ratio is found to be sensitive to the density dependence of the nuclear symmetry energy, but is independent of the compressibility of symmetric nuclear matter and the in-medium nucleon-nucleon cross sections. The experimental measurement of this ratio thus provides a novel means for determining the nuclear equation of state of asymmetric nuclear matter. [S0031-9007(97)02535-0]

PACS numbers: 21.65.+f, 24.10.Nz, 25.60.Pj, 25.70.Lm

Recent advances in radioactive ion beam experiments have opened up a new field of research in nuclear physics (for a recent review, see, e.g., Refs. [1,2]). These experiments have already provided useful information about the structure of unstable nuclei far from the stability valley. However, it has not been generally recognized that they are also useful for extracting information on the equation of state (EOS) of asymmetric nuclear matter, which has not been well determined and is important for understanding both the structure of unstable nuclei and the properties of neutron stars. In relativistic mean-field (RMF) theory, it has been shown recently that the nuclear symmetry energy affects significantly the binding energy and rms radii of neutron-rich nuclei [3]. Also, the chemical composition, the evolution of lepton profiles, and the neutrino fluxes in neutron stars depend strongly on the nuclear symmetry energy [4]. More important but poorly known is the density dependence of the nuclear symmetry energy. Better knowledge on this quantity is required to understand both the matter radii of many neutron-rich isotopes, which has been found to increase faster than $A^{1/3}$ [5], and the central density of neutron-rich nuclei, which is lower than that of stable nuclei [6]. In nuclear astrophysics, the density dependence of nuclear symmetry energy is crucial for understanding the supernova explosion scenarios and the cooling mechanisms of neutron stars [7,8]. In particular, it determines the equilibrium concentration of protons in a neutron star. If the latter is larger than a critical value of about 15%, the direct URCA process can happen, and would then enhance neutrino emissions and the neutron star cooling rate [7].

Müller and Serot have recently shown that the asymmetric EOS has quite distinct new features compared to the symmetric one [9]. In particular, in asymmetric nuclear matter the liquid-gas phase transition is second order rather than first order as in symmetric nuclear matter. Furthermore, the instabilities that produce a liquid-gas phase separation in asymmetric nuclear matter arise from fluctuations in the neutron/proton concentration (chemical instability) instead of arising from fluctuations in the baryon density (mechanical instability) as in symmetric nuclear matter. The latter may be related to the recently observed isospin dependence of nuclear multifragmentation in heavy-ion collisions at intermediate energies [10,11].

Presently available radioactive ion beam facilities and planned isospin laboratories provide the unique opportunity to study experimentally the EOS of asymmetric nuclear matter. Tanihata has recently proposed extracting the EOS of asymmetric nuclear matter by studying the properties of neutron-rich nuclei, such as their density distributions, radii, and the thickness of the neutron skin [6]. In this Letter, we suggest a novel approach for studying the asymmetric part of the nuclear EOS, i.e., via the ratio of the number of pre-equilibrium neutrons to that of protons $[R_{n/p}(E_{\rm kin}) \equiv dN_n/dN_p]$ from collisions of neutron-rich nuclei at intermediate energies. Using a transport model with explicit isospin degrees of freedom, we will show that the ratio $R_{n/p}(E_{\rm kin})$ is sensitive to the density dependence of the symmetry energy, but is almost independent of the compressibility of symmetric nuclear matter and the in-medium nucleon-nucleon cross sections.

Theoretical studies (e.g., [7,12–17]) have shown that the EOS of asymmetric nuclear matter can be approximately expressed as

$$E(\rho, \beta) = E(\rho, \beta = 0) + S(\rho)\beta^2, \qquad (1)$$

where $\rho = \rho_n + \rho_p$ is the baryon density; $\beta = (\rho_n - \rho_p)/(\rho_p + \rho_n)$ is the relative neutron excess; and $E(\rho, \beta = 0)$ is the energy per particle in symmetric nuclear matter. The bulk symmetry energy is denoted by $S(\rho) \equiv E(\rho, \beta = 1) - E(\rho, \beta = 0)$. Its value $S_0 \equiv S(\rho_0)$ at normal nuclear matter density is known to be in the range of 27–36 MeV [18]. In the nonrelativistic Hartree-Fock theory (e.g., [19,20]) and the relativistic mean-field theory (e.g., [21–27]), the predicted values of S_0 are 27–38 and 35–42 MeV, respectively. Also, the density dependence of the symmetry energy varies widely among theoretical studies. A $\rho^{2/3}$ dependence was obtained by Siemens using the Bethe-Goldstone theory for asymmetric nuclear matter [12], while the RMF theory predicts a linear dependence [28,29].

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One of the most sophisticated calculations is that of Wiringa, Fiks, and Fabrocini, using the variational many-body theory [15]. Different density dependences of $S(\rho)$ have been found, depending on the nuclear forces used in the calculation. Typical results of these studies can be parametrized by [16]

$$S(\rho) = (2^{2/3} - 1)\left(\frac{3}{5}E_F^0\right)[u^{2/3} - F(u)] + S_0 F(u), \tag{2}$$

with $F(u)$ having one of the following three forms:

$$F_1(u) = \frac{2u^2}{1+u}, \quad F_2(u) = u, \quad F_3(u) = u^{1/2}, \tag{3}$$

where $u \equiv \rho/\rho_0$ is the reduced baryon density and E_F^0 is the Fermi energy. From Eq. (3) the contribution of nuclear interactions to the symmetry energy density can be obtained, i.e.,

$$w_a(\rho, \beta) = e_a \rho F(u) \beta^2, \tag{4}$$

where $e_a \equiv [(S_0 - (2^{2/3} - 1)(\frac{3}{5}E_F^0)]$ is the contribution of nuclear interactions to the bulk symmetry energy at normal nuclear matter density. The mean-field potentials for neutrons and protons due to the symmetry energy are then

$$V_{\text{asy}}^{n(p)}(\rho, \beta) = \partial w_a(\rho, \beta)/\partial \rho_{n(p)}. \tag{5}$$

To illustrate the magnitude of the symmetry potential, we show in Fig. 1, $V_{\text{asy}}^{n(p)}(\rho, \beta)$, using the three forms of $F(u)$ and $S_0 = 32$ MeV. It is seen that the repulsive (attractive) mean field for neutrons (protons) depends sensitively on the form of $F(u)$, the neutron excess β, and the baryon density ρ. In collisions of neutron-rich nuclei at intermediate energies, both β and ρ can be appreciable in a large space-time region where the isospin-dependent mean fields, which are opposite in sign for neutrons and protons, are strong. This will affect differently the reaction dynamics of neutrons and protons, leading to possible differences in their yields and energy spectra. Since the magnitude of the asymmetric part of the nuclear EOS is small compared to the symmetric part in Eq. (1), to extract $S(\rho)$ requires observables which are sensitive to the asymmetric part but not the symmetric part of the nuclear EOS. Also, these observables should not depend strongly on other factors that affect the reaction dynamics, such as the in-medium nucleon-nucleon cross sections. In the following, we shall demonstrate that the ratio $R_{n/p}(E_{\text{kin}})$ of pre-equilibrium neutrons to protons from collisions of neutron-rich nuclei meets these requirements.

We shall use an isospin-dependent Boltzmann-Uehling-Uhlenbeck (BUU) transport model (e.g., [30–33]). The proton and neutron densities calculated from the nonlinear relativistic mean-field theory [25,34] are used as inputs to initialize the BUU model [35,36]. The isospin dependence is included in the dynamics through nucleon-nucleon collisions by using isospin-dependent cross sections and Pauli blocking factors, the symmetry potential $V_{\text{asy}}^{n(p)}(\rho, \beta)$, and Coulomb potential V_c^p for protons. Besides $V_{\text{asy}}^{n(p)}(\rho, \beta)$ and V_c^p the nucleon mean-field $V^{n(p)}(\rho, \beta)$ also includes a symmetric term for which we use a Skyrme parametrization, i.e.,

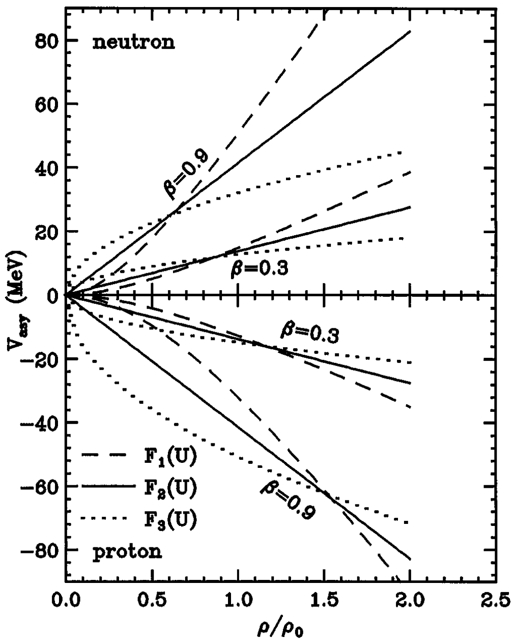

FIG. 1. The symmetry potential for neutrons and protons corresponding to the three forms of $F(u)$ (see text).

$$V^{n(p)}(\rho, \beta) = a(\rho/\rho_0) + b(\rho/\rho_0)^\sigma + V_c^p + V_{\text{asy}}^{n(p)}(\rho, \beta). \tag{6}$$

In the above, the parameters a, b, and σ are determined by the saturation properties and the compressibility K of symmetric nuclear matter [31]. The symmetric term should also contain a momentum-dependent part. However, it is not essential for the present study as we will show that the ratio of pre-equilibrium neutrons to protons from collisions of neutron-rich nuclei is essentially independent of the symmetric part of the nuclear EOS. The isospin-dependent BUU model was used to explain successfully several isospin-dependent phenomena in heavy-ion collisions at intermediate energies [35]. More recently, the isospin dependence of collective flow and balance energy predicted in Ref. [36] using this model was confirmed experimentally at NSCL/MSU by Pak *et al.* [37,38].

We have studied collisions of ^{112}Sn + ^{112}Sn, ^{124}Sn + ^{124}Sn, and ^{132}Sn + ^{132}Sn reactions at a beam energy of 40 MeV/nucleon. The first two reactions have been recently studied experimentally at NSCL/MSU by the MSU-Rochester-Washington-Wisconsin Collaboration [10,11]. Pre-equilibrium particles were measured in these experiments and are now being analyzed [39]. The last system is included only for the purpose of discussions and comparisons. To identify free nucleons from those in clusters, we use in our calculations a phase-space coalescence method at 200 fm/c after the initial contact of the two nuclei, when the quadrupole moment of the nucleon momentum distribution in the heavy residue is almost zero, indicating the approach of thermal equilibrium. A nucleon is considered as free if it is not correlated with other nucleons within a spatial distance of $\triangle r = 3$ fm and a momentum distance of $\triangle p = 300$ MeV/c. We have checked that the results are not sensitive to these parameters if they are varied by less than 30% around the above values.

We first study the effects of the compressibility K of symmetric nuclear matter and the in-medium nucleon-nucleon cross section on the ratio $R_{n/p}(E_{\rm kin})$ by dropping both the Coulomb and symmetry potentials in the BUU model. In Fig. 2 this ratio is shown as a function of nucleon kinetic energy for central (upper panel) and peripheral (lower panel) collisions of ^{132}Sn + ^{132}Sn at a beam energy of 40 MeV/nucleon. When varying the compressibility K from 210 MeV (open squares) to 380 MeV (filled circles), we find that, although the yields of both neutrons and protons increase, their ratio remains almost the same for all impact parameters. This is simply because the effects of symmetric EOS on both neutrons and protons are identical.

The experimental cross section for neutron-proton collisions is about three times that for neutron-neutron (proton-proton) collisions in the energy range studied here. Setting the two cross sections equal (fancy squares), we find that the yields and their ratios change by less than 10% even in peripheral collisions of ^{132}Sn + ^{132}Sn. This result is also easy to understand since both colliding nucleons have the same probability to gain enough energy to become unbound [40]. Thus, the in-medium, isospin-dependent nucleon-nucleon cross sections do not affect much the ratio $R_{n/p}(E_{\rm kin})$. It is important to point out that in the absence of Coulomb and symmetry potentials the ratios are almost independent of the nucleon kinetic energy and have a constant value of about 2.1 ± 0.3 in both central and peripheral collisions of ^{132}Sn + ^{132}Sn.

Including the Coulomb and the asymmetric term of the EOS in Eq. (6), one can then study the effects of the symmetry energy $S(\rho)$ since the Coulomb effect is well known. We expect that the symmetry potential will have the following effects on pre-equilibrium nucleons. First, the symmetry potential $V_{\rm asy}^{n(p)}$ tends to make more neutrons than protons unbound. One therefore expects that a stronger symmetry potential leads to a larger ratio of free neutrons to protons. Second, if both neutrons and protons are already free, the symmetry potential makes neutrons more energetic than protons. These effects are shown in Fig. 3, where we display the ratios calculated using the three forms of $F(u)$ for central (left panels) and peripheral (right panels) collisions of ^{112}Sn + ^{112}Sn, ^{124}Sn + ^{124}Sn, and ^{132}Sn + ^{132}Sn, respectively. The ratios change continuously from central to peripheral collisions; a more detailed study on the impact parameter dependence of the ratio $R_{n/p}(E_{\rm kin})$ will be published elsewhere [41]. The increase of the ratios at lower kinetic energies in all cases is due to Coulomb repulsion which shifts protons from lower to higher kinetic energies. On the other hand, the different ratios calculated using different $F(u)$'s reflect clearly the effect mentioned above, i.e., with a stronger symmetry potential the ratio of pre-equilibrium neutrons to protons becomes larger for more neutron-rich systems.

It is interesting to note that the effects due to different symmetry potentials are seen in different kinetic energy

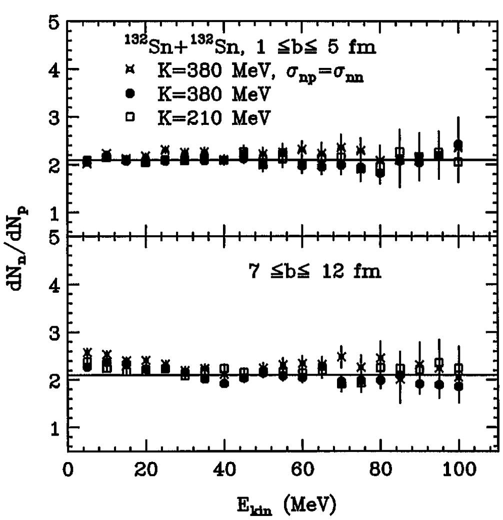

FIG. 2. The ratio of pre-equilibrium neutrons to protons as a function of nucleon kinetic energy for central (upper panel) and peripheral (lower panel) collisions calculated without the Coulomb and symmetry potentials.

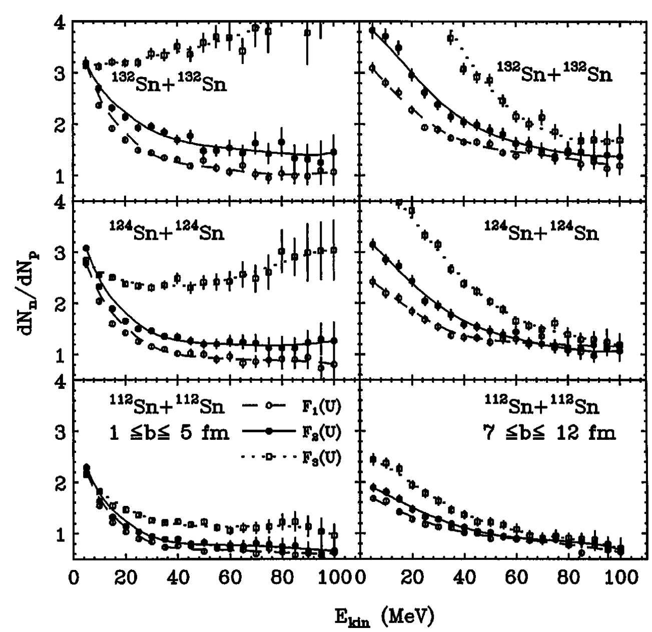

FIG. 3. Same as in Fig. 2 but calculated with both the Coulomb and symmetry potentials.

regions for central and peripheral collisions. In central collisions, the effects of the symmetry potential are most prominent at higher kinetic energies. This is because most of the finally observed free neutrons and protons are already unbound in the early stage of the reaction as a result of violent nucleon-nucleon collisions. The symmetry potential thus affects mainly the nucleon energy spectra by shifting more neutrons to higher kinetic energies with respect to protons. In peripheral collisions, however, there are fewer nucleon-nucleon collisions; whether a nucleon can become unbound depends strongly on its potential energy. With a stronger symmetry potential, more neutrons (protons) become unbound (bound) as a result of a stronger symmetry potential, but they generally have smaller kinetic energies. Therefore, in peripheral collisions, the effects of the symmetry potential show up chiefly at lower kinetic energies. For the more neutron-rich systems the effects of the symmetry potential are so strong that in central (peripheral) collisions different forms of $F(u)$ can be clearly distinguished from the ratio of pre-equilibrium neutrons to protons at higher (lower) kinetic energies. However, because of energy thresholds of detectors, it is difficult to measure low energy nucleons, especially neutrons. Furthermore, the low energy spectrum also has an appreciable contribution from equilibrium emissions. Therefore, the measurement of the ratio $R_{n/p}(E_{\mathrm{kin}})$ in neutron-rich, central heavy-ion collisions, for nucleons with energies higher than about 20 MeV, is more suitable for extracting the EOS of asymmetric nuclear matter.

In conclusion, collisions of neutron-rich nuclei at intermediate energies reveal novel information about the EOS of asymmetric nuclear matter that is of interest to both nuclear physics and astrophysics, such as the properties of radioactive nuclei, supernovas, and neutron stars. The ratio of the number of pre-equilibrium neutrons to that of protons is found to be sensitive to the asymmetric part, but not to the symmetric part of the nuclear EOS. It is also almost independent of the in-medium nucleon-nucleon cross sections. The experimental measurement of this ratio therefore provides a valuable means for determining the EOS of asymmetric nuclear matter.

We would like to thank J. B. Natowitz, W. Q. Shen, and H. M. Xu for helpful discussions. This work was supported in part by the NSF Grant No. PHY-9509266. One of us (Z. R.) was supported in part by grants from the Foundation of National Educational Commission of the Peoples Republic of China.

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