Light Gluino Contribution in Hadronic Decays of the Z Boson and τ Lepton to $O(\alpha_s^3)$

Louis J. Clavelli* and Levan R. Surguladze†

Department of Physics & Astronomy, University of Alabama, Tuscaloosa, Alabama 35487
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The results of calculation of light gluino contributions to hadronic decay widths of the Z boson and τ lepton to $O(\alpha_s^3)$ are presented. The net effect in the case of Z decay is noticeable. For the τ width the effect is very large and, if a light gluino exists, suggests that α_s increases by more than 15% relative to the standard model analysis. [S0031-9007(97)02595-7]

The existence of the light gluino—a color octet, Majorana fermion, of mass a few GeV or less, which is the superpartner of the gluon, is the subject of intensive discussion in the literature [1–3]. The possible impact of light gluinos on LEP/SLC experiments has been discussed recently [2,4].

One of the best ways to look for a trace of a still undiscovered light gluino is to evaluate its contributions to the precisely measured quantities, such as the hadronic decay widths of the Z bozon and the τ lepton, using the perturbation theory. Perturbative QCD (pQCD) calculations of the above quantities up to $O(\alpha_s^3)$ have been completed in recent years [5–9]. For a recent review of the status of these calculations see [10]. For the electroweak contributions see the review [11].

In the present paper we report results of the evaluation of pQCD corrections up to $O(\alpha_s^3)$ to the above quantities due to the gluino. We assume that the gluino is light and its mass is anywhere from less than 1 GeV up to several GeV and the squarks are so heavy that they decouple for the energy range considered ($\sim M_Z$). Note, however, that near the τ mass, the gluino mass has to be below M_τ, in order for our results to be valid for the τ decay rate. However, since the additional corrections due to gluino mass are expected to be suppressed and negligible, our result is relevant for the gluino mass in the GeV region (e.g., at around 1.4 GeV).

We start with a brief outline of the theoretical structure of the Z boson total hadronic decay rate to $O(\alpha_s^3)$

$$\Gamma_Z = \frac{G_F M_Z^3}{8\sqrt{2}\,\pi}\Bigg\{ \sum_f \rho_f \{ v_f^2[\Gamma_0^V(X_f) + \delta_{\mathrm{QED}}^V(\alpha, X_f) + \delta_{\mathrm{QCD}}^V(\alpha_s, X_f, X_t, N_{\tilde{g}})] + a_f^2[\Gamma_0^A(X_f) + \delta_{\mathrm{QED}}^A(\alpha, X_f)$$

$$+\; \delta_{\mathrm{QCD}}^A(\alpha_s, X_f, X_t, N_{\tilde{g}})]\} + \mathcal{L}^V(\alpha_s, X_b, X_t) + \mathcal{L}^A(\alpha_s, X_b, X_t, N_{\tilde{g}}) \Bigg\}. \tag{1}$$

Here the summation index runs over light quark flavors $f = u, d, s, c, b$. We define $X_f = m_f^2(M_Z)/M_Z^2$ and $X_t = m_t^2(M_Z)/M_Z^2$, where we use the $\overline{\mathrm{MS}}$ definition of quark masses. The vector and axial couplings of quark f to the Z boson are $v_f = 2I_f^{(3)} - 4e_f \sin^2\theta_\mathrm{W}$, $a_f = 2I_f^{(3)}$. The electroweak self-energy and vertex corrections are absorbed in the factors ρ_f and k_f. The status report for the electroweak contributions is given in Ref. [11]. The $\Gamma_0^{V/A}(X_f)$ are vector and axial parts of the well known Born approximation of Γ_Z (see, e.g., [11]). The terms $\delta_{\mathrm{QED}}^{V/A}(\alpha, X_f)$ represent contributions due to diagrams involving at least one photon exchange between the quarks (see, e.g., [11]). The QCD contributions are represented by so-called nonsinglet $\delta_{\mathrm{QCD}}^{V/A}()$ and singlet $\mathcal{L}^{V/A}()$ terms. The singlet part is due to Feynman graphs with the electroweak currents in separate quark loops mediated by gluonic states. The other type graphs form the nonsinglet contribution. $N_{\tilde{g}}$ in the above equation is the number of light gluinos that can appear virtually in some topological types of graphs, starting at $O(\alpha_s^2)$. The above QCD terms are calculated up to $O(\alpha_s^3)$ within the standard model, with no gluinos—$N_{\tilde{g}} = 0$ (see, e.g., [10]). In the present work

we calculate the QCD corrections up to the four-loop level that involve light gluino contributions.

The calculational methods are very similar to that for standard QCD (for a detailed description see [12]). The hadronic decay rate of the Z boson in the tree level approximation for the electroweak sector can be evaluated as the imaginary part

$$\Gamma_Z = -\frac{1}{M_Z} \sum_{f=u,d,s,c,b} \mathrm{Im}\,\Pi(m_f, m_t, s + i0)\bigg|_{s=M_Z^2}, \tag{2}$$

where the function Π is defined through a correlation function of two flavor diagonal quark currents

$$i \int d^4x\, e^{iqx} \langle T j_\mu^f(x) j_\nu^f(0) \rangle_0 = g_{\mu\nu}\Pi(m_f, m_t, Q^2)$$
$$- Q_\mu Q_\nu \Pi'(m_f, m_t, Q^2). \tag{3}$$

Here, Q^2 is a large Euclidean momentum $\sim -M_Z^2$. The standard neutral weak current of a quark f coupled to the Z boson is $j_\mu^f = (G_F M_Z^2/2\sqrt{2})^{1/2}(v_f \overline{q}_f \gamma_\mu q_f + a_f \overline{q}_f \gamma_\mu \times \gamma_5 q_f)$. Because of this structure of the neutral weak current, the Π function may be decomposed into vector and axial parts $\Pi(m_f, m_t, Q^2) = \Pi^V(m_f, m_t, Q^2) +$

$\Pi^A(m_f, m_t, Q^2)$. For further calculational convenience we use the approximation

$$\Pi^{V/A}(m_f, m_t, Q^2) = \Pi_0^{V/A}(Q^2, \ln m_t) + O\left(\frac{m_f^2}{Q^2}\right)$$
$$+ O\left(\frac{Q^2}{m_t^2}\right) + \sum_{n \geq 2} \frac{C_n^{V/A} \langle O_n \rangle_0}{Q^{2n}}.$$

(4)

This is a legitimate expansion since our problem scale is M_Z. Thus we work in the limit of zero light quark mass and infinitely heavy top quark mass. The last term in the above equation is the nonperturbative contribution, parametrized by semiphenomenological quantities—the so-called vacuum condensates. $C_n^{V/A}$ are their coefficient functions that can also be evaluated perturbatively (see, e.g., [12] and references therein). In this work we ignore these contributions. The corrections due to light quark masses are known up to the order considered (see, e.g., [10] and references therein). The known mass dependent corrections can simply be added to our result. On the other hand, in the limit of vanishing light quark masses, the vector and axial parts of the nonsinglet contributions are identical. We calculate first the nonsinglet vector (axial) part and then we treat the singlet axial part.

The further steps seem to be straightforward. We write the diagrammatic representation for Π and calculate relevant multiloop Feynman graphs analytically using dimensional regularization [13] and the modified minimal subtraction ($\overline{\text{MS}}$) prescription [14]. After the renormalization of coupling we can get the final result using Eq. (2). Unfortunately this straightforward scheme is realistic only up to three-loop level, since even the advanced computer program HEPLOOPS [15] for evaluation of multiloop Feynman graphs can evaluate diagrams only up to three loops. Fortunately, with the aid of the renormalization group and a unique feature of the $\overline{\text{MS}}$ prescription one can

reduce the four-loop calculation to the evaluation of only one-, two-, and three-loop graphs. The detailed outline of the method and further references are given in Ref. [12].

The running strong coupling obeys the renormalization group equation $\alpha_s \beta(\alpha_s) = \mu^2 d\alpha_s/d\mu^2$, where the QCD β function coefficients for MS type schemes are defined as follows: $\beta(\alpha_s) = -\beta_0(\alpha_s/4\pi) - \beta_1(\alpha_s/4\pi)^2 + O(\alpha_s^3)$, $\beta_0 = 11C_A/3 - 4TN/3 - 2C_AN_{\tilde{g}}/3$, and $\beta_1 = 34C_A^2/3 - 20C_ATN/3 - 4C_FTN - 16C_A^2 N_{\tilde{g}}/3$. In the above coefficients we include light gluino contributions. The corresponding terms are obtained from the analysis of all diagrams contributing to one- and two-loop β-function coefficients, and known results [16] are confirmed.

The evaluation of the three-loop approximation to the decay rate including the light gluino effect is fairly trivial, since there are only two three-loop graphs where the gluino can appear. We use the three-loop graph-by-graph results of [5] and have confirmed three-loop numerical results given in [4]. Note that there are no gluino contributions at one- or two-loop levels. At the four-loop level, the situation is more complex. Although we use still unpublished four-loop diagrammatic results from the first work in [5], it was necessary to reanalyze all 27 four-loop graphs where the light gluino can appear. These graphs contribute with different color weights in the case of gluino. There are several graphs that required recalculation with the help of the HEPLOOPS program [15]. Two of those graphs are shown in Figs. 1(a) and 1(b).

In the previous standard QCD calculations [5], contributions from those and several other graphs were taken into account with the help of the full two-loop gluon propagator inserted into a two-loop graph. In the case of a light gluino it was necessary to treat these graphs individually, because of different color and symmetry weights.

For the nonsinglet part of the four-loop QCD correction to the Z decay rate, including the light gluino contribution, we obtained the following $\overline{\text{MS}}$ analytical result

$$\delta_{\text{QCD}}^{V/A}(\alpha_s, N_{\tilde{g}}) = \left(\frac{\alpha_s(M_Z)}{4\pi}\right)(3C_F) + \left(\frac{\alpha_s(M_Z)}{4\pi}\right)^2 \left\{ C_F^2\left(-\frac{3}{2}\right) + C_F C_A\left[\frac{123}{2} - 44\zeta(3) - [11 - 8\zeta(3)]N_{\tilde{g}}\right] \right.$$
$$\left. - N_f T C_F[22 - 16\zeta(3)] \right\} + \left(\frac{\alpha_s(M_Z)}{4\pi}\right)^3$$
$$\times \left\{ C_F^3\left(-\frac{69}{2}\right) - C_F^2 C_A\{127 + 572\zeta(3) - 880\zeta(5) - [36 + 104\zeta(3) - 160\zeta(5)]N_{\tilde{g}}\} \right.$$
$$+ C_F C_A^2\left[\frac{90\,445}{54} - \frac{10\,948}{9}\zeta(3) - \frac{440}{3}\zeta(5) - \left(\frac{33\,767}{54} - \frac{4016}{9}\zeta(3) - \frac{80}{3}\zeta(5)\right)N_{\tilde{g}}\right.$$
$$\left. + \left(\frac{1208}{27} - \frac{304}{9}\zeta(3)\right)N_{\tilde{g}}^2\right] - N_f T C_F^2[29 - 304\zeta(3) + 320\zeta(5)]$$
$$- N_f T C_F C_A\left[\frac{31\,040}{27} - \frac{7168}{9}\zeta(3) - \frac{160}{3}\zeta(5) - \left(\frac{4832}{27} - \frac{1216}{9}\zeta(3)\right)N_{\tilde{g}}\right]$$
$$+ N_f^2 T^2 C_F\left[\frac{4832}{27} - \frac{1216}{9}\zeta(3)\right] - \pi^2 C_F\left[\left(\frac{11}{3}C_A - \frac{4}{3}N_f T\right) - \frac{2}{3}C_A N_{\tilde{g}}\right]^2 \right\}.$$

(5)

In the above expression the logarithmic contributions are summed up into the running constant by taking $\mu^2 = M_Z^2$. Those contributions can trivially be restored using the renormalization group (see [12]). Inserting the standard $SU_c(3)$ eigenvalues of the Casimir operators for the fundamental and adjoint representations $C_F = 4/3$, $C_A = 3$, and also $T = 1/2$, we obtain the following numerical result

$$
\delta_{\text{QCD}}^{V/A}(\alpha_s, N_{\tilde{g}}) = \frac{\alpha_s(M_Z)}{\pi} + \left(\frac{\alpha_s(M_Z)}{\pi}\right)^2 (1.9857 - 0.1153 N_f - 0.3459 N_{\tilde{g}})
$$
$$
+ \left(\frac{\alpha_s(M_Z)}{\pi}\right)^3 [-6.6369 - 1.2001 N_f - 0.0052 N_f^2 - 2.8505 N_{\tilde{g}} - 0.0311 N_f N_{\tilde{g}} - 0.0466 N_{\tilde{g}}^2)]. \quad (6)
$$

For the singlet part the light gluino contribution to $O(\alpha_s^3)$ shows up in only one single graph [Fig. 1(c)] in the axial channel. In the vector channel, the corresponding graph vanishes due to Furry's theorem [17]. The nonvanishing of the graphs like in Fig. 1(c) is due to a large mass splitting in the top-bottom doublet. This effect was first evaluated exactly in the three-loop level [6] and then extended to four-loop using the large mass expansion method [8]. We use the result of [8] to extract the light gluino contribution to the four-loop axial singlet part. We obtain

$$
\mathcal{L}^A(\alpha_s, X_t, N_{\tilde{g}}) = -\left(\frac{\alpha_s(M_Z)}{\pi}\right)^2 \left[\frac{37}{12} + \ln X_t + O(X_t^{-1})\right]
$$
$$
- \left(\frac{\alpha_s(M_Z)}{\pi}\right)^3 \left[\frac{6401}{216} - \zeta(3) + \frac{7}{6} \ln X_t - \frac{11}{4} \ln^2 X_t - N_f \left(\frac{25}{36} - \frac{1}{9} \ln X_t - \frac{1}{6} \ln^2 X_t\right)\right.
$$
$$
\left. - N_{\tilde{g}}\left(\frac{25}{12} - \frac{1}{3} \ln X_t - \frac{1}{2} \ln^2 X_t\right) - \frac{\pi^2}{3}\left(\frac{11}{4} - \frac{1}{6} N_f - \frac{1}{2} N_{\tilde{g}}\right) + O(X_t^{-1})\right].
$$
$$
(7)
$$

Note that throughout this paper the strong coupling is defined for five flavor effective theory.

For completeness, we also give the result for singlet vector part in the limit of vanishing light quark masses and infinitely heavy top mass [5], although the gluino does not contribute here

$$
\mathcal{L}^V(\alpha_s) = -\left(\frac{\alpha_s(M_Z)}{\pi}\right)^3 [0.4132 + O(X_t^{-1})]
$$
$$
\times \left(\sum_{f=u,d,s,c,b} v_f\right)^2. \quad (8)
$$

The terms of $O(X_t^{-1})$ [8] are about 2 orders of magnitude less than the leading term [5] and are completely negligible.

Next, we use our result to calculate the light gluino contribution to the τ-lepton decay rate to $O(\alpha_s^3)$ in perturbative QCD. We consider the familiar ratio $R_\tau = \Gamma(\tau^- \to \nu_\tau + \text{hadrons})/\Gamma(\tau^- \to \nu_\tau e^- \overline{\nu}_e)$. Here we are interested only in perturbation theory contributions, and we do not consider nonperturbative and instanton corrections. For the calculational method and references see [12]. As before, we work in the limit of vanishing light quark masses and infinitely large heavy quark masses. Note that for the scale $\sim M_\tau$, u, d, s quarks are considered as light and c, b, t are heavy quarks. We use our diagrammatic results obtained for the Z boson case to evaluate correlator of the charged weak currents of quarks coupled to W boson. We obtain the following result for QCD perturbative contributions to R_τ, including a light gluino

$$
R_\tau^{\text{pert}}(M_\tau^2) = 3(0.998 \pm 0.002)\left\{1 + \frac{\alpha_s(M_\tau^2)}{\pi} + \left(\frac{\alpha_s(M_\tau^2)}{\pi}\right)^2\left[\frac{769}{48} - 9\zeta(3) - N_{\tilde{g}}\left(\frac{85}{24} - 2\zeta(3)\right)\right]\right.
$$
$$
+ \left(\frac{\alpha_s(M_\tau^2)}{\pi}\right)^3\left[\frac{363\,247}{1152} - \frac{81}{8}\zeta(2) - \frac{2071}{8}\zeta(3) + \frac{75}{2}\zeta(5)\right.
$$
$$
- N_{\tilde{g}}\left(\frac{10\,787}{72} - \frac{9}{2}\zeta(2) - \frac{649}{6}\zeta(3) + \frac{25}{3}\zeta(5)\right)
$$
$$
\left.\left. + N_{\tilde{g}}^2\left(\frac{3935}{288} - \frac{1}{2}\zeta(2) - \frac{19}{2}\zeta(3)\right)\right] + O\left(\frac{M_\tau^2}{m_c^2}\right) + O\left(\frac{m_{\tilde{g}}^2}{M_\tau^2}\right)\right\}, \quad (9)
$$

and numerically we get

$$
R_\tau^{\text{pert}}(M_\tau^2) = 3(0.998 \pm 0.002)\left[1 + \frac{\alpha_s(M_\tau)}{\pi} + \left(\frac{\alpha_s(M_\tau)}{\pi}\right)^2 (5.2023 - 1.1376 N_{\tilde{g}})\right.
$$
$$
\left. + \left(\frac{\alpha_s(M_\tau)}{\pi}\right)^3 (26.3659 - 21.0358 N_{\tilde{g}} + 1.4212 N_{\tilde{g}}^2) + O\left(\frac{M_\tau^2}{m_c^2}\right) + O\left(\frac{m_{\tilde{g}}^2}{M_\tau^2}\right)\right]. \quad (10)
$$
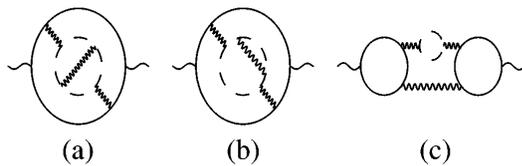
FIG. 1. Four-loop Feynman graphs with light gluino contributing in the nonsinglet (a) and (b) and singlet (c) parts of Γ_Z. The dashed lines correspond to light gluino propagators. The solid and wavy lines correspond to quark and gluon propagators.

The three- and four-loop corrections due to a light gluino are very large, and they increase the extracted $\alpha_s(M_\tau)$ by more than 15%. [On the extraction of $\alpha_s(M_\tau)$ from the τ-lepton decay width see, e.g., [18] and references therein.] For the case $N_{\tilde{g}} = 0$ Eqs. (9) and (10) agree with the known results [5].

Summarizing, we have calculated the light gluino contribution to hadronic decay rates of the Z boson and the τ lepton to four-loop level in perturbative QCD. The corrections in the case of the Z boson decrease the three- and four-loop coefficients by about 25% each. The net effect of a light gluino is to increase $\alpha_s(M_Z)$ by about 2%. In the case of the τ lepton, the corrections are very large and they decrease the three- and four-loop coefficients by 22% and 74%, respectively. Thus, if a light gluino exists, this suggests that for any given experimental value of the τ hadronic width the value of $\alpha_s(M_\tau)$ extracted from our expression that includes a light gluino will be about 15% higher than $\alpha_s(M_\tau)$ extracted from the standard model expression ($N_{\tilde{g}} = 0$). From this, more than 12% increase in $\alpha_s(M_\tau)$ is due to the four-loop gluino contribution that is calculated in the present work. The τ hadronic decay rate remains a major counter-indication to the hypothesis of a light gluino. The value of $\alpha_s(M_\tau)$ extracted from τ decay with a light gluino and extrapolated to M_Z overestimates the direct measurement of $\alpha_s(M_Z)$. Therefore, if a light gluino exists there must be appreciable, positive contributions to the τ hadronic width from, for instance, the nonperturbative region or other as-yet-unknown sources.

Note added.—After this paper was completed, we received a paper [19] where the light gluino contribution was calculated to four loops for the vector part of the Z boson decay rate. The result is in complete agreement with our Eq. (6).

*Electronic address: lclavell@ua1vm.ua.edu
†Electronic address: levan@gluino.ph.ua.edu

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