Quantum Phase Slips and Transport in Ultrathin Superconducting Wires

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We present a microscopic study of the quantum fluctuations of the superconducting order parameter in thin homogeneous superconducting wires at all temperatures below T_c. The rate of quantum phase-slip processes determines the resistance $R(T)$ of the wire, which is observable in very thin wires, even at low temperatures. Furthermore, we predict a new low-temperature metallic phase below a critical wire thickness in the 10-nm range, in which quantum phase slips proliferate. [S0031-9007(97)02502-7]

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The role of fluctuations for the superconducting transition in reduced dimension is well known. Above the critical temperature T_c fluctuations yield an enhanced conductivity [1]. Below T_c one-dimensional (1D) superconductors have a finite resistance due to thermally activated phase slips (TAPS) [2]. Close to T_c the experimental results [3] fully confirm the theoretical predictions [2]. However, as the temperature is lowered the number of TAPS decreases exponentially and no measurable resistance is predicted by the theory [2] at T not very close to T_c. Nevertheless, the experiments by Giordano [4] clearly demonstrate a notable resistivity of ultrathin superconducting wires far below T_c. More recently, other groups reported on strong deviations from the TAPS prediction in thin (quasi-)1D wires as well [5,6].

The natural explanation of these obeservations is in terms of quantum fluctuations which generate quantum phase slips (QPS) in 1D superconducting wires. A first estimate [4] for the QPS tunneling rate $\propto \exp(-S_{\rm QPS})$, however, leads to the disappointing conclusion that the action $S_{\rm QPS}$ roughly equals the number of transverse channels $N_{\rm ch} = k_F^2 S$ in the wire ($S = \pi r_0^2$ is the cross section of the wire), which is very large even for the thinnest wires used in the experiments [4] (e.g., for $r_0 \sim 10^{-6}$ cm we have $S_{\rm QPS} \sim 10^2$-10^3), and therefore QPS effects should be strongly suppressed. This estimate is obtained from the formula $S_{\rm QPS} \sim U_{\rm QPS}/\omega_a$, with energy barrier $U_{\rm QPS}$ and attempt frequency $\omega_a \sim \Delta$. Assuming $U_{\rm QPS}$ to be the condensation energy $N_0\Delta^2/2$ in a volume $\xi_0 S$ during a time Δ^{-1}, one obtains $S_{\rm QPS} \sim \xi_0 S N_0 \Delta^2/2\Delta \sim k_F^2 S/4\pi^2 \sim N_{\rm ch}$. A similar estimate has been obtained using a phenomenological time dependent Ginzburg-Landau (TDGL) free energy with second order time derivatives [7,8]. Furthermore, recently Duan [8] argued that the electromagnetic field yields an additional suppression of the QPS rate by the factor of $\exp(-1/\alpha)$; $\alpha = 1/137$ is the fine structure constant. Even further suppression of this rate—similar to the case of Josephson junctions [9]—can be expected due to dissipative currents in the QPS core. In contrast, the magnitude of the resistance for the thinnest wires measured in Ref. [4] yields $S_{\rm QPS} \sim 10$ with the QPS rate by orders of magnitude larger than that derived from the above estimates.

In this Letter we argue that the above estimate needs *qualitative* improvement. First, the estimate for the potential barrier can be improved upon: as the typical electron mean free path in the wires [4] is very small $l \lesssim 10$ nm, one should rather take $\xi \sim \sqrt{l\xi_0} \ll \xi_0$ for the typical QPS size. Second, we show below that the role of the electromagnetic field for thin wires was overestimated in Ref. [8] (roughly by a factor of $r_0/\lambda_L \sim 10^{-1}$-$10^{-2}$; λ_L is the London length of a bulk superconductor). Third, the dissipative currents turn out not to have a strong impact on the QPS rate, especially in the limit of low T. Also from a general point of view, TDGL-based theories [7,8] are insufficient at T not very close to T_c and fail to give qualitatively correct results. A microscopic theory of QPS is needed that accounts for nonequilibrium, dissipative, and electromagnetic effects during a QPS event.

This theory is reported upon below. Taking into account the above effects we *determine* both the typical size x_0 and time scale τ_0 of a QPS core. For a dirty wire in the Drude limit we obtain $x_0 \approx c_0\tau_0 \approx (\xi^2 c_0/\Delta_0)^{1/3}$ (c_0 is the velocity of the Mooij-Schön mode [10]). For typical parameters the product $x_0\tau_0$ (which enters into $S_{\rm QPS}$) is *smaller* than the naive estimate ξ_0/Δ. The resulting QPS action $S_{\rm QPS} \sim N_0 S(\Delta_0\xi)^{4/3}/c_0^{1/3}$ is also smaller by a factor of $\sim 10^2$, and thus for sufficiently thin wires QPS's are observable for all T. Furthermore, at $T = 0$ we predict a new *metal-superconductor* (MS) phase transition governed by electromagnetic interactions between different QPS's. We also evaluate the effective resistance of a 1D superconducting wire and determine the crossover temperature between the regimes of TAPS and QPS.

The model.—Our calculation is based on the effective action approach for a BCS superconductor [11]. The starting point is the partition function Z expressed as an imaginary time path integral over the electronic fields ψ

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and the gauge fields $V, \mathbf{A}$, with Euclidean action

$$S = \int d^3\mathbf{r} \int_0^\beta d\tau \{\overline{\psi}_\sigma[\partial_\tau - ieV + \xi(\nabla - ie\mathbf{A}/c)]\psi_\sigma - g\overline{\psi}_\uparrow\overline{\psi}_\downarrow\psi_\downarrow\psi_\uparrow + ieVn_i + [\mathbf{E}^2 + \mathbf{B}^2]/8\pi\}.$$

Here $\beta = 1/T$, $\xi(\nabla) \equiv -\nabla^2/2m - \mu$ describes a single conduction band, g is the BCS coupling constant, en_i denotes the background charge density of the ions, and units in which $\hbar = k_B = 1$ are used. A Hubbard-Stratonovich transformation introduces the energy gap Δ as an order parameter and the electronic degrees of freedom can be integrated out. What remains is an expression for the partition function in terms of an effective action for Δ, V, and $\mathbf{A}$, with a saddle-point solution $\Delta = \Delta_{\mathrm{BCS}}$ and $V = \mathbf{A} = 0$

$$S_{\mathrm{eff}} = \int d^3\mathbf{r} \int_0^\beta d\tau \left[\frac{|\Delta|^2}{g} + \frac{\mathbf{E}^2 + \mathbf{B}^2}{8\pi}\right] - \mathrm{Tr}\, \ln \hat{G}^{-1},$$

$$\hat{G}^{-1} = \left(\partial_\tau + \frac{i}{2}\{\nabla, \mathbf{v}_s\}\right)\hat{1} + \Delta_0\hat{\sigma}_1 + \left(\xi(\nabla) + \frac{m\mathbf{v}_s^2}{2} - ie\Phi\right)\hat{\sigma}_3,$$

where the superfluid velocity $\mathbf{v}_s = (1/2m)[\nabla\varphi - 2e\mathbf{A}/c]$, the chemical potential for Cooper pairs $\Phi = V - \dot{\varphi}/2e$, and $\Delta = \Delta_0 e^{i\varphi}$ have been introduced.

Effective action.—The effective theory is constructed by expanding up to second order around the saddle point in Φ and $\mathbf{v}_s$ to obtain the electronic polarization terms. Using the Ward identity from gauge invariance, the result can be written as the sum of terms, related to the normal and superfluid densities n_n and n_s, which describe normal and superconducting "screening," respectively [12]

$$S_{\mathrm{pol}} = \frac{S}{\beta} \sum_{|\omega_\mu| > \Delta_0} \int dx\, \frac{\sigma}{2|\omega_\mu|} E^2 + S \int dx\, d\tau \left(\frac{mn_s}{2} v_s^2 + e^2 N_0\left[\frac{n_n}{n} V^2 + \frac{n_s}{n}\Phi^2\right]\right),$$

where use was made of the one-dimensional nature $r_0 < \xi$ of the problem, and σ is the conductivity of the wire. Transverse screening is irrelevant if the London penetration depth $\lambda_L > r_0$ and we retain only one component of the vector potential [2].

A phase-slip event in imaginary time involves a suppression of the order parameter in the phase-slip core and a winding of the superconducting phase around this core. We now separate the total QPS action S_{QPS} into a core part S_{core} around the phase-slip center for which the condensation energy and dissipation by normal currents are important (scales $x \leq x_0$, $\tau \leq \tau_0$), and a hydrodynamic part outside the core S_{out} which depends on the hydrodynamics of the electromagnetic fields

$$S_{\mathrm{out}} = \int dx\, d\tau \left(\frac{C + C'}{2} V^2 + \frac{\tilde{C}}{2}\Phi^2 + \frac{1}{2Lc^2} A^2 + \frac{m^2\mathbf{v}_s^2}{2e^2\tilde{L}}\right),$$

where the kinetic inductance $\tilde{L} = m/(e^2 n_s S)$ and the kinetic capacitance $\tilde{C} = Se^2 N_0 n_s/n$ have been introduced, as well as $C' = Se^2 N_0 n_n/n$, which we will drop from now on in the limit $n_s \gg n_n$ at low T. The geometry and screening by dielectrics outside the wire are accounted for by the capacitance per length $C = \epsilon_r[2\ln(x_0/r_0)]^{-1}$ and the inductance times length $L = 2\ln(x_0/r_0)/c^2$ that replace the $\mathbf{E}^2 + \mathbf{B}^2$ term. Here c is the velocity of light and ϵ_r the dielectric constant of the substrate.

Single QPS.—Outside the phase-slip core, the equations of motion for V, A, and φ are solved by the saddle point

$$\tilde{\varphi} = \arg(x + ic_0\tau); \quad c_0^2 = \frac{C^{-1} + \tilde{C}^{-1}}{L + \tilde{L}},$$

$$V = \frac{1}{1 + C/\tilde{C}} \frac{\partial_\tau \varphi}{2e}; \quad A = \frac{c}{1 + \tilde{L}/L} \frac{\partial_x \varphi}{2e}.$$

The space-time anisotropy is determined by the plasmon velocity c_0, rather than by v_F. For generic parameters the velocity c_0 reduces to the velocity of the Mooij-Schön mode, which has dispersion $\omega^2 = c_{\mathrm{MS}}^2 k^2$ with $c_{\mathrm{MS}}^2 = S\omega_{pl}^2/4\pi C$ [10], where ω_{pl} is the 3D plasma frequency. The corresponding saddle-point action is

$$S_{\mathrm{out}}^* = \mu \ln[\min(c_0\beta, X)/\max(c_0\tau_0, x_0)], \tag{1}$$

with $\mu = \pi/[4e^2 c_0(L + \tilde{L})]$.

The contribution from the core part is estimated to be

$$S_{\mathrm{core}}^* = \frac{N_0}{2} S\tau_0 x_0 \Delta_0^2 + \frac{S}{\beta} \sum_{|\omega_\mu| > \tau_0^{-1}} \frac{x_0\sigma}{|\omega_\mu|} \left| E\left(\omega_\mu, \frac{x_0}{2}\right)\right|^2. \tag{2}$$

The first part is the condensation energy that is lost inside the core and the second part defines the energy of dissipative currents in the core during a phase-slip event. (We assume that the conductivity equals the normal state value inside the QPS core.) Inserting the saddle-point solution, and minimizing the action with respect to x_0 and τ_0, we find $x_0 \approx c_0\tau_0 \approx (\sigma c_0/2e^2 N_0\Delta_0^2)^{1/3}$ and S_{core}^* as three times the condensation energy in (2). In the Drude limit $\sigma = 2e^2 N_0 v_F l/3$, we obtain $x_0 \approx (\xi^2 c_0/\Delta_0)^{1/3}$ and

$$S_{\mathrm{core}}^* = bN_0 S(\Delta_0\xi)^{4/3}/c_0^{1/3}, \quad b \sim 1. \tag{3}$$

QPS Interactions.—The next step is to consider a gas of QPS's in a superconducting wire. We also assume that an applied current I (much smaller than the depairing current) is flowing through the wire. Substituting the saddle-point solution $\varphi = \sum_i^n \tilde{\varphi}(x - x_i, \tau - \tau_i)$ into the action and keeping track of the additional term $\int d\tau \int dx (I/2e)\partial_x\varphi$ [11], we find

$$S_n = nS_{\mathrm{core}}^* - \mu \sum_{i \neq j} \nu_i \nu_j \ln\left(\frac{\rho_{ij}}{x_0}\right) + \frac{\Phi_0}{c} I \sum_i \nu_i \tau_i. \tag{4}$$

Here $\rho_{ij} = [c_0^2(\tau_i - \tau_j)^2 + (x_i - x_j)^2]^{1/2}$ defines the distance between the ith and jth QPS in the (x, τ) plane, $\nu_i = +1$ (-1) are the QPS (anti-QPS) "charges," and $\Phi_0 = hc/2e$ is the flux quantum. Only neutral QPS configurations with $\nu_{\text{tot}} = \sum_i^n \nu_i = 0$ (and hence n even) contribute to the partition function. This is a consequence of the boundary condition $\varphi(x, \tau) = \varphi(x, \tau + \beta)$ in the path integral for the partition function [11].

Metal-superconductor phase transition.—For $I = 0$ Eq. (4) defines the standard model of a 2D gas of logarithmically interacting charges ν_i. The effective (small) fugacity y of these charges (or the QPS rate $y/x_0\tau_0$) is

$$y = x_0\tau_0 B \exp(-S^*_{\text{core}})\,, \tag{5}$$

where B is the usual fluctuation determinant with zero modes excluded. From the Coulomb gas analogy, we conclude that a Kosterlitz-Thouless-Berezinskii (KTB) phase transition [13] for QPS's occurs in a superconducting wire at $\mu = \mu^* \equiv 2 + 4\pi y \approx 2$: for $\mu < \mu^*$ the density of free QPS in the wire (and therefore its resistance) always remains finite, whereas for $\mu > \mu^*$ QPS's and anti-QPS's (AQPS) are bound in pairs and the *linear* resistance of a superconducting wire is strongly suppressed and T dependent. We arrive at an *important conclusion*: at $T = 0$ a 1D superconducting wire *is* in a superconducting state, with vanishing linear resistance, provided the electromagnetic interaction between phase slips is sufficiently strong, i.e., $\mu > \mu^*$.

The above analysis is valid for sufficiently long wires. For typical experimental parameters, however, $X < c_0\beta$ (or even $X \ll c_0\beta$), and the finite wire size needs to be accounted for. To this end, we first apply the standard 2D scaling analysis [13] $\partial_l \mu = -4\pi^2\mu^2 y^2$ and $\partial_l y = (2 - \mu)y$, where μ and y depend on the scaling parameter l. Solving these equations up to $l = l_X = \ln(X/x_0)$ we obtain the renormalized values $\tilde{\mu} = \mu(l_X)$ and $\tilde{y} = y(l_X)$. For larger scales $l > l_X$ only the time coordinate matters and the problem reduces to a that of a 1D Coulomb gas with logarithmic interaction. Therefore, (for $\tilde{y} \ll 1$) further scaling for $l > l_X$ is defined by [11,14] $\partial_l \mu = 0$ and $\partial_l y = (1 - \mu)y$. For $\tilde{\mu} > 1$ the fugacity scales down to zero, which again corresponds to a superconducting phase, whereas for $\tilde{\mu} < 1$ it increases indicating a resistive phase in complete analogy to a single Josephson junction with Ohmic dissipation. Thus, our above conclusion about the presence of a MS phase transition at $T = 0$ remains valid also for relatively short wires, although the phase transition point is given by the somewhat different condition $\tilde{\mu} = 1$. In practice, both conditions $\mu = \mu^*$ and $\tilde{\mu} = 1$ are realized in wires with diameter $2r_0 \sim 10$–20 nm; see also the discussion below.

Wire resistance at low T.—At any nonzero T the wire has a nonzero resistance $R(T, I)$ even in the "ordered" phase $\mu > \mu^*$ (or $\tilde{\mu} > 1$). In order to evaluate $R(T)$ in this phase for a long wire we proceed perturbatively and first calculate the free energy correction δF due to one bound QPS-AQPS pair. [See Ref. [11] (Chap. 5.3) for a discussion of a similar procedure.] The one QPS-AQPS pair contribution δF to the free energy of the wire is

$$\delta F = \frac{X y^2}{x_0\tau_0} \int_{\tau_0}^{\beta} \frac{d\tau}{\tau_0} \int_{x_0}^{X} \frac{dx}{x_0}\, e^{(\Phi_0 I\tau/c) - 2\mu \ln[\rho(\tau,x)/x_0]}, \tag{6}$$

where $\rho = (c_0^2\tau^2 + x^2)^{1/2}$. It is convenient to first integrate over the spatial coordinate x and take the wire length $X \to \infty$. For nonzero I the expression in Eq. (6) is formally divergent for $\beta \to \infty$ and acquires an imaginary part Im δF after analytic continuation of the integral over the temporal coordinate τ [11,15]. This indicates a QPS-induced instability of the superconducting state of the wire: the state with a zero phase difference $\delta\varphi(X) = \varphi(X) - \varphi(0) = 0$ decays into a lower energy state with $\delta\varphi(X) = 2\pi$. The corresponding decay rate is $\Gamma_{2\pi} = 2\text{Im}\,\delta F$. The rate for the opposite transition (which is nonzero at nonzero T) is defined analogously with $I \to -I$. The average voltage drop $V = (\Phi_0/c)[\Gamma_{2\pi}(I) - \Gamma_{2\pi}(-I)]$ across the wire is

$$V = \frac{\Phi_0 X y^2}{c\tau_0 x_0} \frac{\sqrt{\pi}\,\Gamma(\mu - \frac{1}{2})}{\Gamma(\mu)\Gamma(2\mu - 1)} \sinh\!\left(\frac{\Phi_0 I}{2cT}\right) \times \left|\Gamma\!\left(\mu - \frac{1}{2} + \frac{i}{\pi}\frac{\Phi_0 I}{2cT}\right)\right|^2 \left[\frac{2\pi\tau_0}{\beta}\right]^{2\mu-2}. \tag{7}$$

$\Gamma(x)$ is the Euler gamma function. For the wire resistance $R(T, I) = V/I$ this yields $R \propto T^{2\mu-3}$ and $R \propto I^{2\mu-3}$ for $T \gg \Phi_0 I$ and $T \ll \Phi_0 I$, respectively. For thick wires with $\mu > \mu^*$, we expect a strong temperature dependence of the resistivity. For thinner wires the temperature dependence of the resistivity becomes linear at the transition to the disordered phase in which our analysis is not valid. At $T \ll \Phi_0 I/c$ we expect a strongly nonlinear I-V characteristic $V \sim I^a$ in thick wires, and a universal $a(\mu^*) = 2$ in thin wires at the transition into the resistive state with $V \sim I$, i.e., $a = 1$. Note that in contrast to the KTB transition in 2D superconducting films, the jump is from $a = 2$ to 1, instead of $a = 3$ to 1.

For a short wire $X < c_0/T$ we again proceed in two steps. A 2D scaling analysis yields the "global" parameters $\tilde{y}$, $\tilde{\mu}$, and the microscopic cutoff $\tilde{\tau}_0 = \tau_0 X/x_0$. In analogy with the resistively shunted Josephson junction [11], the voltage drop from the imaginary part of the free energy reads

$$V = \frac{2\Phi_0\tilde{y}^2}{\Gamma(2\tilde{\mu})c\tilde{\tau}_0} \sinh\!\left(\frac{\Phi_0 I}{2cT}\right) \times \left|\Gamma\!\left(\tilde{\mu} + \frac{i\Phi_0 I}{2\pi cT}\right)\right|^2 \left[\frac{2\pi\tilde{\tau}_0}{\beta}\right]^{2\tilde{\mu}-1},$$

giving $R \propto T^{2\tilde{\mu}-2}$ and $R \propto I^{2\tilde{\mu}-2}$, respectively, at high and low T. This result is valid for $\tilde{\mu} > 1$ and also for smaller $\tilde{\mu}$ at not very small T [11]. At $T \to 0$ in the metallic phase the resistance flattens off and becomes [11]

$$R = R_q/\tilde{\mu}\,, \tag{8}$$

where $R_q = \pi/2e^2 \simeq 6.5\ \mathrm{k}\Omega$ is the quantum resistance. Note that in this limit the size dependence of the resistance enters only through the renormalized value $\tilde{\mu}$.

Crossover temperature.—The crossover between the TAPS and the QPS regime occurs at a temperature T^* that can be estimated by comparing the corresponding resistivities R_{TAPS} and R_{QPS}. The standard result of Ref. [2] is $R_{\mathrm{TAPS}} = \beta\Omega(h/4e^2)\exp(-\beta\Delta F)$, with $\Delta F = \sqrt{2}\,H_c^2 S\xi/3\pi$, attempt frequency $\Omega = (X/\xi)\sqrt{\beta\Delta F}\tau_s^{-1}$, and relaxation time $\tau_s^{-1} = 8(T_c - T)/\pi$. For thin wires with $\mu < S^*_{\mathrm{core}}$ the value T^* which follows from a comparison of the exponents $\beta\Delta F$ and $2S^*_{\mathrm{core}}$ is

$$T^* = \frac{\Delta F}{2S^*_{\mathrm{core}}} \approx \Delta_0^{2/3} c_0^{1/3}/\xi^{1/3}. \qquad (9)$$

The pre-exponential factor B in Eq. (5) can be estimated by matching the pre-exponential factors of R_{QPS} and R_{TAPS} at $T = T^*$. A more detailed analysis of the pre-exponent will be published elsewhere.

Discussion.—For typical system parameters $k_F^{-1} \sim 0.2$ nm $< l \sim 7$ nm $< \xi \sim 10$ nm $< \xi_0 \sim \lambda_L \sim 100$ nm, we find that L and $\tilde{C}$ drop out of the problem and $\tilde{L}$ and C determine the physics (unless $T \sim T_c$ or $\epsilon_r \gg 1$). We will also take the length X of the wire to be smaller than the localization length, so that localization effects do not play a role. Taking $r_0 \sim 10$ nm and $\epsilon_r = 1$, we obtain the velocity $c_0/c = c_{\mathrm{MS}}/c \approx (r_0/6\lambda_L)$,

$$\mu = (\pi\sqrt{\epsilon_r}/8\alpha)(r_0/\lambda_L) \approx 50(r_0/\lambda_L),$$

and $2S^*_{\mathrm{core}} \lesssim 10$. Thus—in contrast to previous studies [8]—quantum fluctuations in thin superconducting wires are not negligibly small and can be well observed in experiment. Furthermore, our estimate for the classical-to-quantum crossover temperature Eq. (9) yields $T^* \sim 10\Delta(T^*)$, i.e., for thin wires one expects this crossover to happen quite close to T_c. These features are in good agreement with the experimental findings [4].

For the quoted parameters, we predict the superconductor to metal transition at a wire thickness $r_0 \simeq \lambda_L/25 \approx 5-10$ nm. This prediction agrees with the results of Giordano, who finds that wires with $r_0 \approx 8$ nm have a resistivity that saturates at a measurable level at low T, whereas the resistivity of thicker wires $r_0 \gtrsim 13$ nm decreases with T even at the lowest temperatures [4]. Also the saturation value $R = R_q/\tilde{\mu} \sim 10-20\ \mathrm{k}\Omega$ (8) is consistent with that measured in [4].

Independent measurements of $R(T)$ for superconducting wires have been reported in Ref. [5]. Whereas the data [5] for thicker wires agree with the TAPS theory [2], the resistance of *thinner* wires was found to be systematically *higher* than R_{TAPS}. This behavior is qualitatively similar to that observed in [4] and can be also attributed to the effect of QPS discussed here. Furthermore, the resistivity of the thinnest wires used in [5] (with $S \approx 10^{-13}$ cm^2) extrapolates to a finite $T = 0$ value. This is consistent with our prediction that wires with cross section $S \lesssim 10^{-13}$ cm^2 (i.e., $\mu < 1-2$) exhibit metallic behavior even at $T = 0$. Further measurements at lower T would be desirable to verify our conclusions.

Note that superconductivity in wires with radius $r_0 \sim 5$ nm is *not* destroyed by finite size and level spacing effects; particles of radius down to 1–3 nm do turn superconducting [16]. Finally, the MS phase transition discussed here is in many respects different from that in granular wires [17]. In the latter case the onsite Coulomb interaction drives the transition into an insulating phase. For homogeneous wires, in contrast, the transition is into a metallic state.

In conclusion, we have studied QPS's starting from microscopic theory and find a measurable resistivity in superconducting ultrathin wires at temperatures $T \ll T_c$, as well as a new superconductor to metal phase transition as a function of the wire thickness.

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