Family Replication in the Dual Standard Model

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The family replication problem is addressed in the context of the dual standard model. The breaking of a simple grand unified group to $(G_{\rm low} \times H_1 \times H_2 \times H_3)/Z_5^3$, and then further to $G_{\rm low}$, produces a spectrum of stable monopoles that falls into three families, each of whose magnetic quantum numbers correspond to the electric charges on the fermions of the standard model. Here $G_{\rm low} = [{\rm SU}(3) \times {\rm SU}(2) \times {\rm U}(1)]/Z_6$ is the symmetry group of the standard model above the weak scale, and H_i are simple Lie groups which each have a Z_5 symmetry in common with $G_{\rm low}$. [S0031-9007(97)02401-0]

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In his 1962 paper, Skyrme [1] made the radical suggestion that baryons and mesons may be seen as the solitons of what is now known as the Skyrme model. More recently, it has been realized [2,3] that one can take Skyrme's program further and construct a "dual standard model" in which the magnetic monopoles correspond to the quarks and leptons of the standard model. In this program, there is no freedom to add particles to the spectrum since the spectrum of monopoles is completely determined by the topology of the model. If successful, such a program would reproduce the charge spectrum of standard-model fermions, their group representations, their space-time transformation properties, and, ultimately, the mass spectrum and dynamics of interacting fermions, making it possible to answer questions that have long eluded current particle physics models.

In [2,3] it was pointed out that breaking a grand unified SU(5) symmetry results in a charge spectrum of stable magnetic monopoles in one-to-one correspondence with one family of standard-model fermions. The symmetry breaking under consideration was

$$\mathrm{SU}(5) \to G_{\rm low} = [\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}_Y(1)]/Z_6\,, \quad (1)$$

and the scalar field masses were chosen so that the long range SU(3) × SU(2) interactions between monopoles is stronger than the $\mathrm{U}_Y(1)$ interactions. The charge spectrum of the stable monopoles in this model is shown in Table I.

While the correspondence between the monopoles of the SU(5) model and a family of standard-model fermions is remarkable, it is by no means complete since the standard model has three families of light fermions, not one. The existence of three families of light fermions has long been an outstanding problem in particle physics [4]. It is this problem that we now address: We find a symmetry breaking that yields three families of monopoles with the magnetic charge spectrum of each of these families corresponding exactly to the electric "charge" spectrum of a family of standard-model fermions.

The strategy we adopt for obtaining three families of monopoles is to build upon the correspondence of SU(5) monopoles with quarks and leptons. In essence, we want the SU(5) monopoles three times over. For this, we consider a symmetry breaking pattern of the kind

$$G \to K \equiv \frac{[\mathrm{SU}_0(5) \times H_1 \times H_2 \times H_3]}{[Z_5^{(1)} \times Z_5^{(2)} \times Z_5^{(3)}]}\,, \quad (2)$$

where G and H_i are all simply connected groups. (The Z_5 factors in the denominator can be generalized to Z_n where $n \geq 5$ but $n \neq 6$. We consider only $n = 5$ as it is the simplest.) The $Z_5^{(i)}$ $(i = 1,2,3)$ contain group elements that are common to $\mathrm{SU}_0(5)$ and H_i. A specific example is $H_i = \mathrm{SU}_i(5)$, with $Z_5^{(i)}$ the center of H_i. Then $K = \mathrm{SU}(5)^4/Z_5^3$, that is, all four SU(5)'s share a common Z_5 center. A possible choice for G is $\mathrm{SU}(5^4) = \mathrm{SU}(625)$, but smaller groups may also work [5]. Since the spectrum of monopoles depends only on the incontractable closed paths in K, the actual choice of G is immaterial as long as G is simply connected.

Consider the incontractable paths in K. An example of such a path is one that starts on the identity, traverses $\mathrm{SU}_0(5)$ to an element of $Z_5^{(i)}$, then returns to the identity through H_i. This is a closed path that is incontractable because of the discrete nature of $Z_5^{(i)}$, and corresponds to a monopole with $\mathrm{SU}_0(5)$ and H_i charge. We call this monopole a "digit." Similarly, there are paths that pass through H_i and H_j $(i \neq j)$ and avoid $\mathrm{SU}_0(5)$ altogether; these correspond to a monopole which is a singlet of $\mathrm{SU}_0(5)$ but which has H_i and H_j charge. We refer to these as "sterile" monopoles. All other incontractable paths (and hence all other monopoles), such as those that pass through $\mathrm{SU}_0(5)$ and several of the H_i, can be built out of these two types of paths.

TABLE I. "Charges" on stable SU(5) monopoles and their corresponding standard-model fermions. Monopole and fermion-representation degeneracies d_m and d_f are also given.

n	SU(3)	SU(2)	$\mathrm{U}(1)_Y$	$d_m = d_f$	
+1	1/3	1/2	+1/6	6	$(u,d)_L$
−2	1/3	0	−1/3	3	d_R
−3	0	1/2	−1/2	2	$(\nu,e)_L$
+4	1/3	0	+2/3	3	u_R
−6	0	0	−1	1	e_R

We next break the $SU_0(5)$ to $G_{low} = [SU(3) \times SU(2) \times U_Y(1)]/Z_6$, the low energy symmetry group. The pre-existing digit monopoles from the symmetry breaking in (2) will now get SU(3), SU(2), and $U_Y(1)$ charges. In addition, new monopoles lying entirely in the $SU_0(5)$ sector will be produced since G_{low} has its own incontractable closed paths. We refer to these as "pure" monopoles. The $U_Y(1)$ charge on a pure monopole is five times the $U_Y(1)$ charge on the digit with the same $SU(3) \times SU(2)$ charges. To see this, note that the incontractable path for the digit (produced during $G \to K$) need only traverse between elements of $Z_5^{(i)}$ shared with the $U_Y(1)$. For example, there is a pure monopole corresponding to the path that traverses the entire $U_Y(1)$ circle; but there is a digit with the same $SU(3) \times SU(2)$ charges whose path traverses only one-fifth of the $U_Y(1)$ circle and then closes by traversing a path in the H_i factors. So, at this stage, there are two types of monopoles: digits, with nonzero 3-2-1 and H_i charges, and pure monopoles with zero H_i charge and a $U_Y(1)$ charge that is five times the charge of the digit with the same 3–2 charges.

The next step is to break each of the H_i to $Z_5^{(i)}$ since we want the low energy symmetry to be the usual G_{low}. This symmetry breaking does not yield any new monopoles, but it does produce Z_5 strings that confine the digits into clusters of 5 with each cluster being a singlet of the H_i. Since this cluster is a singlet of all the H_i, its topological charge agrees with the topological charge of the corresponding pure monopole. Hence the SU(3) and SU(2) charges and the hypercharge on all the monopoles are given by the usual values shown in Table I. At this stage, the sterile monopoles also get connected by strings into H_i singlets. But since the sterile monopoles have no $SU_0(5)$ charge, the clusters of sterile monopoles are topologically trivial and can decay to the vacuum. The exception to this statement would be if the cluster is fermionic (as of course we must imagine all the other clusters to ultimately be if they are to correspond to standard-model fermions). Fermionic clusters of sterile monopoles would correspond to right-handed neutrinos.

Now that the charge spectrum of the monopoles agrees with that shown in Table I, we need to count the different monopoles of each 3-2-1 charge. For this we look at the interactions of the digits in a cluster. The digits interact by exchange of 3-2-1 gauge and scalar fields and, by an argument identical to that in [6], a cluster of 5 digits would be unstable to declustering in the absence of Z_5 strings. But the Z_5 strings provide a confining potential and do not allow the cluster to disperse. This shows that the pure monopoles are unstable to decaying into a cluster of digits that are confined by strings.

The digit clusters confined by Z_5 strings in each of the three H_i's will turn out to be the three families of monopoles corresponding to the three families of standard-model fermions. A cluster composed of digits having charges in different H_i's is unstable to decay into a cluster with digits having charge in a single H_i. We show this by explicit construction in a concrete example.

We realize that nothing changes if $SU_0(5)$ is replaced by $G_{low} = [SU(3) \times SU(2) \times U_Y(1)]/Z_6$ directly in Eq. (2) since the Z_5 center of $SU_0(5)$ is contained in $U_Y(1)$. Consider now the specific symmetry breaking,

$$G \to [G_{low} \times SU(5)^3]/Z_5^3 \to G_{low} . \qquad (3)$$

The monopoles formed in the first stage of symmetry breaking correspond to all closed incontractable paths in the unbroken group which have the form,

$$P[s] = \exp\left[is\left(n_3 T_8 + n_2 \lambda_3 + n_1 Y + \sum_{i=1}^{3} m_i \Lambda_{24}^i \right)\right], \quad s \in [0, 4\pi],$$

where n_i and m_i are integers. The generators T_8, λ_3, Y, and Λ_{24}^i of SU(3), SU(2), $U_Y(1)$, and $SU_i(5)$ $(i = 1, 2, 3)$, respectively, generate the centers of these groups. They are normalized to satisfy

$$e^{i4\pi n T_8} = e^{-i2\pi n/3}\mathbf{1}, \qquad e^{i4\pi n \lambda_3} = e^{i2\pi n/2}\mathbf{1},$$
$$e^{i4\pi n Y} = e^{i2\pi n/30}\mathbf{1}, \qquad e^{i4\pi n \Lambda_{24}^j} = e^{i2\pi n/5}\mathbf{1},$$
$$j = 1, 2, 3,$$

where n is any integer and $\mathbf{1}$ is the identity element of G. Note the normalization of $U_Y(1)$ generator Y here differs from that of Table I by a factor of 5. For $P[s]$ to be closed we need $P[0] = P[4\pi]$, and so we have the following constraint on the integers n_i, m_i:

$$\frac{n_1}{30} + \frac{n_2}{2} - \frac{n_3}{3} + \frac{m}{5} = \text{integer}, \qquad (4)$$

where $m = m_1 + m_2 + m_3$. The only monopoles in which we are interested are those with nontrivial hypercharge $(n_1 \neq 0)$ since those with $n_1 = 0$ will be topologically equivalent to the vacuum once the SU(5)'s break down in the second stage of symmetry breaking. Now we want to find all possible n_i, m_i so as to satisfy (4) with $n_1 \neq 0$.

We want to restrict our attention to those solutions that lead to stable monopoles. Following [2,3,6], we consider scalar field masses such that the long range $SU(3) \times SU(2)$ interactions are much stronger than the $U_Y(1)$ interactions. We also assume mass parameters such that the $U_Y(1)$ interactions are much stronger than the $SU(5)^3$ interactions, so that the $SU(5)^3$ interactions play no role in the stability analysis of monopoles with $n_1 \neq 0$ and the results in [2,3,6] apply directly. Hence the monopoles with $n_1 = 5$ and $n_1 > 6$ are unstable to decay (similarly, for negative n_1). We can therefore restrict our attention to $n_1 = 1, \ldots, 6$.

Note that if we do find a solution, adding 3 to n_3, 2 to n_2, 30 to n_1, or 5 to m will also yield a solution. These correspond to adding closed paths that are trivial in the case of n_3 and n_2 and nontrivial in the case of n_1. In the case of adding 5 to m, the additional closed path may be trivial or nontrivial depending on how the 5 is split between the m_i. But, since the SU(5)'s will ultimately be broken, the monopoles corresponding to the nontrivial closed paths in the case of m will cluster in topologically trivial configurations. So we restrict our attention to

$$n_3 = 0, \pm 1, \qquad n_2 = 0, 1,$$
$$n_1 = 1, \ldots, 6, \quad \text{and} \quad m = 0, \pm 1, \pm 2.$$

Note that, at this stage, these monopoles have only one-fifth of the desired values of $U_Y(1)$ charge. The monopoles from the first stage of symmetry breaking are shown in Table II.

Consider the masses of the digits shown in Table II. The $n_1 = 1$ digit has $m = -1$ and so could be any one of $(m_1, m_2, m_3) = (-1, 0, 0)$, $(-1, -1, 1)$, $(-1, -2, 2)$ (or permutations thereof). Assuming that the monopole mass is proportional to its charge as in the Bogomolny-Prasad-Sommerfield (BPS) case, this tells us that the square of the masses goes like

$$M_1^2 \sim \mathrm{Tr}[(\Lambda_a^{(1)})^2],$$
$$M_2^2 \sim \mathrm{Tr}[(\Lambda_a^{(1)})^2 + (\Lambda_{a'}^{(2)})^2 + (\Lambda_{a''}^{(3)})^2] = 3M_1^2,$$
$$M_3^2 \sim \mathrm{Tr}[(\Lambda_a^{(1)})^2 + (\Lambda_{a'}^{(2)} + \Lambda_{b'}^{(2)})^2 + (\Lambda_{a''}^{(3)} + \Lambda_{b''}^{(3)})^2]$$
$$= 5M_1^2 + 4\mathrm{Tr}(\Lambda_a^{(i)}\Lambda_b^{(i)}).$$

While $\mathrm{Tr}(\Lambda_a^{(i)}\Lambda_b^{(i)}) < 0$, $(a \neq b)$, $|\mathrm{Tr}(\Lambda_a^{(i)}\Lambda_b^{(i)})| \leq \mathrm{Tr}[(\Lambda_a^{(i)})^2] = M_1^2$ [in SU(5), this inequality is $5 < 20$], so $M_2, M_3 > M_1$. Thus the lightest $n_1 = 1$ digit is indeed $(m_1, m_2, m_3) = (-1, 0, 0)$, $(0, -1, 0)$, or $(0, 0, -1)$. Equivalent calculations for higher charge monopoles show that the lightest digits have charge in a single H_i. For example, for $n_1 = 3$, $(m_1, m_2, m_3) = (-2, 0, 0)$ (up to the three permutations). Therefore the lightest digits come in three families with the family identified by the i for which m_i is nonzero.

In addition to the digits shown in Table II, there are sterile monopoles for which $n_i = 0$ but $m_i \neq 0$. These will form topologically trivial clusters once the SU(5)'s break. There are also pure monopoles for which $m_i = 0$ but $n_i \neq 0$. For these, $n_1 = 5$, 10, 15, 20, 25, or 30 only. We have already argued [below Eq. (4)] that all of these are unstable because the $n_1 = 5$ monopole is unstable and the others have $n_1 > 6$. The $n_1 = 5$ monopole is unstable to fragmentation into an $n_1 = 2$ and an $n_1 = 3$ monopole since these two monopoles interact mainly by the repulsive hypercharge interaction. Similarly, the $n_1 = n_* > 6$ are unstable to fragmentation into an $n_1 = 6$ and an $n_1 = n_* - 6$ monopole. The instability of $n_1 = 5$ monopoles in our model is crucial to the realization that it contains three families of monopoles.

TABLE II. n_i, m for the digits and digit stability.

n_1	n_2	n_3	m	Stable?
1	1	1	−1	yes
2	0	−1	−2	yes
3	1	0	2	yes
4	0	1	1	yes
5	1	−1	0	no
6	0	0	−1	yes

When the $SU_i(5)$ break to Z_5, the digits must bind into clusters with trivial $SU_i(5)$ charge, i.e., $SU_i(5)$ charge which is a multiple of 5. Note the clustering will multiply the $U_Y(1)$ charges of the monopoles by a factor of 5, bringing them to the desired values. In the cluster of $|m| = 1$ digits, the m's on each digit will all live in the same $SU_i(5)$ since they have to be confined by Z_5 strings belonging to the same $SU_i(5)$ factor. For the case $|m| = 2$, since the lightest digits have charges in a single $SU_i(5)$, we expect the stable $m = 2$ clusters to be composed of five $m = 2$ digits having charges in the same $SU_i(5)$. In general, a cluster with higher energy will decay into a cluster of lightest digits since these are related by differences of 5 sterile monopoles which are equivalent to the vacuum.

Now that we have three families of stable monopoles (clusters of digits) with the proper $U_Y(1)$ charges, we would next like to determine the monopole degeneracies within each family from SU(2) and SU(3) arrangements of the clusters.

Consider first the SU(2) arrangement of a cluster of five $n_1 = 1$ digits. These could take any one of the five forms: $(UUUUU)_i \equiv 5U_i$, $4U_i + D_i$, $3U_i + 2D_i$, $2U_i + 3D_i$, $U_i + 4D_i$, $5D_i$, where U_i is the $n_2 = +1$, $m_i \neq 0$, and D_i is the $n_2 = -1$, $m_i \neq 0$ digit. (We have suppressed the SU(3) labels for convenience.) However, while there is an attractive SU(2) force between both two U's and between a U and a D, the latter is stronger [3], and hence the lowest energy configurations will be $3U_i + 2D_i$ and $2U_i + 3D_i$. We identify the former as being dual to the u_L, c_L, and t_L quarks (for $i = 1, 2, 3$), the latter to the d_L, s_L, and b_L. Similarly, we could consider the SU(3) arrangement of the $3U_i + 2D_i$ cluster. Labeling the SU(3) charges by b, g and r, the most tightly bound cluster will have the color arrangements $2b + 2g + r = \bar{r}$, $2b + g + 2r = \bar{g}$, or $b + 2g + 2r = \bar{b}$. Hence we see that the $n_1 = 1$ cluster does have the desired degeneracy. The above arguments apply straightforwardly to the $n_1 = 2$, 3, 4, and 6 clusters, and it can be explicitly checked that they all have the desired degeneracies as indicated in Table I. The digits and stable clusters are tabulated in Table III.

We have now identified the fundamental fermions of the standard model and demonstrated how triplication occurs dynamically. Notice that there is no clear prediction of the existence or absence of right-handed neutrinos, since these are topologically trivial (at least in the 3-2-1 sector). However, there are certainly many potential candidates, namely, the clusters of sterile monopoles.

TABLE III. Arrangement of digits and clusters and correspondence to standard model fermions. (The generational and color indices are suppressed.)

n_1	Digit	Clusters	SM
1	$\binom{U}{D}$	$\binom{(3U+2D)}{(2U+3D)}$	$\binom{u_L}{d_L}$
2	$\overline{D}_L \equiv U + D$	$5\overline{D}_L$	$\overline{d}_L$
3	$\binom{\overline{E}_R}{\overline{N}_R} \equiv \binom{2U+D}{U+2D}$	$\binom{3\overline{E}_R+2\overline{N}_R}{2\overline{E}_R+3\overline{N}_R}$	$\binom{\overline{e}_R}{\overline{\nu}_{eR}}$
4	$U_R \equiv 2U + 2D$	$5U_R$	$\overline{u}_R$
6	$E_L^+ \equiv 3U + 3D$	$5E_L^+$	e_L^+

In [2,3] several issues not resolved in the earlier (or present) version of the dual standard model were pointed out. These had to do with the spin and chirality of monopoles. Conceivably, a resolution of these problems will indicate that the monopole spectrum we have found will have additional degeneracies. For example, there could be monopoles with the same internal charges but different spins. Such a degeneracy might account for the electroweak Higgs, since it has the same internal charges as the electron-neutrino doublet. The issue then would be to investigate why the monopole field dual to the electroweak Higgs acquires a vacuum expectation value. These issues are hard to address since they are nonperturbative, but we hope that they can be addressed within a lattice formulation and studied analytically in a supersymmetric context.

Some phenomenological issues arise in the dual standard model that we now address. The first issue is that the monopoles corresponding to the proton and to the positron have the same charges, and so topology does not forbid this transition: Proton decay should be possible. But baryon and lepton numbers are approximately conserved in the standard model, and so the proton decay rate in the dual standard model had better be suppressed. This suppression can only come from dynamical arguments. This is possible classically if there is an energy barrier that prevents three loosely clustered monopoles, which we would identify with a proton, from collapsing and forming a tightly bound monopole, which we would identify with a positron. Such a barrier is indeed present, as can be seen by constructing the interaction potential between an $n = 1$ and an $n = 2$ SU(5) monopole as done in [3,6]. It would be of interest to see if this barrier survives when going beyond the classical level calculation. Another issue is that of the rate of flavor changing processes. In the present model, a t quark monopole can convert to a u quark, but the process requires an intermediate state that is a pure monopole with the charges of a u quark. We know that the pure monopole has higher energy than both the t and u quarks. So the decay of the t monopole to the u monopole is a classically forbidden but quantum mechanically allowed process.

Within the philosophy of the dual standard model, it is interesting to note that SU(5) cannot be the ultimate symmetry of particle physics since it does not yield the three families of particles that we know to exist. If the model described in this paper is the only way to get three families, it tells us that the true symmetry group must be large enough to contain

$$[G_{\rm low} \times H_1 \times H_2 \times H_3]/[Z_5^{(1)} \times Z_5^{(2)} \times Z_5^{(3)}].$$

Another important prediction of the current model is that the digits, and not the quarks and leptons, are the fundamental building blocks of matter. Ultimately, we should see these preonic components in the laboratory.

The successful resolution of the family replication problem in the dual standard model offers a glimmer of hope that the spectrum of standard-model fermions can be understood in terms of the topology of certain manifolds. To us it seems that this is not unlike the classification of baryons and mesons in terms of group representations [7]. It is too early to say, however, if the present attempt will meet with the same degree of success.

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