

Pointlike D-brane Dynamics and Space-Time Uncertainty Relation

Miao Li*

Enrico Fermi Institute, University of Chicago, 5640 Ellis Avenue, Chicago, Illinois 60637

Tamiaki Yoneya†

Institute of Physics, University of Tokyo, Komaba, Meguro-ku, 153 Tokyo

(Received 12 November 1996)

We argue that the space-time uncertainty relation of the form $\Delta X \Delta T \geq \alpha'$ for the observability of the distances with respect to time, ΔT , and space, ΔX , is universally valid in string theory including D-branes. This relation has been previously proposed by one (T. Y.) of the present authors as a simple qualitative representation of the perturbative short-distance structure of fundamental string theory. We show that the relation, combined with the usual quantum mechanical uncertainty principle, explains the key qualitative features of D-particle dynamics. [S0031-9007(97)02389-2]

PACS numbers: 11.25.Mj, 04.60.-m

It is often stated that in the fundamental string theory there exists a minimum length of order of $\sqrt{\alpha'} \equiv \ell_s$ beyond which we cannot probe the structure of space-time. This comes about from the properties of string amplitudes in the high-energy limit [1,2] and also in the high-temperature limit [3]. Such a statement is indeed quite natural when we have only the ordinary string states as possible probes for short distances, since string states themselves have an intrinsic extension of the order of length ℓ_s .

Recently, however, we found that string theory, in fact, allows a variety of objects of various dimensions as solitonic excitations and that they are bound to play crucial roles in nonperturbative formulations of string theory. In particular, we have even pointlike objects called D0-branes [4] or D-particles. Recent studies [5–12] of D-particle dynamics revealed the possibility of probing the distance scales of eleven-dimensional (11D) Planck scale of the order $g_s^{1/3} \ell_s$, the natural scale of the M-theory [13], which is indeed much shorter than the string scale ℓ_s for weak string coupling (the importance of shorter length scales in string theory has been suggested earlier in [14]). If we remember that the string scale represents the unique fundamental constant of Nature in the natural unit $c = \hbar = 1$, its precise significance must certainly be clarified. The purpose of the present paper is to remark that a simple space-time uncertainty relation [15] proposed in 1987 by one of the present authors is an appropriate interpretation for the meaning of the string scale ℓ_s , since it is universally valid for both ordinary string scattering and D-particle dynamics.

Let us begin by briefly recalling the arguments of Ref. [15] which motivate the space-time uncertainty relation. Consider a high-energy scattering of arbitrary objects whose interactions are mediated by strings. If the energy scale is of order E , the smallest time scale probed by this scattering event is of order $\Delta T \sim \frac{1}{E}$. Now, what is the typical spatial length scale probed by this scattering event? If both the scattering objects and their interactions

were described by the usual local field theories neglecting quantum gravity, we would be allowed to state that it is determined by the typical wavelength of the objects, namely, $\frac{1}{E}$ for sufficiently high energies. Hence, in principle, we would have no limitation for probing the short-distance scale, provided we neglect quantum gravity. If, on the other hand, the interactions are mediated by fundamental strings, high energies do not necessarily imply that the typical spatial scale is given by the wavelength of the scattering objects, since higher energies dominantly cause larger fluctuations with respect to string excitations during interactions than with respect to the center of mass motion because of the huge degeneracy of string excitation modes. It is easy to see [15,16] that the typical (smeared-out) spatial extension ΔX of strings with energy E is of order $\Delta X \sim \ell_s^2 E$. This implies the simple relation for the indeterminacies of the space and time lengths,

$$\Delta X \Delta T \geq \ell_s^2, \quad (1)$$

which we call the space-time uncertainty relation. In Ref. [15], this relation was proposed as a natural space-time representation of the *st*-duality properties of string scattering amplitudes. As discussed later in Ref. [16], it can also be derived as a direct consequence of the world-sheet conformal invariance.

From the viewpoint of the space-time uncertainty relation, the usual argument [17] for the minimal length essentially amounts to assuming that the observable length is the average of the spatial and time distances; we would then have the lower bound $\frac{\Delta X + \Delta T}{2} \geq \ell_s$. Clearly, however, what is the dominant scale measured by scattering experiments depends on which kinematical regions we are interested in. For example, a high-energy low-momentum transfer (peripheral) scattering experiment can probe the small time scale, but the spatial scale is not necessarily small. Thus, it corresponds to $\Delta T \rightarrow 0$, and our relation (1) implies that the spatial length scale grows as $\frac{\ell_s^2}{\Delta T} \sim \ell_s^2 E$ as the laboratory energy E increases. By adapting the string-bit argument due to Susskind [18],

this longitudinal length scale of a string, growing linearly with energy, leads to the Regge intercept $\alpha(0) = 2$ of the probability amplitude $A(E) \sim E^{\alpha(t)-1}$, in conformity with the existence of a graviton. [Here, t is the invariant momentum transfer whose dependence is determined basically by the effective transverse size of the string.] On the other hand, a high-energy fixed-angle scattering as studied in [2] tries to probe short distances with respect to both space and time. In this case, since relation (1) shows there is no such degrees of freedom, the amplitude damps exponentially in the high-energy limit.

As long as we only use the strings as probes, it seems difficult to imagine a scattering experiment which makes it possible to measure directly the region $\Delta X \rightarrow 0$, because of the intrinsic extension of the string. In Refs. [15,16], it was suggested that we interpret relation (1) in the limit $\Delta X \rightarrow 0$ as an explanation as to why it is possible to treat the asymptotic string states, propagating an infinitely long time $\Delta T \rightarrow \infty$, as *local* external fields in the sigma-model approach to world-sheet string theory. The asymptotic states correspond to the s -channel poles. Combined with the Regge-pole exchange picture for the limit $\Delta T \sim \frac{1}{\Delta X} \rightarrow 0$, relation (1) is thus interpreted as a natural space-time interpretation of [Regge-pole] \leftrightarrow [resonance-pole] duality. Now it is clear that the D0-branes are ideal objects for the purpose of directly probing the short spatial length scale and testing relation (1) beyond such formal arguments. Fortunately, there already appeared several works cited above which studied the dynamics of D0-branes in the low-energy limit. In the following, we show that all the results so far are consistent with relation (1), and the most crucial feature behind these results can be naturally understood on the basis of (1).

First of all, we note that the slow velocity limit studied in these works is appropriate just for probing the small ΔX regions where (1) implies that ΔT grows. Moreover, it seems fairly clear that the space-time uncertainty relation conforms just to the basic principles emphasized in Ref. [10] that the leading singular behavior in the short spatial distance limit is determined by the infrared behavior of brane world-volume quantum theory. In fact, this statement is a direct consequence of the duality between s and t channels, which was nothing but the original interpretation of relation (1) as discussed above.

Now, let us consider the scattering of two heavy D-particles of mass $m = \frac{1}{g_s \ell_s}$ with slow typical velocity v . If we assume that there is a limitation for the meaningful space-time lengths in the form (1), what is the smallest possible spatial length scale b probed by this scattering? Let

$$b \sim v^\eta \ell_s.$$

The typical time length of the scattering is $t = \frac{b}{v}$. Substituting these relations to (1) with $\Delta X \sim b$, $\Delta T \sim t$, we must have $\eta = 1/2$,

$$b \sim \sqrt{v} \ell_s. \quad (2)$$

This length scale first appeared in [6], where an annulus amplitude of open strings is computed, and was further

analyzed in detail in [10] using the effective world line theories [19] ($0 + 1$ -dimensional super Yang-Mills matrix quantum mechanics) of D-particles. Relation (2) constitutes one of the most crucial relations in all previous discussions of short-distance structure in D-particle dynamics. Following [10], we often call the scale $\sqrt{v} \ell_s$ the stadium size. From the point of view of the effective super Yang-Mills matrix model, the stadium size is the limit for the spatial scale where the Born-Oppenheimer approximation for the coupling between the D-particle coordinates and the short open-string excitation connecting them ceases to be valid. The dynamics of the latter, corresponding to the off-diagonal part of the adjoint Higgs fields originating from the 10D gauge fields by dimensional reduction, is governed by the time scale $\mathcal{O}(\ell_s^2/b)$ while that of the former, the diagonal part of the Higgs, is by $\mathcal{O}(b/v)$. If the Born-Oppenheimer approximation is not valid, we cannot clearly separate the diagonal part as the spatial coordinates of D-particles, and hence we must have (2). Our proposal is thus to interpret this result as a consequence of the universal space-time uncertainty relation (1).

Now, once relation (2) is known, the important fact that the characteristic spatial length scale of D-particle dynamics is nothing but the eleven-dimensional Planck scale associated with M-theory is understood from the usual uncertainty relation. If the time duration of the scattering is of order $\Delta T \sim \frac{b}{v} \sim v^{-1/2} \ell_s$, the usual time-energy uncertainty relation applied for a pointlike D-particle implies an uncertainty with respect to the D-particle velocity of order $\Delta v \sim g_s v^{-1/2}$ which leads to the spread of the wave packet of order $g_s v^{-1} \ell_s$ during the time interval $v^{-1/2} \ell_s$. Here, we used the fact that the kinetic energy of a D-particle is $\frac{1}{2g_s \ell_s} v^2$ for weak string coupling. [For the present order estimate, we can neglect the potential energy of order $\mathcal{O}(\frac{v^4}{(\Delta X)^7}) \ell_s^6$ which is smaller than the kinetic energy when $\Delta X \gtrsim b$.] In order that the minimum length scale $b \sim \sqrt{v} \ell_s$ be meaningful, b must be larger than this spread. Thus we have the lower bound for the velocity [10]

$$v \gtrsim g_s^{2/3}$$

which leads to the 11D Planck scale $b \sim g_s^{1/3} \ell_s$ of the M-theory as a meaningful smallest distance probed by D-particle scattering at low velocities.

From the viewpoint of the effective super Yang-Mills matrix model, the 11D Planck scale is easily understood from a scaling argument [8,9] which says that coupling constant g_s can be eliminated from the dynamics by making a rescaling, $X_i \rightarrow g_s^{1/3} X_i$ and $t \rightarrow g_s^{-1/3} t$, for the D-particle coordinates X_i and time t , respectively. We here emphasize a trivial but crucial fact that the *opposite* scalings for the space and time coordinates just conform to a necessary requirement for the validity of relation (1).

Actually, as pointed out in [10], when we consider a D-particle in the presence of a large number of D4-branes, it becomes possible to probe arbitrary short spatial

distance scale. This is basically due to the fact that the D4-branes produce an effective metric for the moduli space of a D0-particle which makes the effective mass of the D0-particle much heavier than that in the flat space at short distances: The effective action in the presence of N coincident parallel D4-branes is given by

$$S_{\text{eff}} = \int dt \left[\frac{1}{2g_s \ell_s} \left(1 + \frac{Ng_s \ell_s^3}{r^3} \right) v^2 + \mathcal{O}(Nv^4 \ell_s^6 / r^7) \right],$$

where r is the distance between the D-particle and D4-branes. According to [10], this metric is likely to be exact without α' and instanton corrections. When the distance r is much shorter than $(Ng_s)^{1/3} \ell_s$, the mass of the D-particle is effectively given by $m \sim N \ell_s^2 / r^3 \gg \frac{1}{g_s \ell_s}$. We can then easily check that the spread of the D-particle wave packet can be neglected during the time $t \sim v^{-1/2} \ell_s$ compared with the stadium size $\sqrt{v} \ell_s$ for large N (if r itself is the stadium size $\sqrt{v} \ell_s$). This allows us to probe arbitrary short lengths with respect to the distance between the D-particle and D4-branes and hence indicates that the singular spatial metric in the D0-particle moduli space is meaningful even in the limit $r \rightarrow 0$. On the other hand, since the time scale grows indefinitely, we cannot talk about the interaction time in any meaningful way in the limit of short spatial distance.

The space-time uncertainty relation (1), in general, says that, to probe the short spatial distances, an inversely large time interval is necessarily required and vice versa. Thus, if it is universally valid, we would not be able to introduce the concept of the space-time event which is local with respect to both space and time.

Although we expect that the space-time locality is lost eventually in any theory including quantum gravity, relation (1) suggests a specific manner on how this happens in fundamental string theories. It is very important to see whether this way of expressing the significance of the string constant ℓ_s is useful in the dynamics of more general branes and strings, including D-instantons. Since the interaction between general D-branes is governed by the fundamental strings and relation (1) originates from the conformal invariance of the fundamental string dynamics, it is reasonable to expect its general validity, if we interpret the relation appropriately. Here, the case of D-instantons (D-1-branes) is very special since we cannot talk about time evolution in their dynamics. It is known [20] that the invariant scattering amplitude of massless string states off a fixed D-instanton for arbitrary energy is simply given by $\frac{1}{t}$ apart from the kinematical factor where t is the invariant t -channel energy, provided we supply the appropriate fermion contribution to cancel possible fermion zero modes. The pole $t = 0$ represents the massless dilaton exchange contribution. Namely, the amplitude for weak string coupling is reproduced by a local field theory without any α' correction for any energy. This is *not* a contradiction to our space-time uncertainty relation, since the above behavior can be interpreted to correspond to the special case where $\Delta T \sim \frac{\ell_s^2}{\Delta X} \sim 0$: $\Delta T \sim 0$ reflects the

pointlike nature of the D-instanton in space-time, while $\Delta X \rightarrow \infty$ is associated with the long-range propagation of a virtual dilaton exchange. Indeed, after the integration over the position of a single D-instanton, we would have $t = 0$. From the viewpoint of (1), the appearance of the long-range exchange of massless dilaton is a necessary condition for the existence of the pointlike instanton ($\Delta T \sim 0$) contribution. Therefore, the apparent loss of the stringy property of the D-instanton-string interaction may be regarded as yet another piece of evidence for the universal nature of the space-time uncertainty relation. Of course, this argument is restricted to one instanton case. The multi-instanton dynamics is much more complicated due to stringy interactions and the integration over the collective coordinates. Further investigations on general D-brane dynamics including instantons from our viewpoint will be useful for clarification towards more precise and general interpretation of the space-time uncertainty relation.

Our discussions so far assumed weak string coupling. However, given that, except for type IIA theory and heterotic $E_8 \times E_8$ theory, the S-duals of string theories are again string theories, it is natural to expect that the relation is valid even at strong string coupling. Note that the string constant ℓ_s can be regarded as invariant under the S-dual transformations, and therefore the string tension with respect to the correctly rescaled space-time coordinates remains the same as well. In the case of type IIA (which we are mainly concerned with in the present paper) and heterotic $E_8 \times E_8$, the dual is the M-theory, whose strong (string) coupling limit is believed to be described by 11D supergravity in the long-distance limit. If the recent interesting conjecture [21] that the microscopic M-theory is described *exactly* by the $0 + 1$ D Yang-Mills matrix model in the infinite-momentum frame is correct, it is plausible that our uncertainty relation continues to be valid even for strong string coupling, since the time scale in the Galilean dynamics of the off-diagonal elements is then always given by the difference of the diagonal elements $\frac{\ell_s}{|x_i - x_j|}$ for the D-particle coordinates x_i, x_j , which is the basis for relation (1) from the viewpoint of the super Yang-Mills matrix model as a world line theory of D-particles.

Another relevant question towards a possible generalization of the space-time uncertainty relation would be whether we can have a similar relation for spatial domains without including the time length. As a simple example, let us consider a measurement of the position of a D-particle inside D4-branes by scattering with an external D-particle. Classically, a bound state of a D-particle with N coincident D4-branes can be described by an instanton solution of the D4-brane gauge field [22]. Probing the D-particle inside these D4-branes is equivalent to probing the localization of the background field A . The localization of A is disturbed by the massless D4-brane open string modes as the decay product of pairs of open strings connecting the D-particle probe and D4-branes. We note that D4-brane

open string modes are the inevitable product if the impact parameter is sufficiently small. The dissipation rate is calculated in [6] for D-particle–D-particle scattering, and in [10] for D-particle–D4-brane scattering. When the impact parameter is of order ΔX , the space-time uncertainty relation shows that the typical energy transferred to the massless open string modes of the D4-brane is of order $\Delta E \sim \Delta T^{-1} \sim \frac{\Delta X}{\ell_s^2}$. If this energy is used for the interaction with the D-particle inside the D4-brane to probe its position, it would contribute to an uncertainty of the velocity of the D-particle inside, of order $\Delta v = \frac{g_s \ell_s}{v \Delta T}$, which implies uncertainty of the position of order $\frac{g_s \ell_s}{v}$ during the interaction time ΔT . On the other hand, the D-particle inside the D4-brane travels the distance of the order $v \Delta T$. Thus the net uncertainty of the position of the D-particle inside the D4-brane is at least of order $\Delta Y \sim (\frac{g_s \ell_s}{v} + v \Delta T)/2 \geq \sqrt{\Delta T \ell_s g_s}$. [This argument can be made more accurate by using the distribution $\int p^3 dp$ for a localized wave function, the result will remain the same.] Combined with (1), this suggests the validity of a strange relation of the following type: $\Delta X \Delta Y^2 \geq g_s \ell_s^3$. When the D-particle–4-brane impact parameter ΔX is very small, a large lump of massless 4-brane modes is produced, and the D-particle inside the 4-branes becomes difficult to locate, $\Delta Y \geq (\frac{g_s \ell_s^3}{\Delta X})^{1/2}$. The 11D Planck scale appeared here again, but the argument is apparently independent of the discussion for the minimum stadium size for the projectile D-particle. We do not know whether this new relation has a universal meaning beyond the situation discussed here. The origin of this relation might be related to the holographic principle [23]. We should note here that, depending on various situations such as the dimensionalities of D-branes and the directions of the spatial distances we are interested in, the characteristic spatial scale can vary from case to case. This is easily seen from the scaling arguments of the supersymmetric Yang-Mills (SYM) matrix models. We want to emphasize that, in contrast to this, relation (1) for the transverse distances between D-branes and the time scale is universal since the transverse distances and the time always have opposite scaling behavior.

From a formal point of view, what is lacking in our arguments is a mathematical foundation for the appearance of the uncertainties. For example, we cannot at present give any precise quantitative definitions for the space and time uncertainties. We feel that there should be a formulation of the theory which surpasses strings and D-branes. In this respect, a very urgent and challenging problem seems to construct a completely covariant formulation of interacting D-particle dynamics as an extension of the SYM models towards its quantum-geometrical reformulation. Covariance would require us to treat even the time variables as noncommutative objects and also to include all brane-antibrane dynamics

automatically. It would hopefully lay the mathematical foundation for the space-time uncertainty relation.

The present work grew out of discussions between us, begun when T. Y. was visiting Brown University under the U.S.-Japan Collaborative Program for Scientific Research supported by the Japan Society for the Promotion of Science. T. Y. would like to thank Professor A. Jevicki for warm hospitality and discussions during his stay. We would like to thank M. Douglas and E. Martinec for reading the manuscript. The work of M. L. was supported by DOE Grant No. DE-FG02-90ER-40560 and NSF Grant No. PHY 91-23780.

*Electronic address: mli@curie.uchicago.edu

†Electronic address: tam@hep1.c.u-tokyo.ac.jp

- [1] D. Amati, M. Ciafaloni, and G. Veneziano, *Phys. Lett. B* **197**, 81 (1987); see also G. Veneziano, *Europhys. Lett.* **2**, 199 (1986).
- [2] D. Gross and P. Mende, *Nucl. Phys.* **B303**, 407 (1988).
- [3] J. J. Atick and E. Witten, *Nucl. Phys.* **B310**, 291 (1988).
- [4] J. Polchinski, *Phys. Rev. Lett.* **75**, 4724 (1995).
- [5] J. L. F. Barbón, Report No. hep-th/9601098 (to be published).
- [6] C. Bachas, *Phys. Lett. B* **374**, 37 (1996).
- [7] G. Lifschytz, Report No. hep-th/9604156 (to be published).
- [8] U. H. Danielsson, G. Feretti, and B. Sundborg, Report No. hep-th/9603081 (to be published).
- [9] D. Kabat and P. Pouliot, Report No. hep-th/9603127 (to be published).
- [10] M. R. Douglas, D. Kabat, P. Pouliot, and S. H. Shenker, Report No. hep-th/9608042 (to be published).
- [11] U. H. Danielsson and G. Ferretti, Report No. hep-th/96010082 (to be published).
- [12] G. Lifschytz, Report No. hep-th/96010125 (to be published).
- [13] E. Witten, *Nucl. Phys.* **B443**, 85 (1995).
- [14] S. H. Shenker, Report No. hep-th/9509132 (to be published).
- [15] T. Yoneya, in *Wandering in the Fields*, edited by K. Kawarabayashi and A. Ukawa (World Scientific, Singapore, 1987), p. 419; see also *Quantum String Theory*, edited by N. Kawamoto and T. Kugo (Springer, New York, 1988), p. 23.
- [16] T. Yoneya, *Mod. Phys. Lett.* **A4**, 1587 (1989).
- [17] For a list of references related to this subject, see, e.g., L. J. Garay, *Int. J. Mod. Phys. A* **10**, 145 (1995).
- [18] L. Susskind, *Phys. Rev. D* **49**, 6606 (1994).
- [19] E. Witten, *Nucl. Phys.* **B460**, 335 (1996).
- [20] I. Klebanov and L. Thorlacius, *Phys. Lett. B* **371**, 51 (1996); S. S. Gruber, A. Hashimoto, I. R. Klebanov, and J. M. Maldacena, Report No. hep-th/9601057 (to be published).
- [21] T. Banks, W. Fischler, S. H. Shenker, and L. Susskind, Report No. hep-th/96010043 (to be published).
- [22] M. R. Douglas, Report No. hep-th/9512077 (to be published).
- [23] G. 't Hooft, Report No. gr-qc/9310026 (to be published); L. Susskind, *J. Math. Phys.* **36**, 6377 (1995).