Meson Masses in Nuclear Matter

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Mass shifts Δm of particles in nuclear matter relative to their vacuum values are considered. A general formula relating $\Delta m(E)$ (E is the particle energy) to the real part of the forward particle-nucleon scattering amplitude $\mathrm{Re}\, f(E)$ is presented and its applicability domain is formulated. The ρ-meson mass shift in nuclear matter is calculated at $2 \lesssim E_\rho \lesssim 7$ GeV for transversally and longitudinally polarized ρ mesons. [S0031-9007(97)02366-1]

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The problem of how the properties of mesons and baryons change in nuclear matter in comparison to their free values has attracted a lot of attention recently. Among these properties the first of interest are mass shifts of particles in nuclear matter. The values of particle masses can be measured experimentally, and some data started to appear. In this aspect experiments on heavy ion collisions, in which the dependence of particle masses on nuclear density can be found, are very promising.

In early theoretical investigations of this problem [1,2], one or another model of strong interaction of particles in nuclear matter was used. In the pioneering work by Drukarev and Levin [3] the use of QCD sum rules for the calculation of nucleon mass shift in nuclear matter was suggested. Later this method was applied also to the calculation of meson masses (for recent reviews see [4]). Among the latter the most interesting is the case of light vector mesons. Theoretically clean measurements of vector meson masses in nuclei would be possible in electroproduction experiments, where mesons are created uniformly inside the nucleous, and not predominantly at its surface as in hadroproduction or photoproduction. The masses, energies, and widths of vector mesons can be obtained by measuring momenta of e^+ and e^- in $V \to e^+e^-$ decay.

The masses of vector mesons in nuclear matter were calculated in [2,5–15]. (In Ref. [7] a universal ratio of particle masses in nuclear matter to their vacuum values was suggested.) However, the results do not coincide and are rather model dependent as emphasized, e.g., in Ref. [11]. Since the interaction of ρ meson with nucleons in medium is energy dependent, one may also expect that the mass shift is energy dependent. This problem was considered only for a rather narrow energy interval in the Walecka model [12,13]. In model independent QCD sum rule calculations [8,14,15] only mass shifts of vector mesons at rest were studied. Here we present a calculation of meson mass shifts in nuclear matter for a wide range of meson energies not considered before. In the case of pions the results are completely model independent and general. In the case of vector mesons the only hypothesis used is the vector dominance model (VMD), which for the energies we consider is confirmed by experiment with the accuracy quoted below. Transverse and longitudinal vector mesons are treated separately. We believe that the possibility to compare data with the theory in a wide energy interval will essentially extend the field of experimental investigations.

We start with general considerations applicable to any particle imbedded in nuclear matter. Assume that the interaction of the particle with a nucleon in matter is not affected by other nucleons, i.e., the nuclear matter can be considered as an inhomogeneous macroscopic medium. This restricts the particle wavelength: $\lambda = k^{-1} \ll d$, where d is the mean internucleon distance. This means that the particle momentum k must be larger than a few hundred MeV. Since we assume that the particle is created inside the nucleus, we must require that its formation length $l_{\mathrm{form}} \sim (E/m)/m_{\mathrm{char}}$ is less than the nucleus radius R, where $m_{\mathrm{char}} \sim m_\rho$ is the characteristic strong interaction scale. This implies an upper limit on the particle energy, $E/m < 15$, for middle weight nuclei. An additional restriction on the upper value of the particle momentum k arises from the requirement that for the mass shift to be observable the particle must decay mainly inside the nucleous, $k/\Gamma m < R$. This gives $k_\rho < 6$ GeV for the vacuum value of Γ_ρ. For ω and ϕ the restrictions are less certain, since their in-medium widths may be very different from the vacuum ones.

To calculate particle mass shifts in nuclear matter we use the general method suggested long ago for treatment of propagation of fast neutrons in nuclei [16] (see also [17]). The main idea is that for $\lambda \ll d \ll R$ the effect of medium on the particle propagation can be described by attenuation and refraction indices. Attenuation of particles moving in the direction of z axis at a distance z is equal to $\exp(-\rho\sigma z)$, where $\rho = A/V$ is the nuclear density, A is the atomic number, V is the nucleus volume, and σ is the total cross section of the interaction of the particle with nucleons. [Strictly speaking, $\rho\sigma = (Z\sigma_p + N\sigma_n)/V$.]

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Using the optical theorem

$$k\sigma = 4\pi \,\mathrm{Im}\, f(E)\,, \tag{1}$$

where $f(E)$ is the forward scattering amplitude, we have for the modulus of the particle wave function in matter

$$|\psi| \sim \exp\left[-\rho\, \frac{2\pi z}{k}\, \mathrm{Im}\, f(E)\right]. \tag{2}$$

This formula is evidently generalized to the wave function itself

$$\psi \sim \exp\left[i\rho\, \frac{2\pi z}{k}\, f(E)\right]. \tag{3}$$

Equation (3) is correct if $|\mathrm{Re}\, f| < d = (V/A)^{1/3}$: only in this case the scattering on each nucleon can be considered as independent and interference effects can be neglected [17]. $\mathrm{Re}\, f(E)$ is related to the refraction index of matter for particle propagation [16]. We want to describe the propagation of a particle through nuclear matter introducing an effective mass $m_{\mathrm{eff}} = m + \Delta m$. This means that (leaving absorption aside) $\psi \sim e^{ik_{\mathrm{eff}} z}$, with $k_{\mathrm{eff}} = \sqrt{E^2 - m_{\mathrm{eff}}^2} \approx k - (m/k)\Delta m$. Then from Eq. (3) we get

$$\Delta m(E) = -2\pi\, \frac{\rho}{m}\, \mathrm{Re}\, f(E)\,. \tag{4}$$

The expression in Eq. (4) for Δm has the meaning of an effective potential acting on the particle in medium [16,17]. For the correction to the particle width we have in a similar way

$$\Delta\Gamma(E) = \frac{\rho}{m}\, k\sigma(E)\,. \tag{5}$$

All of the above statements are general and can be applied to any particle in nuclear matter.

Let us now turn to the case of vector mesons. Interactions of ρ and ω with isospin symmetric nuclear matter are identical. Then, according to Eq. (4), $\delta m_\rho = \delta m_\omega$.

In order to find ρN forward scattering amplitude we use the VDM and the relation which follows from VDM (see, e.g., [18])

$$f_{\gamma N} = 4\pi\alpha\left(\frac{1}{g_\rho^2}\, f_{\rho N} + \frac{1}{g_\omega^2}\, f_{\omega N} + \frac{1}{g_\phi^2}\, f_{\phi N}\right). \tag{6}$$

The last term on the right hand side (rhs) of Eq. (6) can be safely neglected: as follows from ϕ-photoproduction data, it is small. Basing on the quark model, assume $f_{\omega N} \approx f_{\rho N}$. (This assumption is supported by ρ and ω photoproduction data, particularly on deuterium [18,19].) Since $g_\omega^2/g_\rho^2 \approx 8$, the contribution of ω to the rhs of Eq. (6) is also small. Therefore, according to Eq. (6), $\mathrm{Re}\, f_{\rho N}(E)$ is expressed through $\mathrm{Re}\, f_{\gamma N}(E)$. The latter can be found from the photoproduction data using the dispersion relation with one subtraction,

$$\mathrm{Re}\, f_{\gamma N}(E) = f_{\gamma N}(0) + \frac{E^2}{(2\pi)^2}\, P \int_{E_{\mathrm{th}}}^{\infty} dE'\, \frac{\sigma_{\gamma N}(E')}{E'^2 - E^2}\,, \tag{7}$$

where P denotes principle value, $\sigma_{\gamma N}(E)$ is the total photoproduction cross section, $E_{\mathrm{th}} = \mu + \mu^2/2m_N$, μ and m_N are the pion and nucleon masses, and $f_{\gamma N}(0)$ is given by the Thompson formula, $f_{\gamma p}(0) = -\alpha/m_p$, $f_{\gamma n} = 0$.

The VDM relation Eq. (6) holds only for the amplitude of a transverse vector meson $f_{\rho N}^T$, since $f_{\gamma N}$ is the scattering amplitude of a real transverse photon. In Eq. (6) the ρ-meson energy E_ρ is related to the photon energy by the requirement that the masses of hadronic states produced in ρN and γN scattering should be equal, $E_\rho = E_\gamma - m_\rho^2/2m_N$.

It is known that VDM works well starting from γ energies about 2 GeV, where one may expect the VDM accuracy of about 30% and better at higher energies (see, e.g., [18]). At these energies the nucleon Fermi motion can be neglected. In calculation of $\mathrm{Re}\, f_{\gamma N}(E)$, according to Eq. (7), we used the PDG data [20] on photoproduction on deuteron. For the high-energy tail the Donnachie-Landshoff fitting formula [21] for $\sigma_{\gamma p}$ was used, and it was assumed that $\sigma_{\gamma D}/\sigma_{\gamma p} =$ const starting from $E_\gamma = 20$ GeV. The results for $Re\, f_{\rho N}^T$ and Δm_ρ^T at normal nuclear density $\rho = (4\pi r_0^3/3)^{-1}$, $r_0 = 1.25$ fm, are shown in Fig. 1 as functions of E_ρ. The mass shift in the energy region, where our consideration is valid, 2 GeV $\lesssim E_\rho \lesssim$ 7 GeV, is positive (ρ mass increases in nuclear matter) and is of order of 50 MeV. However, the condition $|\mathrm{Re}\, f| < d \sim 2$ fm is not well fulfilled. Probably the main effect of interference of different nucleons is screening and the true values of Δm_ρ are a bit smaller than our results.

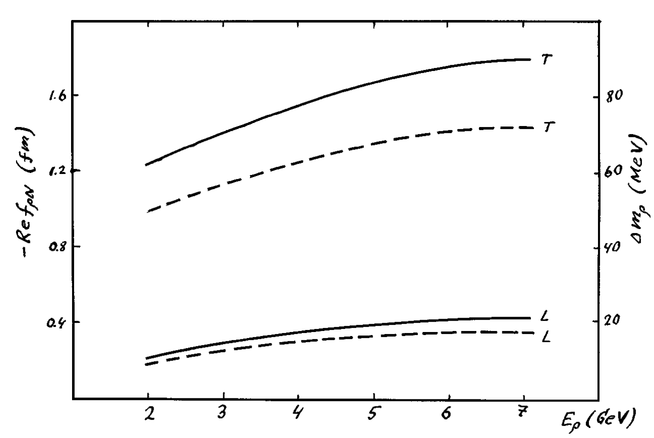

FIG. 1. Energy dependence of $-\mathrm{Re}\, f_{\rho N}^T$ and $-\mathrm{Re}\, f_{\rho N}^L$ (upper and lower solid curves, left scale) and of Δm_ρ^T and Δm_ρ^L (upper and lower dashed curves, right scale) at normal nuclear density.

In the case of longitudinal ρ mesons it is impossible to relate the forward scattering amplitude of the ρ to that of the real photon, but it is still possible to have such a relation for the virtual photon. We assume that VDM holds for virtual photons with virtualities less or of order of m_ρ^2. For the transverse scattering amplitude the generalization of Eq. (6) to the virtual photon is

$$f_{\gamma N}^T(E_\gamma, q^2) = 4\pi\alpha \sum_{V=\rho,\omega,\phi} \frac{m_V^4}{(q^2 - m_V^2)^2} \frac{1}{g_V^2} f_{VN}^T(E_V). \tag{8}$$

For the longitudinal scattering amplitude the generalization of VDM has the form

$$f_{\gamma N}^L(E_\gamma, q^2) = 4\pi\alpha \sum_{V=\rho,\omega,\phi} \frac{|q^2| m_V^4}{(q^2 - m_V^2)^2} \frac{1}{g_V^2} f_{VN}^L(E_V). \tag{9}$$

Equations (8) and (9) can be proved in models incorporating direct γN interaction. These equations correspond to the assumption that at $Q^2 = -q^2 \lesssim m_V^2$ the dominant intermediate states in the γ channel are vector mesons and the contributions of higher states can be neglected. The factor q^2 in the numerator of Eq. (9) is a kinematical factor that evidently follows from the requirement of vanishing $f_{\gamma N}^L$ at $q^2 = 0$. The absolute value $|q^2|$ arises, since $\mathrm{Im}\, f_{\gamma N}^L$ is positive at $q^2 < 0$ as well as at $q^2 > 0$. This corresponds to the fact that while for a transverse photon the polarization vector squared is $e^2 = -1$, for a longitudinal virtual photon we put $e^2 = 1$ in order to get a positive cross section (see [18]). The relation between E_ρ and E_γ is now $E_\rho = E_\gamma - (m_\rho^2 + Q^2)/2m_N$.

$\mathrm{Re}\, f_{\gamma N}^{T,L}(E, Q^2)$ can be found from the data on deep inelastic scattering in the same way as was done for the real photon. The dispersion relation takes the form

$$\mathrm{Re}\, f_{\gamma N}^{T,L}(E, Q^2) = f_{\gamma N}^{T,L}(0, Q^2) - \frac{\alpha}{m_N} P \times \int_0^1 dx' \frac{1 + 4m_N^2 x'^2/Q^2}{x'^2 - x^2} \times F_2(x', Q^2) \frac{(1, R)}{1 + R}, \tag{10}$$

where $x = Q^2/2\nu$, $\nu = m_N E$, $F_2(x, Q^2)$ is the nucleon structure function, and $R = \sigma_L/\sigma_T$ is the ratio of longitudinal to transverse photon cross sections.

Consider first the case of transverse photons and check whether starting from the deep inelastic scattering data we can get the values of $\mathrm{Re}\, f_{\rho N}^T(E)$ close to those we have already found from photoproduction. We choose $Q^2 = 0.5\ \mathrm{GeV}^2$ and take $F_2^p(x, 0.5\ \mathrm{GeV}^2)$ from the data compilation done by Ji and Unrau [22]. The ratio F_2^n/F_2^p was taken from [23] for $x < 0.2$. For $x > 0.2$, where the data at small Q^2 are absent, we assume $F_2^n/F_2^p = 0.75$. The information about R at small Q^2 is scarce. Based on the data from Refs. [23,24] we assume $R_p = R_n = 0.3$. We also assume that at $Q^2 = 0.5\ \mathrm{GeV}^2$ the subtraction term in Eq. (10) is given by the one-nucleon intermediate state, as it takes place in the Thompson formula. The one-nucleon intermediate state contributes also to the integral in Eq. (10). Its total contribution to Eq. (10) is

$$\mathrm{Re}\, f_{\gamma N}^T(\nu, Q^2)_{\text{one-nucl}} = -\frac{\alpha}{m_N}\left[F_E^2(Q^2) + \frac{1}{4} Q^4 G_M^2(Q^2) \times \frac{1}{\nu^2 - Q^4/4}\right], \tag{11}$$

where F_E and G_M are the nucleon electric Pauli and magnetic Sachs form factors. The results of our calculation show that the shape of the curve for $\mathrm{Re}\, f_{\rho N}^T(E_\rho)$ obtained from the data at $Q^2 = 0.5\ \mathrm{GeV}^2$ is similar to the curve $\mathrm{Re}\, f_{\rho N}^T(E_\rho)$ in Fig. 1, but the absolute values are (30–40)% smaller. Since the factor $(Q^2 + m_\rho^2)^2/m_\rho^4 \approx 3.4$ connecting the values of $f_{\gamma N}^T(E_\gamma, Q^2)$ and $f_{\rho N}^T(E_\rho)$ is rather large, this fact can be considered as an indication that the accuracy of VDM for the problem considered is of order (30–40)%.

The calculation of $\mathrm{Re}\, f_{\gamma N}^L(E, Q^2)$ is similar. The only difference appears in the subtraction term in Eq. (10). In [25] it was proved that $f_{\gamma N}^L(0, Q^2)$ at small Q^2 is given by the one-nucleon intermediate state, and it was argued that its contribution dominates up to $Q^2 = 0.5\ \mathrm{GeV}^2$. The contribution of one-nucleon intermediate state to $f_{\gamma N}^L(\nu, Q^2)$ is

$$\mathrm{Re}\, f_{\gamma N}^L(\nu, Q^2)_{\text{one-nucl}} = -\alpha m_N Q^2 \left[\frac{1}{4m_N^4} F_M^2(Q^2) + \frac{1}{\nu^2 - Q^4/4} G_E^2(Q^2)\right], \tag{12}$$

where F_M and G_E are the nucleon magnetic Pauli and electric Sachs form factors.

The results of calculation of $\mathrm{Re}\, f_{\rho N}^L(E_\rho)$ and $\Delta m_\rho^L(E_\rho)$ are plotted in Fig. 1. It is seen that in the energy range $E_\rho = 2$–7 GeV Δm_ρ^L is essentially smaller than Δm_ρ^T. Although the uncertainty in the determination of Δm_ρ^L is rather large, we believe that this qualitative conclusion will be intact in a true theory. Since at rest $\Delta m_\rho^T = \Delta m_\rho^L$, one should expect a strong energy dependence of Δm_ρ^T and/or Δm_ρ^L in the domain $m_\rho < E_\rho < 2$ GeV. This is not surprising in the framework of our approach, since there are resonances in this domain and strong variations of $\mathrm{Re}\, f_{\rho N}(E_\rho)$ and $\Delta m_\rho(E_\rho)$ are very likely. The main sources of uncertainty in our approach are the assumption of independent scattering on nucleons in the nucleus (Fermi gas approximation) and the use of VDM, especially for the virtual photon. We estimate the uncertainty as $\sim$30%–50% for Δm_ρ^T and as a factor of $\sim$2 for Δm_ρ^L. $\Delta\Gamma_\rho$ calculated according to Eq. (5) is large: $\Delta\Gamma_\rho^T \approx 300$ MeV, $\Delta\Gamma_\rho^L \approx 100$ MeV at $E_\rho = 3$ GeV and normal nuclear density. However, these $\Delta\Gamma$'s so not characterize the broadening of the ρ peak observed experimentally. For example, they get contributions from elastic and diffraction scattering which do not result in broadening of the peak. One may expect experimentally observable broadening to be about 2 times smaller than these numbers.

We would like to note that a similar treatment of in-medium pions using the data on πN forward scattering amplitudes extracted from the phase analysis in Ref. [26] shows a strong energy dependence of the pion mass shift for $400 < E_\pi < 1500$ MeV: $\Delta m_\pi = 30-70$ MeV for normal nuclear density.

No direct comparison of our results with the previous ones can be made, since all earlier calculations refer to the mass shift of ρ-meson with the energy $E_\rho \lesssim 1$ GeV, while the applicability domain of our results is $E_\rho > 2$ GeV. As was mentioned above, one may expect a strong energy dependence of $\Delta m_\rho(E)$ in the interval $m_\rho < E_\rho < 2$ GeV. [Even the sign difference in Δm_ρ obtained here and in Refs. [7,8,15] cannot be considered as a contradiction, since $\mathrm{Re}\, f_{\rho N}(E)$ may change sign going through resonances, as it indeed happens with $\mathrm{Re}\, f_{\gamma N}(E)$.] The basic physical content of our approach is the statement that the meson mass shift in nuclear matter is determined by the meson-nucleon interaction and scattering proceeding at rather large distances ~ 1 fm. As follows from our basic formula Eq. (4), one may expect strong variation of δm_ρ at low E_ρ, since this is the resonance region in the ρN system. This would also indicate that large distances are important in this problem. The main point of Refs. [3,4,8,14,15] was the assumption that the mass shifts are determined by small distances and that the QCD sum rule method developed for the calculation of small distance contributions can be applied to this problem. In the calculations of Refs. [8,14,15] the operator product expansion (OPE) for the virtual photon-nucleon forward scattering amplitude was used and a few terms in OPE were kept. As is well known the OPE in this case is a light-cone expansion, and the expansion parameter along the light-cone is $1/x = 2\nu/Q^2$. For the ρ meson at rest $\nu \sim m_N m_\rho$, $Q^2 \sim m_\rho^2$, and $1/x \sim 2m_N/m_\rho \approx 2.5$. Therefore there are no reasons to keep only a few terms in this expansion, as was done in [8,14,15]. This fact, of course, is the manifestation of importance of large distances in the problem discussed. It should also be noted that our approach is phenomenological and thus takes into account all possible intermediate states used in model calculations of vector meson mass shifts.

Let us summarize our main results. The mass shift $\Delta m(E)$ of a particle in nuclear matter is given by the general Eq. (4). For the case of ρ meson with energies $2 \lesssim E \lesssim 7$ GeV the mass shift is positive, $\Delta m \sim 50$ MeV for transverse ρ mesons and 3 times smaller for longitudinal ones. At lower energies the mass shift is strongly energy dependent due to competition of various ρN resonances in Eq. (4).

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