Haussmann and Dohm Reply: In their Comment, Goodstein, Chui, and Harter (GCH) [1] calculate the specific heat $C(T,Q)$ of ^{4}He near T_λ in a heat current Q using thermodynamic and scaling arguments. They find an enhancement $\Delta C = C(T,Q) - C(T,0)$ that is considerably larger than ΔC calculated previously by us [2] within a renormalization-group (RG) calculation. GCH claim that we may not have calculated the proper specific heat anomaly but do not say why. Here we clarify this issue and explain the *origin* and *size* of the apparent discrepancy.

It is known from thermodynamics that any specific heat $C_\gamma = T(dS/dT)_\gamma$ depends on which variable γ is kept fixed when taking the derivative of the entropy S with respect to the temperature T. In superfluid ^{4}He at finite Q there exist both a finite superfluid velocity v_s and a superfluid current J_s. Consequently, there exist at least three different specific heats C_{v_s}, C_{J_s}, and C_Q. Close to T_λ we have [1,2] $Q \sim J_s$ which implies $C_Q = C_{J_s}$ while C_{v_s} is different. Previously we found it natural to consider a plane-wave structure of the order parameter $\langle\Psi(\mathbf{x})\rangle = \eta \exp(i\mathbf{k}\cdot\mathbf{x})$ where $\hbar\mathbf{k}/m = \mathbf{v}_s$ is fixed. Thus we calculated [2] $C_{v_s}(T,Q)$. An appealing feature of C_{v_s} is that, unlike ΔC_{J_s}, ΔC_{v_s} constitutes a pure fluctuation effect with a vanishing mean-field contribution [2]. On the other hand, GCH calculated $C_{J_s}(T,Q)$ which differs from C_{v_s} even at the mean-field level. Thus it is not surprising that the GCH result differs from ours.

The *size* of the apparent discrepancy is determined by a proper RG calculation of C_{J_s}. As previously [2] we may calculate C_{J_s} via the temperature derivative of $\langle|\Psi|^2\rangle(T,\kappa)$ where now $J_s(T,\kappa) = [\xi(-2t)]^{-2} f_J(\kappa)$ is kept fixed, with $t = (T - T_\lambda)/T_\lambda$, $\kappa = k\xi(-2t)$, and $\xi(t) = \xi_0 t^{-\nu}$. The scaling function $f_J(\kappa)$ is known [3] in one-loop order. Inverting $f_J(\kappa)$ yields κ as a function of T at fixed J_s. The resulting enhancement

$$\Delta C_{J_s}(T,Q) = (-t)^{-\alpha} f_{J_s}(Q/Q_c) \qquad (1)$$

is best illustrated in terms of the scaling function f_{J_s} shown in Fig. 1 (upper solid line). It is considerably larger than f_{v_s} calculated previously [2] (lower solid line). The analytic expression of f_{J_s} will be given elsewhere [4]. In the limit $Q \ll Q_c$ corresponding to the approximation of GCH we find the structure $f(Q/Q_c) = a(Q/Q_c)^2$ where the coefficients a_{J_s} and a_{v_s} constitute a universal ratio

$$a_{J_s}/a_{v_s} = (1+\nu)/(1-\nu) = 5.08\,, \qquad (2)$$

with the correlation length exponent $\nu = 0.671$. This explains the *size* of the apparent discrepancy. Our calculation yields $a_{J_s} = 6.25$ and $a_{v_s} = 1.23$ (in J/mol K) whereas GCH obtain $a_{J_s} = 9.2$. For larger Q/Q_c the GCH approximation (dashed line in Fig. 1) fails qualitatively.

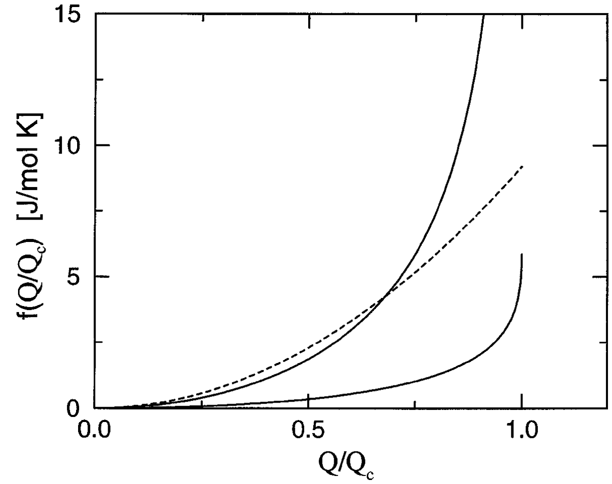

FIG. 1. Scaling function of the specific-heat enhancement $f(Q/Q_c) = (-t)^\alpha \Delta C(T,Q)$ at constant v_s (lower solid line) and constant J_s (upper solid line). The result of Ref. [1] at constant J_s is shown as a dashed line.

The question of an experimental verification of these predictions has not yet been discussed [1,2]. Since Q is controllable quite accurately, a *thermodynamic* measurement of $C_Q = C_{J_s}$ is presumably easier than that of C_{v_s}. The latter, however, may be measurable in a *dynamic* (second-sound) experiment with the second-sound velocity $\mathbf{c}_2$ being perpendicular to $\mathbf{Q}$ as indicated by inspection of the linearized hydrodynamic equations of Ref. [5]. As a general reservation we note, however, that the predicted specific-heat anomaly ΔC near $T_\lambda(Q)$ (corresponding to Q_c) may be partly masked by the onset of dissipation at $T_c(Q) < T_\lambda(Q)$ as detected in recent experiments [6].

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