Suppression of Rayleigh-Taylor Instability in Z-Pinch Loads with Tailored Density Profiles

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A load structure with tailored density profile which delays the onset of the Rayleigh-Taylor instability development in imploding Z pinches by inverting acceleration of the magnetic field/plasma interface is proposed and studied numerically. This approach makes it possible to start gas-puff implosions from large radii (say, 8 cm) and produce significant K-shell yield with current pulse duration of 250 ns and longer. It could also be used to mitigate imprinting of initial perturbations into laser fusion targets. [S0031-9007(96)00707-7]

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Rayleigh-Taylor (RT) instability is known to be a major problem for a wide range of applications, from pulsed power technology to inertial confinement fusion. In this Letter, we describe a simple method that makes it possible to suppress the exponential RT instability growth for a certain time interval. This method, having been initially developed to improve the performance of plasma radiation sources (PRS), as described below, is equally usable for any applications that would benefit from increased uniformity of plasma acceleration.

In plasma radiation sources, Z-pinch loads are imploded to convert implosion kinetic energy to soft x-ray radiation [1]. Conventional load configurations are wire arrays and cylindrical annular gas puffs produced by high Mach number nozzles. These loads are imploded by the magnetic pressure which is produced by the axial current. At stagnation, the plasma's kinetic energy is thermalized and can be further converted to K-shell radiation.

Rayleigh-Taylor hydrodynamic instability of a pinch plasma accelerated by the massless magnetic field is a major factor limiting PRS radiative performance. All kinds of perturbations grow during the implosion, from necks, $m = 0$ [these are the fastest, and the only ones which could be simulated with (r, z) codes], and kinks, $m = 1$, to instability modes with arbitrary combinations of azimuthal and axial wave numbers, which do not normally develop in a steady diffuse Z pinch [2,3]. Since the linear growth rate of the classical RT instability is $\Gamma = \sqrt{gk}$, the distance L traveled by the plasma shell during the implosion is related to the ratio A of final to initial perturbation amplitudes according to $L = (\ln A)^2 \lambda / 4\pi$ (here, g is the inward acceleration, $\lambda = 2\pi/k$ is the perturbation wavelength). The distance L available for acceleration is therefore limited by the RT instability; e.g., for a wavelength $\lambda = 0.5$ cm and amplification A below 1000, L must be less than 2 cm. Having started from a larger radius, a plasma shell would not arrive at the axis in one piece (see Ref. [4]) and, hence, would be an inefficient radiator.

To produce a K-shell yield at stagnation, the plasma has to be accelerated to high velocities—hundreds to thousands of km/s. Therefore the limitation on initial radius translates into severe requirements on the design of the pulsed power machine driving the implosion. Suppression of the RT instability imposing this limitation would make it possible to implode PRS with larger radii and longer implosion times. This means that there is a considerable potential for improving radiative performance of existing machines and that new parameter ranges will be open for the next generation of machines.

The RT instability could be mitigated by a so-called snowplow mechanism [5] responsible for enhanced efficiency of double-puff Z-pinch loads [6]. This mechanism employs superstability of the shock wave driven into the load by magnetic pressure. Although exponential growth of the RT instability at the magnetic field/plasma interface, once it starts, is not suppressed by the snowplow mechanism, the stagnating front part of the load is perturbed much less than in the case of a thin shell. For this reason, as predicted in Ref. [4] (see also Ref. [7]), large diameter uniform-fill loads can be good radiators. Recent experiments on the Saturn generator in Sandia National Laboratories [8] have confirmed the viability of uniform fill loads.

In this Letter we demonstrate that exponential RT growth could be fully suppressed as long as a shock wave propagates into a load with an appropriately tailored density profile. Therefore, radii of the structured loads and the corresponding implosion times could be increased to a greater extent than is possible even for uniform-fill loads, without sacrificing the implosion quality.

The idea of this approach is very simple. Let magnetic pressure drive a shock wave into a stratified plasma layer with increasing density. This causes the shock wave to slow down. Since the magnetic field/plasma interface also slows down, the effective gravity vector **g** is directed from the magnetic field (massless fluid) to the plasma. Once there is no light fluid supporting a heavy fluid in a gravitational field, there is no reason for exponential growth of perturbations of the interface—rather, they would oscillate. Although magnetic pressure continues to perform work, accelerating an increasing plasma mass, the interface feels a deceleration, and this is all that counts.

As an elementary example, consider a Z-pinch snowplow model in which plasma extends from $r = R_0$ to some much larger radius $R_1 \gg R_0$, and is imploded by constant current I. The density profile is of the form $\rho(r) = \rho_0(r/R_0)^{-s}$ (to have finite total plasma mass for large R_1, we assume $s > 2$). Although the density profile is specified as a function of radius, the snowplow model assumes that the plasma is collected in an infinitely thin shell whose radius is given by $R(t)$. Then the equation of motion is

$$\frac{d}{dt}\,\mu\,\frac{dR}{dt} = -\frac{I^2}{c^2 R}\,, \tag{1}$$

where $\mu = 2\pi \int_R^{R1} \rho(r) r\, dr$ is the line mass collected by the shell. Solving Eq. (1), we find the effective gravity acceleration experienced by the shell:

$$g = -\frac{d^2R}{dt^2} \propto -(s-2)\left(\frac{R}{R_0}\right)^{s-3}\left[1 + C\left(\frac{R}{R_0}\right)^{s-2}\right] < 0\,, \tag{2}$$

where C is a positive integration constant determined by the ratio R_1/R_0.

Since the acceleration g in Eq. (2) is negative, that is, directed from magnetic field to plasma, we expect oscillations instead of exponential perturbation growth. It does not mean, however, that perturbations do not grow. As seen from Eq. (2), for $s \geq 3$, $|g|$ decreases with time. Consider a pendulum oscillating in a decreasing gravitational field. Going down to the equilibrium position, the pendulum is accelerated by a stronger field than that decelerating it after it passes the equilibrium point on the way up. The amplitude of its oscillations is thus increased. We can evaluate its growth, supposing the gravitational field to vary at a sufficiently slow rate, the energy-to-frequency ratio E/ω thus being an adiabatic invariant of the motion ("number of quanta"). Recalling that $E = M|g|l\xi^2/2$ and $\omega = \sqrt{|g|/l}$, where M, l, and ξ are mass, length, and amplitude of oscillations of the pendulum, respectively, we find that

$$\langle\xi\rangle^2 |g|^{1/2} = \text{inv}\,, \tag{3}$$

and hence, the mean-square amplitude $\langle\xi\rangle$ grows as $|g|^{-1/4}$.

Small-amplitude, single-mode perturbations on the surface of an incompressible fluid in a time-dependent gravitational field are described by the same equation as a pendulum, with l replaced by $k^{-1} = \lambda/2\pi$. As an example, we present solutions of this equation for a power-law time dependence of gravity acceleration, $|g| = -\ddot{L} > 0$, where $L = g_0 t^n/n$ and $n < 1$:

$$\xi = t^{1/2} H_{1/n}^{(1,2)}\left[\sqrt{g_0 k(1-n)}\,\frac{t^{n/2}}{n}\right]\exp(ikx)\,, \tag{4}$$

where $H_\nu^{(1,2)}(z)$ are Hankel functions. These solutions describe gravitation waves running across the surface, whose amplitudes, in the limit $t \to \infty$, follow (3), as it should be. For the case of a load with a tailored density profile, a similar, relatively slow, nonexponential perturbation growth is predicted.

Numerical simulation results presented below have been obtained with the same two-dimensional magnetohydrodynamic code PRISM (which stands for plasma radiating imploding source model) as used in Ref. [4] and in previous simulations of X-pinch and Z-pinch implosions [9,10]. The physical model incorporated into this code includes electrical resistivity, separate energy equations for ions and electrons which include their respective thermal conductivities, and lookup tables for equation of state properties and radiative power. A "local" approximation or single zone opacity and transport scheme is used for the radiation. Argon was the only material considered in the simulations discussed here. Initial perturbations were seeded as small random density variations. The axial length of the computation field was 0.333 cm.

Figures 1(a) and 2 illustrate the numerical results obtained for constant current $I = 5$ MA and the initial density of the load $\rho_0(r)$ varied as $1/r^3$ from 2 to 8 cm, with a line density of 300 μg/cm. The usable portion of the imploding plasma's kinetic energy is its radial kinetic energy KE_r, whereas its axial kinetic energy KE_z (which would be identically zero in a one-dimensional simulation) is an integral measure of perturbation growth. In Fig. 1(a), perturbations are demonstrated to behave as predicted. While the shock wave propagates from 8 to 2 cm, compressing and accelerating the load mass, the acceleration of magnetic field/plasma interface is inverted. Perturbations should then run across the back surface of the load as waves with slowly growing amplitudes. This is indeed the case. The slow growth phase lasts about 190 ns, and during this time interval, KE_z behaves exactly as expected [see Fig. 1(a)]. Small-amplitude waves running across the back surface are discernible in Fig. 2(a). As shown in Fig. 2(a), the plasma shell remains almost cylindrical during the slow growth stage. Although only the $m = 0$ modes are simulated here, the same —and for the same reason—must be true for all perturbation modes and all wavelengths.

When the shock wave reaches the inner, high-density part of the load, and the reflected rarefaction wave transmits decreased pressure to the back surface, its acceleration is inverted again, with the result that the

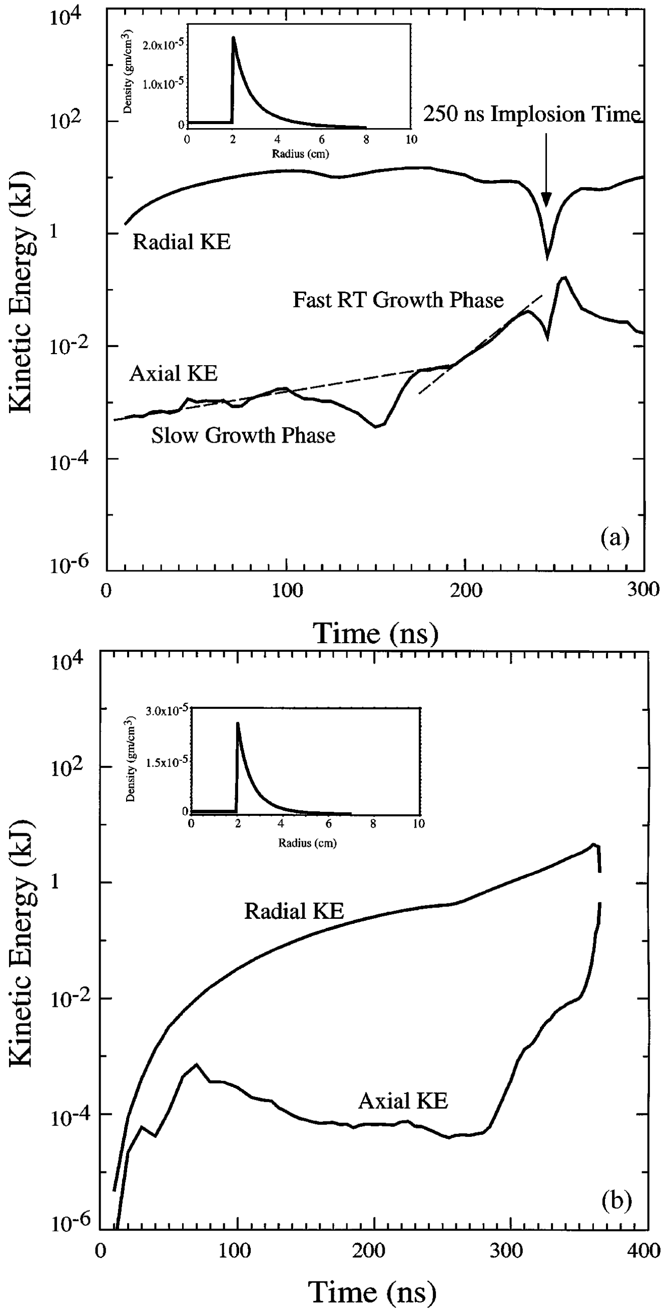

FIG. 1. Radial (KE_r) and axial (KE_z) kinetic energies vs time for implosion of a load with a constant current (a) and constant voltage (b). Initial density profiles are shown in the insets. Stagnation occurs at $t = 250$ ns (a) and at $t = 360$ ns (b).

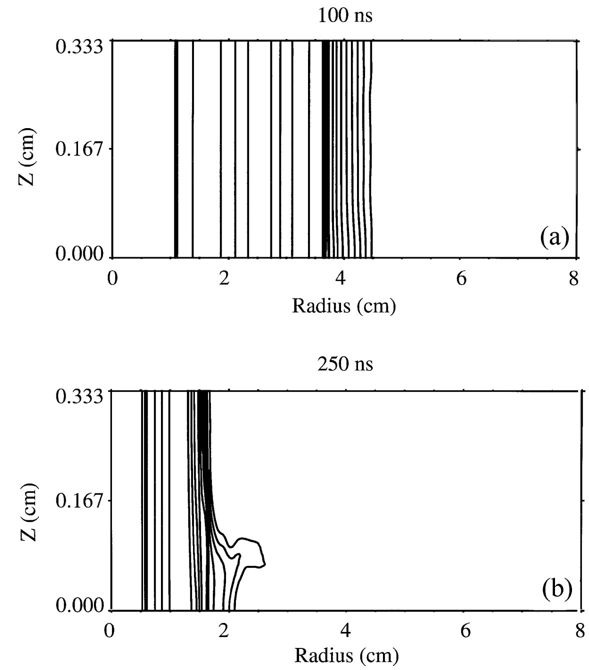

FIG. 2. Density contours corresponding to slow growth phase $t = 100$ ns (a) and rapid growth phase $t = 250$ ns (b) in Fig. 1(a).

classical RT exponential growth starts to distort the shell at the back surface. If, as in this simulation, the inner surface is sufficiently close to the axis, the central regions of the implosion could remain relatively unaffected by the instability, as seen in Fig. 2(b).

Figure 1(b) presents the results obtained with a more realistic current wave form, one produced by constant voltage V applied to the Z-pinch column. The voltage was chosen such that the current would rise linearly to 5 MA over 200 ns, or $V/L = 2.5 \times 10^7$ MA/s, where L is the inductance. This translates into linear growth of the current at early time. Rapid rise of magnetic pressure makes the density growth provided by the $1/r^3$ initial profile of Figs. 1(a) and 2 too slow to invert acceleration. This could be improved by choosing a sharper density profile near the outer boundary of the load; see the inset in Fig. 1(b). Once the density profile has been appropriately tailored for the given current wave form, the results are similar to those given in Figs. 1(a) and 2.

Having started with a tailored density profile, after the slow growth phase we assemble a plasma shell, which is RT unstable like any conventional load. What is the advantage of this shell, say, over a gas puff with the same initial radius?

(1) Appreciable magnetic energy is converted to kinetic energy during the slow growth phase. For instance, the center of mass of the $1/r^3$ profile (Fig. 1) is located at $r = 3.7$ cm, hence the implosion should produce radiation yield equivalent to that of a shell implosion from 3.5 to 4 cm initial radius, although no shell can survive such an implosion. This is illustrated by Fig. 3, where the density contours for a 0.5 cm thick shell which extends from 2 to 2.5 cm is imploded under the assumption of the constant voltage used for the simulations of Fig. 1(b). The mass was chosen such that MR^2, where M is the line mass and R is the mean radius, is equal to that for the simulation shown in Fig. 1(a). Contours are shown at the beginning of the simulation and at 220 ns, just after the shell has been completely destroyed by the RT instability in-flight, prior to stagnation (see also Ref. [4]).

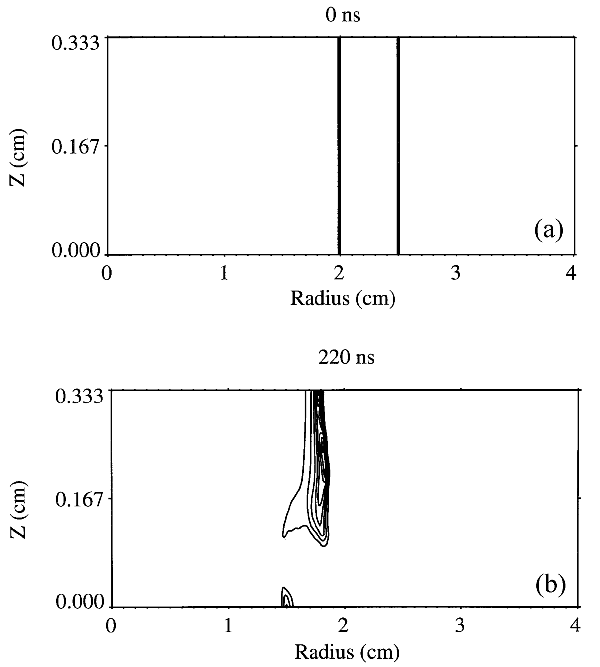

FIG. 3. Density contours for a shell which initially extends from 2 to 2.5 cm: before implosion (a) and at $t = 220$ ns (b).

(2) A tailored density profile operates as a switching device, making it possible to work with longer current pulses. The shell is assembled during the current rise time and imploded by the peak current when the peak is reached. This cannot be done with a conventional load, which would be either imploded too early or distorted too much by a long current pulse. In addition, it is easier to couple magnetic energy to a load whose initial radius is large.

(3) After the passage of the shock wave, the shell already has some, albeit relatively small, velocity directed to the axis. This increases hydrodynamic efficiency of its acceleration, since the rate of magnetic-to-kinetic energy conversion is proportional to this velocity.

Suppression of the RT instability by inverting acceleration is a robust, purely hydrodynamic effect. Formally, there are no limitations on its efficiency, and one can demonstrate the feasibility of implosions generating K-shell yield with arbitrarily large outer radii of the loads and arbitrarily long current pulses: 16 cm is as good as 8 cm, and 500 ns is as good as 250 ns. In fact, the density in the outer layers of the load should not be too small—otherwise, a diode regime, beam generation would take place instead of an implosion. Moreover, the hydromagnetic RT instability is more volatile for low-density plasmas due to the Hall effect [11,12]. The lower limits on the density of the load and on the level of the gas preionization have yet to be established. A practical method of tailoring initial density profiles (with specially designed gas-puff nozzles, or otherwise) should also be developed. Although this issue is beyond the scope of the present Letter, one can hardly doubt that efforts and costs required for this development would be minimal compared with those involved in building and operating major pulsed power facilities.

Applications of the stabilization method described here are not limited to PRS load design. A plasma annulus with a tailored density profile could be used for switching the current, once it approaches its peak, to a compact wire-array load located inside it, as demonstrated in Ref. [13]. It could be utilized with a conventional plasma opening switch (POS), or possibly in place of the POS. This idea seems to be worth pursuing, since this method of switching, being based on much simpler physics than POS operation, might be made more reliable. Structured density profiles could also be helpful in designing low-isentrope targets relevant for laser fusion. Appropriately tailored density increase in a planar or spherical target would mitigate the imprinting of the initial target surface and laser beam nonuniformities into the flow by the low-energy foot of the laser pulse.

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